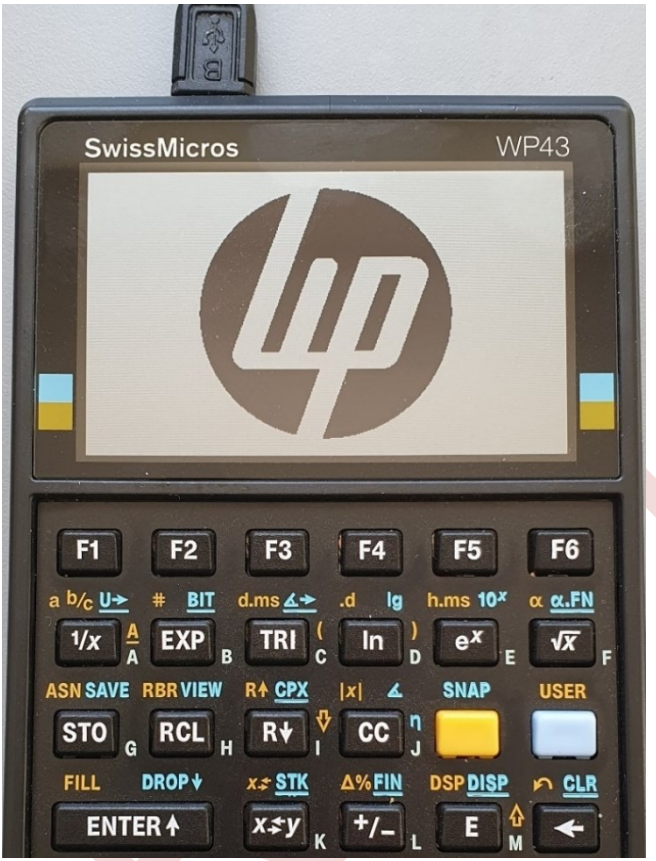

WP43 OWNER'S MANUAL
AND PROBLEM SOLVING GUIDE

This manual explains *WP43*. **(***WP43* is a free scientific software for *DM42* calculator hardware built by *SwissMicros*. You can redistribute that software and/or modify it under the terms of the *GNU General Public License* as published by the *Free Software Foundation*, either version 3 of the *License*, or (at your option) any later version. *WP43* is published and distributed in the hope that it will be useful, but without any warranty; without even the implied warranty of merchantability or fitness for a particular purpose. Please see *http://www.gnu.org/licenses/* for more details.**)**

This manual is preliminary still; it will change while we develop *WP43* in the course of this project. We reserve the right to do so any time. The fundamental principles of *WP43* will remain constant, however. You can stay informed by watching *https://gitlab.com/rpncalculators/wp43*.

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All rights reserved. No part of this manual may be copied, displayed, extracted, reproduced, utilized, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical or otherwise including but not limited to photocopying, recording, or scanning without the prior written consent of the author. As long as it is just a draft, this manual may be downloaded but not changed in any way. The locations highlighted cyan are open issues – information is missing there or needs further discussion or investigation to be determined and fixed. Any contributions in these matters are appreciated.

HP is a registered trade mark of *Hewlett-Packard*. *Excel* and *WordPad* are applications of *Microsoft*.

The photographs on p. 19 were published by *NASA* in 1969. All paintings, the drawings on pp. 36 (top), 89, 148-145, 167, 170ff, 225, 263, 271, 290ff, 340f, and 344-348, as well as the pictures on pp. 56, 212f, 256, and 358f were taken from various calculator manuals and advertisements published by *HP* between 1974 and 1989 (some were refined by me). The picture on p. 293 and the diagrams on p. 106 are based on material found in *Wikipedia*. The sources of the maps on pp. 99, 305, and 307 are stated there – I found them by chance browsing the internet. All *WP43* keyboard graphics as well as the other photographs, pictures, graphics, and diagrams printed in this manual were created by the author.

Internet addresses and links are specified as found and verified at 2023-11-16 (just the map on p. 99 is no longer online). Please note internet addresses may change or vanish without notice at any time.

This manual is published in English since it became the *lingua franca* of our time (after Greek, Latin, and French in Europe) – using it we can reach the maximum number of people without further translations. I apologize to the people of other languages and inserted some 'translator's notes' where applicable.

(It is worth mentioning that the two most widespread written languages on this planet nowadays are the most ambiguous ones: English and Chinese. Both rely heavily on context since they have a reduced grammar and flexion compared to, e.g., Spanish (Chinese lacks both completely). Older 'common' languages were more elaborated (compare Latin). What can we learn from that about mankind? – You may find more barbed remarks below, reflecting the author's impressions watching mankind for seven decades so far. No personal offense intended but overconfident people might get some.)

Printed in the *USA*

ISBN-13: 978-172950098-9
ISBN-10: 172950098-6

WP43 would neither have been created without our love for *Classics, Woodstocks, Stings, Spices, Nuts, Voyagers*, and *Pioneers* nor without our ability to imagine better products on a way towards a better world. Thus we want to quote what was printed in *HP* pocket calculator manuals until 1980, so it will not fade:

"The success and prosperity of our company will be assured only if we offer our customers superior products that fill real needs and provide lasting value, and that are supported by a wide variety of useful services, both before and after sales."

Statement of Corporate Objectives
Hewlett-Packard

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Just in Case You Don't Have Your Own *WP43* Calculator yet …

A pilot batch of *WP43* calculators was made on *SwissMicros'* production line in China in the summer of 2022. These were sold out in few weeks – thus, please turn to *https://www.swissmicros.com/* expressing your wishes. Maybe they will listen to their customers.

Alternatively, you can create a *WP43* most easily by flashing a *DM42* you own – this way will cost you next to nothing but some key stickers and an overlay will be inevitable. Please consult *App. F* of the *WP43 Reference Manual* for what and how to do.

The *WP43* software package includes a free full-size *Simulator* (see *App. E* of said manual), so you can check it out on your computer before buying any hardware. And later on, you may test and edit your programs there as well before sending them to your *WP43* calculator. Function sets are 99% identical on both platforms.

For the following, we assume you hold your own *WP43* in your hands – or have at least access to its *Simulator*.

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WELCOME!

Dear user, we congratulate you on your new *WP43.* It will serve you well, being an accurate, reliable, fast, and compact problem solver like you have never owned before – programmable, user-customizable, connecting to your computer, incorporating a high-resolution display and *RPN*[1] – a serious scientific instrument supporting you in your studies and professional activities, be it in mathematics, science, engineering, computing, or business. It readily provides advanced capabilities never before combined so conveniently in a true pocket-size calculator:[2]

+ A **wealth of functions** operating on integers, fractions, real and complex numbers, matrices, angles, dates, times, and text strings. Programmable sums, products, integrals, and derivatives.
+ A comprehensive set of **statistical operations**: probability distributions, functions for data analysis, curve fitting, forecasting, and field proven measuring system analysis (*MSA*).
+ **Vector and matrix operations** on real and complex arrays of arbitrary size; dot and cross products, eigenvalues and eigenvectors, and a solver for systems of linear equations, all supported by a **full screen *Matrix Editor***.
+ **Integer arithmetic** in 15 bases from binary to hexadecimal; base conversions and extensive **bit manipulation** (shift, rotate, mask, count, flip, etc.) in words of up to 64 *bits*.
+ **Long decimal integers** of up to 1001 digits for precise numeric applications. Finds and checks primes. Factorizes integers.
+ The ***Solver*** (root[3] finder) can solve for any variable in any equation you entered using the ***Equation Editor***. Numeric integration, differentia-

[1] *RPN* stands for *Reverse Polish Notation*, a fast, effective, and coherent method for solving mathematical problems efficiently. It is based on a logic known as *Polish Notation* (since proposed by the Polish logician *Jan Łukasiewicz* in 1951; see https://www.youtube.com/watch?v=qRrAj-GCTQM). He put operators before operands instead between them as in usual mathematical notation. *RPN* places operators after operands. See *Section 1* of this manual.

[2] Per definition, a pocket calculator is a device fitting comfortably in your shirt pocket. Marketing people see this feature more elastic – our pockets are not elastic enough for that. Being at it, we recommend not to put your *WP43* in the back pocket of your jeans – beyond frazzling your pocket, it may break or multiply there.

[3] Translator's note for German readers: *Root* bedeutet hier *Nullstelle*.

tion, and basic plotting of equations are supported as well.

+ Field proven **keystroke programming** incl. conditional tests and branching, finite and infinite loops, user and system flags, subroutines, recursions, global and local data. Create and refine your programs using a **multi-line editor** on your calculator or computer.
+ A convenient **menu system** assigning up to 18 operations to the top row of calculator keys according to your actual needs.
+ **Alphabetically searchable *catalogs*** for easy finding all the items stored in memory, be they provided by us or created by you.
+ A **timer** (a.k.a. stopwatch) based on a real-time clock.
+ Easy system **control by named system flags and variables**.
+ **Battery-fail-safe on-board backup** for all your data and programs; calculator state files you can exchange with your fellow *WP43* users.
+ A ***USB* socket** allowing for auxiliary external power supply as well as for program exchange with your computer – you can transmit, edit, and test routines there and return them verified to your *WP43*.
+ A **user interface you can customize**: Create your own menus, specify your personal keyboards and calculator states, save all of them on-board and recall them one by one as you need them. Dedicated keyboard overlays are supported.

WP43 provides the amplest function set ever seen in an *RPN* calculator:

+ **786 functions**, including *Euler's Beta* and *Riemann's Zeta*, *Lambert's W*, *Bessel functions of 1st and 2nd kind*, *Bernoulli* and *Fibonacci numbers*, *Jacobi elliptic functions* and *integrals*, as well as *Chebyshev*, *Hermite*, *Laguerre*, and *Legendre polynomials* – get independent of table books and computer software on these matters.
+ **14 probability distributions**: *normal* (*Gaussian*), *Student's t*, *chi-square*, *Fisher's F*, *Poisson*, *binomial*, *geometric*, *hypergeometric*, *Cauchy-Lorentz*, *exponential*, *Weibull*, and more.
+ **10 curve fit models** (linear, orthogonal, exponential, logarithmic, power, root, hyperbolic, parabolic, as well as Cauchy and Gauss peak). Data plots let you assess your fits, draw histograms, and even analyze and qualify your measuring instruments on site. Just do it! ™

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- **54 fundamental physical constants** as precise as used today by national standards institutes (following *CODATA 2022*) plus 25 important mathematical, astronomical, and surveying constants.
- **162 unit conversions** (for times, distances, areas, volumes, masses, forces and pressures, energies, and powers), mainly from *British Imperial* or *US customary* to universal *SI* units and vice versa.
- A **compact set of financial functions** and applications for the business matters you may experience as a technical manager.

WP43 features lots of space for your data, results, programs, and ideas:

- A high contrast ultra-low power **240 × 400 pixel *LCD*** allowing for crisp results and menus, plots, natural display of matrices, a wide variety of mathematical symbols, Greek and international Latin letters.
- Your choice of **workspace comprising 4 or 8 stack registers**; up to **107 global general purpose registers** and **1000 named variables.** Each register or variable can take any object of arbitrary data type (be it a matrix, vector, text string, or any number of arbitrary kind).
- **100 global user flags** and some 40 named system flags.
- Additionally, **32 local user flags** and up to **99 local registers** per (sub-) routine, allowing, e.g., for recursive programs.
- Space for some **20 000 program steps in *RAM***, up to **10 000 per program**. Battery-fail-safe flash memory for saving and exchanging your routines, data, settings, configurations, layouts, and entire calculator states.
- Last but not least, your *WP43* computes with **34-digit precision** – the most precise pocket calculator available worldwide, second to none.[4]

Your *WP43* is a true pioneer: the very first entirely community-designed and ‑built *RPN* pocket calculator. All of its hardware, firmware, user interface, and layout were thoroughly thought through, discussed, designed and assembled, written and tested by us over and over again since 2012. *WP43* is the result of a temporary collaboration of two independent international teams: ***SwissMicros*** (*SM*, i.e. Czech ***David Jedelsky***, Swiss ***Michael Steinmann*** and ***Emy Amstein***) created the

[4] There are a few pocket calculators sharing this top position, actually. Good for advertising! See the *Reference Manual* for more about precision and accuracy.

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hardware and *DMCP*;[5] French ***Martin Lorang*** and ***Didier Lachieze***, ***Mihail*** from Japan, South African ***Jaco Mostert***, British ***Benjamin Titmus***, Australian ***Paul Dale***, and I designed the user interface (*UI*) and wrote the firmware.[6] Our overarching goals were the coherence and internationality of the *UI* as well as precision, accuracy, and reliability of the calculation results.

As our *WP 34S* and *WP 31S* before, also *WP43* is just a hobbyist's project. I published a first *WP 43S* draft layout in 2012; it was then discussed on two online forums. Prototypes of *SM's DM42* hardware were presented first on *HHC2016* (*USA*) and the *Allschwil Meeting (AM) 2016* (Switzerland). *Martin* started coding *WP 43S* firmware in 2017. He and I presented the first version of the *WP 43S Simulator* on *AM2018*. Later, the narrow blank and the *scientific* '*S*' were sacrificed for better alignment with *SM's* calculator labels, and *AM2022* saw the pilot batch of *WP43* calculators made and distributed. We thank all participants of said meetings and all members of the international calculator community who contributed their ideas, put their votes, and in particular those who lent their support in various phases of this project. We greatly appreciate your contributions! Feel free to monitor said forums for further news.

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[5] *DMCP* = *David's* and *Michael's Control Program*. They made this basic firmware layer for their *DM42* launched in 2017, whose layout is very closely linked to the *HP-42S* produced from 1988 to 1995. Its firmware uses *Thomas Okken's* outstanding *Free42* software simulating the *HP-42S*.

[6] … with major contributions of Italian *Gianluca Puggelli*, Dutch *Harald Overbeek*, German *Gert Menke* and *Friedrich Mütschele*.

The firmware of your *WP43* is based on the experience *Paul* and I gained with the *WP 34S RPN Scientific Calculator* (that project started in 2008 with the availability of the repurposable *HP-20b*). Together with *Marcus von Cube* (Germany), we launched the *WP 34S* in 2011. You find specific information about it and its minor derivative, the *WP 31S* of 2014 (created by Sanjeev Visvanatha (Canada), Jonathan Cameron (USA), and us) at *https://sourceforge.net/projects/wp34s/*. *WP 34S* and *31S* run on *HP* hardware (*HP-20b* or preferably *HP-30b*, both out of production for years meanwhile).

There is a video of the 2022 *Allschwil Meeting* describing our path from *WP 34S* to *WP43* in a nutshell (see *https://www.youtube.com/watch?v=gPzCMss_imE*).

If you are interested in the entire long and winding road how your *WP43* got the features and layout you are facing now, look up *https://www.hpmuseum.org/forum/forum-8.html* until 2016 and *https://forum.swissmicros.com/viewtopic.php?f=41&t=1816* since 2017. See also the *WP43 Reference Manual* for the full *Release Notes*. And feel free to check *https://gitlab.com/rpncalculators/wp43* to find out who contributed when and what – there is quite some fluctuation in such a hobbyists' project.

We baptized our baby in honor of the *HP-42S* of 1988, the most powerful *RPN* scientific pocket calculator made before our activities. May it be a worthy and valiant successor of the *HP-42S*[7] – though we would have preferred *HP* making it (*HP* as we remember it controlled by *Bill Hewlett* and *David Packard* until the Eighties of last century). In any way, *WP43* stands in the tradition of over 50 years of *RPN* pocket computing.

We carefully checked *WP43* to the best of our abilities, so we are confident it is free of severe bugs. This cannot be guaranteed, however, so we will improve *WP43* whenever necessary. Should you discover any strange result, please check its reproducibility and the *Troubleshooting Guide* in the *Reference Manual*; if it persists report it to us (SNAP will take a snapshot of your *WP43* screen, see p. 252). We will investigate, correct if necessary, and provide you with a free updated firmware as soon as possible – you can flash it yourself! As we have done for our *WP 34S* and *WP 31S* since 2011, we will continue maintaining short response times.

Enjoy!

Walter

P.S.: There are several offsprings of our *WP43*, called *C43, C47*, *R47*, etc. While documented on their own site (just search for the respective models), they lack user manuals as far as I am aware. Since they originated from an earlier stage of *WP 43S*, the fundamental ideas (e.g., display layout, the three menu rows, the overwhelming part of their fonts) and most of their functions are identical. Thus, you can profit from this *WP43 Owner's Manual* here also for those offsprings for the time being, until their team manages to write their own. We will not, however, explicitly refer to those offsprings in the following text – we simply don't know enough about them to be sufficiently competent in those matters.

[7] *HP-42S* is our reference – if in doubt we follow it. *WP43* is not (and was never planned to be) *HP-42S* compatible, however. In a nutshell, it comprises the functionalities of *WP 34S*, *WP 31S*, *HP-42S*, *HP-16C*, *HP-21S*, and more. See the *WP43 Reference Manual* for details.

Print Conventions and Common Abbreviations

- Standard text font in our manuals is *Arial*. Emphasis is added by underlining or **bold** printing. COMMANDS, MENUS, predefined VARIABLES and SYSTEM FLAGS (SET or ~~CLEAR~~) are generally called by their *names*, printed capitalized in running text.

  Examples are printed in green, quoted text in blue (as well as translator's notes). Bold italic letters (like ***n***) denote variables, bold regular characters (like **b**) represent constant values (e.g., specific labels, numbers, or symbols).

  Specific *terms, titles, trademarks, names*, or *abbreviations* are printed in italics, *hyperlinks in blue underlined italics*. For best use in print, such links will generally refer to a page number, the *Table of Contents*, or an external address fully specified.

- *Times New Roman* regular letters are used for unit symbols and mathematical functions, italics for *unit names* in running text.

  Bold capitals denote **REGISTERS**, lower case bold italics their ***contents***.[8] E.g., *register* **Y** contains the value ***y***, and **R45** contains ***r45***. *Stack* contents are generally quoted in order [***x***, ***y***, ***z***, …].

- This KEY font (created by *Luiz Vieira* of Brazil) denotes calculator KEYS, incl. **softkeys** in general. For shifted operations like GTO or SF, the respective colors are used. `Text` and numeric outputs (like 1.234×10^{-56} or $7{,}089\cdot10^{12}$) are printed as seen on your calculator screen.

- *Courier* is used for `filenames` and describing `numeric formats`.

- Regarding mathematical notation, we follow *ISO 80000-2* to comply with international standards. For the manuals, we chose decimal points, multiplication crosses, and division slashes – you may choose decimal commas and multiplication dots on your *WP43* as well, of course.

All this holds unless stated otherwise locally.

The abbreviations below, sorted alphabetically, are used throughout this manual:

1D, 2D, 3D	=	one-, two-, three-dimensional.
4LS, 8LS	=	4-level or 8-level stack.
ADM	=	angular display mode.
AIM	=	alpha input mode.
App.	=	Appendix.

[8] We kept the term *register* for the space where an object is stored, although the actual size of such a *register* may vary widely following the size of the object stored therein.

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DT	=	data type.
FD	=	the *USB FAT* disk.
FM	=	flash memory, encompassing the *FD*.
f.	=	footnote.
GP	=	general purpose.
HP	=	*Hewlett-Packard.*
IOI	=	*Index of all Items* provided in *WP43* (see *ReM* below).
MSA	=	measuring system analysis.
OH	=	*Owner's Handbook.*
OHPG	=	*Owner's Handbook and Programming Guide.*
OM	=	*Owner's Manual.*
PC	=	computer of arbitrary kind and brand.
PEM	=	program entry mode (see also *RUM*).
PM	=	program memory (in *RAM* for the calculator).
PP	=	program pointer.
RAM	=	random access memory, allowing read and write operations.
ReM	=	*WP43 Reference Manual*, comprising the *IOI* and more.
ReMA…	=	*ReM App. ...*
ReMS…	=	*ReM Section ...* Note *ReMS 1* contains the *IOI*.
RoV	=	register or variable.
RPN	=	*Reverse Polish Notation* (cf. f. 1 on p. 9).
RUM	=	run mode (cf. *PEM*).
SDC	=	startup default configuration (e.g., after RESET).
SI	=	*système international d'unités*.[9]
TAM	=	transient alpha mode.
TI	=	temporary information.
TN	=	translator's note (and *TNG* = *TN* for German readers).
USA	=	*United States of America.*

A few further abbreviations may be introduced and used locally.

Finally: WARNING indicates risk of severe errors. There are only four warnings printed in this manual (although *WP43* may be used in the *USA* as well). Resetting your calculator by actuating the RESET button on its back will help in almost all cases – though this will erase all your data in *RAM*. You find a *Troubleshooting Guide* in *ReMA G*.

[9] *SI* is a coherent system of units of measurement agreed on internationally and adopted by almost all countries on this planet. The last three not participating yet are Liberia, Myanmar, and the *USA*, obviously not knowing what they miss. Find basic information about *SI* in https://en.wikipedia.org/wiki/International_System_of_Units. See also the chapters about *Constants* and *Unit Conversions* in *Section 5*.

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SECTION 1: GETTING STARTED

At its heart, your *WP43* is an extremely powerful, versatile problem-solving tool. It allows you to solve even very elaborate mathematical and computational problems in either of two ways:

- **Manual problem solving:** Using the *RPN* logic system of your *WP43*, you can manually work step-by-step through the toughest problems while seeing intermediate answers after each and every step of the way. The advantages of *RPN* become particularly apparent when working with exploratory type problems where intermediate answers are an important part of the problem solving process.
- **Programmed problem solving:** Your *WP43* can remember any sequence of keystrokes you entered, and it can then run it repeatedly as often as you need it. This simple programming paradigm is particularly useful in providing answers to repetitive problems that require different inputs. Advanced programs may also be written for solving more elaborate tasks, e.g., iterative computations containing automatic decisions and branching. Thousands of keystrokes can be recorded in your *WP43* and exchanged with and through your *PC*.

SAFETY WARNING: Your *WP43* is not designed to be used by children under 3 years. Keep toddlers away from it.[10]

This is not a toy. Don't use it before you can read, tell left from right, and do mental arithmetic reliably. It contains small parts which if swallowed are hazardous for health. Don't ingest the battery.[10] **Chemical Burn Hazard: Swallowing the battery can be fatal within 2 hours – seek medical attention and help immediately!**

Follow the operating instructions in this manual carefully. Do not use your *WP43* for any other purpose than specified therein (neither as a hammer nor as a lever nor as a door stop, neither kick nor throw

[10] No, we don't think you are stupid, irresponsible, or lacking any life experience. Alas, however, such a waste of print space became inevitable due to the legal system, lawyers, and courts of that great nation where such warnings appeared first in user manuals a few decades ago. Advise for dummies.

Battery bays must be secured by a small Phillips screw in Europe. In the USA, extended warnings are printed. Guess what will keep toddlers away from batteries more reliably?

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it); else you may destroy your *WP43* and/or other objects easily, even hurt animals or human beings including yourself.[10]

ATTENTION: Your *WP43* shall be operated with care in a clean, dry environment (relative humidity less than 70%, non-condensing) at ambient temperatures from 5 to 40 °C (41 to 104 °F).

Do not drop your *WP43* on solid ground – it may break. Do not leave it lying in the sun; its dark surface may become hot, and hot surfaces may cause burns. Do not leave your *WP43* lying in the cold; humidity may condense on its surface when a cold *WP43* is put in warmer space.[10]

Should your *WP43* become wet, turn it off, remove the battery (see *Disassembly* below), and let your *WP43* dry for sufficient time before re-inserting the battery and turning it on again. Do not try to accelerate drying by blowing hot air (exceeding 60 °C or 140 °F) into your *WP43* or putting it into an oven (neither conventional nor microwave) or the like – you may destroy it.[10]

Disassembly: Do not disassemble your *WP43* unless you are qualified for such work and have the proper tools handy. You will need one small Phillips screwdriver and two hands for opening it.[10]

Replacing the Battery: Your *WP43* contains a battery (surprise!). When it runs out of power, the symbol 🔋 will appear top right on its screen. Please open your *WP43* (cf. *Disassembly* above) and replace its old battery by a fresh quality cell.[11] Do not operate your *WP43* without a good battery installed – old cells may leak and the acid may oxidize and destroy your *WP43*. Dispose of the old battery responsibly in the appropriate in-store containers or at your local disposal center; neither take it apart nor throw it in a bin nor in fire nor in your environment.

Disposal of the calculator: Your *WP43* must not be disposed of along with household waste. Remove its battery (cf. *Replacing the Battery* above) and dispose of your *WP43* according to applicable laws and regulations at your electronics collection point.

When you have taken that and **you know how to deal with an *HP RPN* scientific calculator, feel free to start using your *WP43* right away now.** You will find it easy and intuitive to use. Browse this manual to learn about some fundamental design concepts putting your *WP43* ahead of previous *RPN* scientific pocket calculators.

[11] See *https://technical.swissmicros.com/dm32/doc/dm32_user_manual.html#_battery*

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If, on the other hand, *RPN* is new to you, we recommend going through *Sections 1* and *2* thoroughly. This will empower you to easily solve problems manually, benefitting from this unique logic system implemented. Once learned, *RPN* forms a lifelong lasting, reliable basis of your work.

Most commands of your *WP43* work as they did on its antecessors, in particular on *WP 34S* and *HP-42S*. Its manuals are designed to supplement your prior knowledge, focusing on the new features of your *WP43* and providing information about them. They may include some formulas and technical explanations; though they are not intended to replace textbooks on mathematics, statistics, science, engineering, economy, computer science, or programming.

Section 1 of this *Owner's Manual* starts introducing the keyboard of your *WP43* to you, so you learn where to find what you are looking for. It continues demonstrating basic calculations, shows the memory of your *WP43* and how to address objects therein.

Section 2 covers the display and its indicators giving you feedback about what is going on in your *WP43*. Furthermore, it introduces and presents the various mathematical objects and *data types* supported.

Section 3 explains programming your *WP43* for automatic problem solving, following tried and tested concepts which are known from successful earlier pocket calculators up to and including *HP-42S* and *WP 34S*. It also tells how your *WP43* can communicate with your *PC*.

Sections 4 and ***5*** present advanced functionalities implemented.

Section 6 covers everything you will need for customizing the user interface of your *WP43* according to your very personal preferences.

This *Owner's Manual* is supplemented by the ***Reference Manual***. A major part of the latter is taken by the *Index of all Items* available *(IOI)*, what they do, and how to call them. It contains full information about all *menus*, *system flags*, constants, and *variables* provided. It closes with appendices covering special topics (like memory management, the free *WP43 Simulator* for your *PC*, the advanced mathematical functions implemented, and includes information on how to keep your *WP43* up-to-date whenever we will release new firmware revisions).

Continue using both manuals for your reference.

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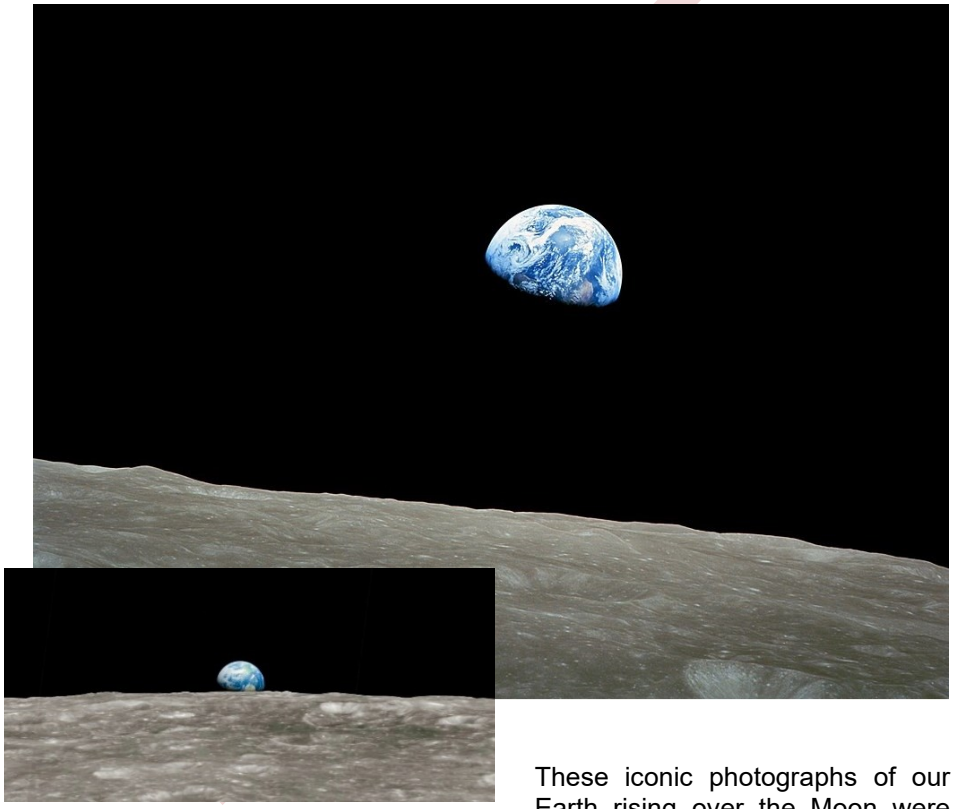
Before diving into this *Owner's Manual*, here is something we ask you to keep in mind always:

Your *WP43* is designed to support you solving analytical and calculatory problems. But it **is just a tool:** it can neither think for you nor can it check the sensibility of a problem you apply it to**. Gather information, think before and while keying in and calculating, and check your results: these tasks will stay your responsibility always. We will not be liable for any of your results, nor their consequences.**

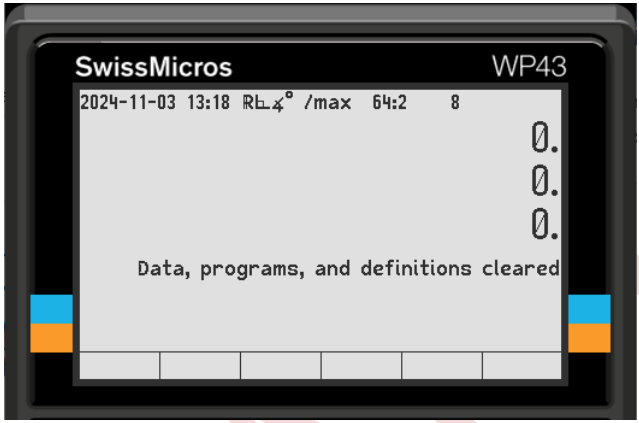


These iconic photographs of our Earth rising over the Moon were taken by the 'astronauts' of *Apollo 8* in 1968. Remember orbiting the Moon and even landing on it multiple times was accomplished before a single scientific pocket calculator was launched on our planet (most results were achieved by careful slide rule calculations then – cf. the movie '*Apollo 13*').

Imagine how much more is within your powers using precise tools like your *WP43*. Apply it wisely, taking care of our planet – it's our only home we have in this breathtaking, black, and bitter cold gigantic universe. Distant worlds are just too far away – most probably exist but definitely beyond our reach.

Problem Solving, Part 1: First Steps

Start exploring your *WP43*: Press its bottom right key – notice **ON** is printed below that key. Turned on the very first time (or after a reset), it will show a display like this:



To turn it off again, press , then **EXIT** (note **OFF** printed below it). Since your *WP43* features *Continuous Memory*, turning it off and on doesn't affect the information contained. For conserving battery power, your *WP43* will shut down automatically after ten minutes of inactivity – turn it on again and you can resume your work right where you left off.

This works as on preceding calculators (e.g., *WP 34S* or *HP-42S*). Your *WP43*, however, looks cleaner than a *WP 34S* while more colorful than an *HP-42S*. This is since your *WP43* features two general *prefixes* and as well as *menus* – offering you up to six functions per key.

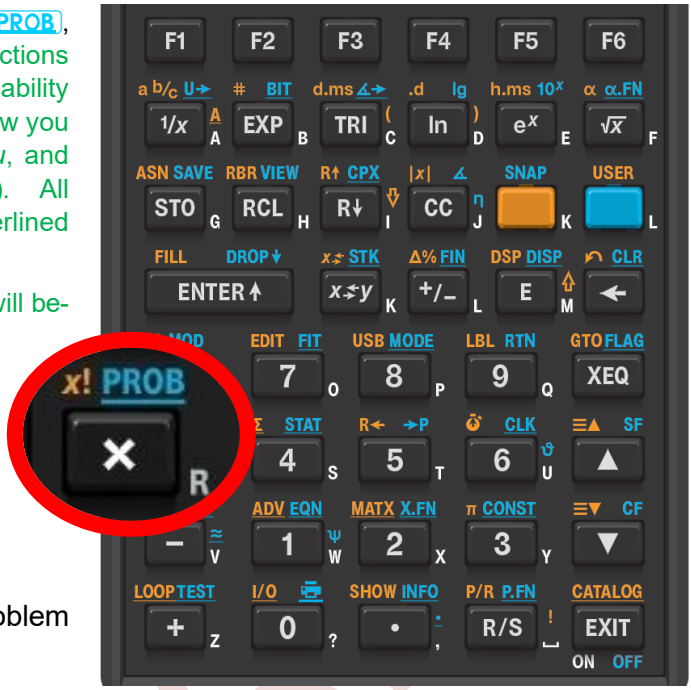
To call a *primary* function (printed in white), just press the corresponding key. For a golden or blue function, press or before.

Take the key [×],[12] for **example** (see the keyboard overleaf). Pressing...

- [×] alone executes a multiplication.
- + [×] calls the *factorial* [x!] (you will learn more about it in *Part 2*).

[12] For better readability, we will refer to keys using dark print on light background from here on. And for shifted functions like [x!] or **OFF**, we will omit or most times since redundant by color print.

- [blue] + [×] opens [PROB], a *menu* of functions related to probability ([▲] or [▼] will show you more of this *menu*, and [EXIT] will leave it). All labels printed underlined refer to *menus*.[13]
- The grey letter R will become relevant with text data.



Find all keyboard labels explained on pp. 320ff.

Time for a first problem solving **example**:

Now, let's assume you want to fence a little patch of land,[14] rectangular, 40 *yards* long and 30 wide.[15] You have already set the 1st corner post (**A**), and also the 2nd (**B**) in a distance of 40 *yards* from **A**. Where do you set the posts **C** and **D** to warrant the fence forming a proper rectangle? Just key in:

[3] [0]

```
                0.
30_
```

[DISP] FIX [0] [1] [16]

```
               0.0
30
```

Note the input cursor _ vanished and 30 is adjusted to the right now,

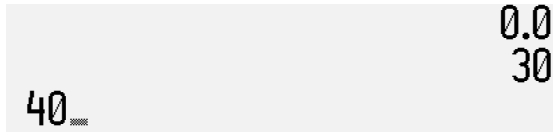
[13] A most clear and compact way of signaling menus, also used throughout this text.

[14] Assume it in suburbia. Remember?

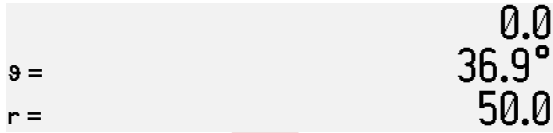
[15] Here, we use outdated *British Imperial* units since they are still spread in some British colonies and ex-territories. Although dealing with them is far more cumbersome than using rational units, the archconservative heirs of the progressive immigrants there will just not get rid of them. But solving the problem above will work with *meters* as well.

indicating this input is closed; so the next number can be entered:

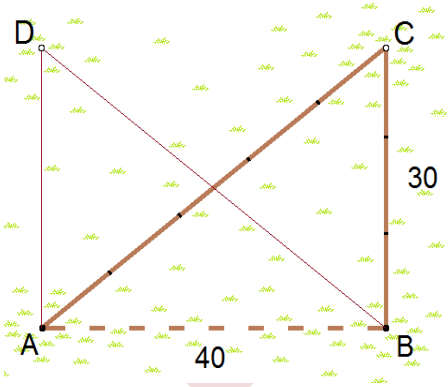
4 0



→P



Now, all you need is the bottom number, a friend, 80 *yards* of rope, and a black marker: Ask your friend to hold both ends of the rope firmly, take the loose loop and walk away as far as you can – when the loop is stretched, mark that position on the rope and return. Ask him or her to hold this marked point of the rope as well, fetch the two loose loops and walk again as far as possible: with both loops stretched, mark these positions on the rope. Return again; hand over the two new marked points and walk away once more, now with four loose loops. After marking as before, your rope will show regular marks every 10 *yards*.[17]



Take this rope now, nail its one end on post **A**, its other end on **B**, fetch the loose loop, a hammer, and a post, and walk 5 marks away as calculated. When both sections of the rope are tightly stretched, place post **C** there. Set **D** the same way on the other side.

This solution will work for arbitrary rectangles: As soon as you press →P, your *WP43* will calculate the diagonal automatically for you. You just provide the land, posts, hammer, nails, and the rope (or rather a long tapeline) – and it will remain up to you to set the posts.

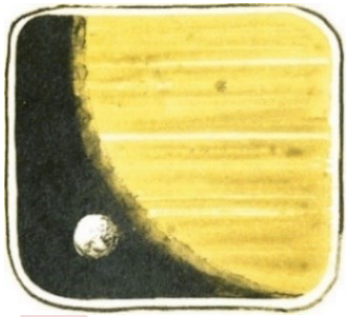
---

[16] Press [blue], then E to access DISP; then press F1 for FIX. Key in 0 1, telling your *WP43* to display only one decimal digit (internally, everything is handled with full precision always). You will learn more about output formatting in *Section 2*.

Generally, we will print no more than one display row comprising just **0.** from here on.

[17] Something alike, a knotted cord or rope, is known as one of the oldest surveying tools of mankind, evidently used, e.g., in Egypt some 5000 years ago.

Another **example**, with greater areas: [18]



To calculate the surface area of a sphere, the formula $A = \pi\, d^2$ can be used, where ***A*** is the surface area, π is 3.141 5..., and ***d*** is the diameter of the sphere.

Ganymede, one of *Jupiter's* 95 moons, has a diameter of 5262 km. To use your *WP43* to manually compute the area of *Ganymede*, you can press the following keys in order:

5 2 6 2	5 262	diameter of *Ganymede*
ENTER↑ ×	27 688 644	square of the diameter
π	3.1	the constant π (shown with only 1 decimal as chosen above)
×	86 986 440.6	area of *Ganymede* (in km^2)

If you wanted the surface areas of each of *Jupiter's* 95 moons, you could repeat the above procedure 95 times. However, you might wish to write a *program* that would calculate the area of a sphere from its diameter, instead of pressing all the keys for each moon.

To calculate the area of a sphere using a program, you should first

write the program, then you must

record the program into the calculator, and finally you

run the program to calculate the answer.

Writing the program: You have already written it! A program is nothing more than the sequence of keystrokes you would execute to solve the same problem manually.

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[18] Quoted from the *HP-25 OH* though updated – only 12 moons of *Jupiter* were known in 1975 (not counting the black cuboidal monolith seen in *Stanley Kubrick's* masterpiece '*2001 – a Space Odyssey*' of 1968, an amazingly realistic science fiction movie shot before even *Neil Armstrong* set foot on the Moon). Emphases were added by me.

Reading *HP*'s vintage calculator manuals may be real fun. *Eric Rechlin* collected them (and more literature in various languages about *HP* and 'nearly *HP*' pocket calculators from 1972 to today) and put them at *https://literature.hpcalc.org/* where you can browse and download them for free. But take care: copyright is not always observed there!

Recording the program: To record the keystrokes of the program into the calculator, press the following keys in order:

P/R turns your *WP43* to *program entry mode*. The display will change in this course – simply don't bother for the time being.

GTO . . goes to the location where free *program memory* space begins.

LBL T is the opening step of your program (labelled **T** for total surface – for T press F6 as you see the letter T displayed above it).

ENTER↑
×
π
×

These keys are the same you pressed to solve this problem manually above.

RTN is the closing step of your program. Finally,

P/R returns from *program entry mode* to *run mode*.

Any straight program on your *WP43* will contain an opening LBL and a closing RTN, framing the keystrokes needed for solving the problem manually.

Running the program: Now all you have to do to calculate the area of any sphere is keying in the value for its diameter and press XEQ T meaning 'execute program **T**'.

When you press XEQ T the sequence of keystrokes you recorded is automatically executed by the calculator, giving you the same answer you would have obtained manually:

For instance, to calculate the surface area of *Ganymede*, press

5 2 6 2 — 5 262 — *Ganymede's* diameter

XEQ T — 86 986 440.6 — its surface area – as you had calculated manually above; so you have verified your routine works properly.

With the program you have recorded, you can now calculate the respective surface area of any of *Jupiter's* moons – in fact, of any sphere – using its diameter. You have only to leave the calculator in *run mode* and key in the diameter of each sphere that you wish to compute, and then press XEQ T .

For instance, to compute the surface area of *Jupiter's* moon *Io* with a diameter of 3643 km:

3 6 4 3 — 3 643 — *Io's* diameter

XEQ T — 41 693 486.7 — its surface area;

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3 1 2 2	3 122_	*Europa's* diameter
XEQ T	30 620 739.2	its surface area;
4 8 2 1	4 821_	*Callisto's* diameter
XEQ T	73 017 025.3	its surface area; etc.

Programming your *WP43* is *that* easy! It remembers a series of keystrokes and then executes them automatically when you press XEQ ... [19]

There is no need memorizing any formula you keyed in once – your *WP43* can remember it for you (and provides space for hundreds more). And you can even define shortcuts to your favorite formulas by customizing the surface of your *WP43* (see *Section 6*) following your very personal requirements and wishes.

The early portions of this handbook show you how easy it is to manually use the power of the *WP43*; while in *Section 3: Programming* you will find a complete guide to *WP43* calculator programming.

… you will want to take a good look at this handbook: It explains the unique *HP* logic system that makes simple answers out of complex problems, and *WP43* features that make programming painless. When you see the simple power of your *WP43*, you'll become an apostle just as have some twenty million *RPN* calculator owners before you.

How to Enter Numbers (and How to Edit Them)

Numeric entry is as easy as typing. Key in 1 2 3 4 5 . 6 7 8 9, e.g., in *SDC* and you will see

12 345.678 9_

Note the gap inserted automatically after each group of three digits for easier reading. You may enter up to 43 digits for one number. As long as

[19] Program **T** as recorded above is very short and simple: Its center part contains five keystrokes only (ENTER↑ × f π ×). You may store far, far more keystrokes in *program memory* – the general procedure of recording and running programs will remain unchanged, however; only the center parts of programs will grow.

input is open, any digit mistyped may be erased by ⬅ and can be replaced then (when no digit is left, your *WP43* will return to the *calculator state* it was in before this input).

For entering a negative number like −7.5, type 7 +/− . 5 or 7 . +/− 5 or 7 . 5 +/−: pressing +/− flips the sign of the open number. Only negative signs will be displayed.

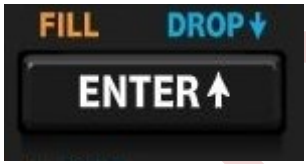
For a huge figure like the age of our universe in years, just type 1 3 . 8 E 9. It is echoed

13.8×10^{9}

i.e. 13.8 *billion*[20] years in scientific '*mantissa plus exponent*' format. E opens entry for the exponent of 10 – your *WP43* translates this shortcut into naturally readable display.

Input is closed and released for interpretation by a <u>command</u> – e.g., by ENTER↑. Here, ENTER↑ will change the display to the equivalent

13 800 000 000.



The closed number is displayed right adjusted and also duplicated by ENTER↑ (cf. p. 21).

Really tiny values such as a typical diameter of an atom (i.e. $^{1}/_{10\ 000\ 000\ 000}$ *meter*) are entered in analogy:

E +/− 1 0 is echoed $1.\times10^{-10}$

Closed, this will be displayed as 0.000 000 000 1

> Note there's no need to key in 1 . E +/− 1 0 here: if no digit is heading E, **1** is assumed for the mantissa. And pressing +/− after E flips the sign of the exponent – if you want to flip the sign of the mantissa, press +/− before E or after closing the entire input.[21]

The entire bottom numeric row may be dropped by DROP↓ at any time; subsequent input will then fill this row again, pushing old content up.

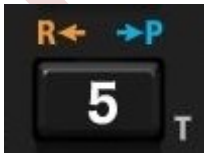
[20] A *billion* for the *USA*. Many other people call this *milliard* – a *billion* is 10^{12} there. Thus, exponents of 10 are actually a better measure since unambiguous.

[21] If you prefer output in *scientific* format as above, *Section 2* will show you how to get it.

A closed bottom row is cleared by ⬅ at once, resetting it to 0. – subsequent input will overwrite this zero then.

How to Enter and Execute Commands

For calling any operation you wish executed, just enter the keystrokes to access its label (cf. p. 20). Pending input will be echoed at left end of the top numeric row on your screen until the command is completed. Therein, pending 🟧 will be echoed by f and 🟦 by g, if applicable;[22] these symbols will be replaced by the *name* of the command called as soon as it can be decoded.



For many commands, f or g will be the only echo you will see since the next keystroke may well terminate entry already, call the command, execute it, and display its result, all done in a fraction of a second (as observed with 🟦 →P above).

There are, however, commands requiring parameters trailing – they will hence stay on the screen for longer. STO and RCL are of this kind, and there are many more (see pp. 63ff).

🟧, 🟦, and the *menus* in particular allow for easily accessing a multiple of the 43 primary labels the keyboard of your *WP43* can take.

How the Keyboard is Organized

You might have noticed already that the labels on your *WP43* are grouped according to their purposes. Beyond the four basic arithmetic operations +, −, ×, and /, six sets of four or more keys are provided as pictured overleaf: [23]

[22] If you pressed 🟧 or 🟦 erroneously, just press the same key once more to cancel it. And if you inadvertently executed a command you did not want to, press 🟧 ↶ to undo this last command immediately (see the chapter about *Error Recovery* further below). Mistakes can be undone easily on your *WP43*.

Keys for numeric input (cf. pp. 25ff).

General navigation and control functions like ⬅ for deleting, ↶ for undoing the last command, ▲ and ▼ for browsing, and EXIT for escaping.

[23] The multiplication and division keys on the *WP43* calculator are labelled × and ÷ for sake of tradition only – although a printed ÷ looks very similar to + (check vintage program listings!) and may irritate a great group of users initially (note *ISO 80000-2* recommends not to use ÷ in print at all). Instead, those two keys could read • and : as well which would be visually unambiguous at least – but this would have upset another great group of users.

As a compromise, we printed / in the manuals whenever we talk about the division key. It is displayed as / in *WP43* programs and equations for sake of clarity as well. This /, however, dashed against rigid resistance of *SM* although *HP* had used it together with a multiplication star on the *HP-71B* and *HP-38G* already.

For the multiplication key, × is printed in the manuals although this symbol choice causes ambiguities in *vector* calculus, see *Section 2*.

Users preferring • and : over × and ÷ (for good reasons) can achieve this easily using a black permanent marker.

Softkeys calling *items* from the *menu* displayed above (see next chapter).

Modes, *data types*, and 'common' trans-cendental functions (SIN, COS, TAN, and their inverses are collected in the *menu* TRI displayed here).

Stack and register operations.

Functions for programming and calling routines, like XEQ for executing.

Find the basic functionalities of all primary keys of your *WP43* explained overleaf, top left to bottom right on the keyboard. See the pages mentioned (and the *IOI*) for details about these functions. There may be further functionalities assigned to these keys not printed here, depending on particular types of operands; they will be covered in *Section 2*.

Note the lowest numeric row displayed shows x, the next y (see *Part 2* of this *Section* for the reasons).

1/x Inverts x (see p. 34 for an instance).

EXP Opens a *menu* containing exponential functions like y^x, 2^x, x^2, x^3, roots, logarithms, and hyperbolics (shown on p. 32).

TRI Opens a *menu* of trigonometric and hyperbolic functions as well as their inverses (cf. previous page, see also pp. 102f and 139ff).

ln Returns the natural logarithm of x

e^x Raises Euler's e to the power of x

√x Returns the square root of x

see pp. 34, 88ff, and 94ff.

STO Stores (copies) x in the destination specified (see pp. 57ff).

RCL Recalls (copies) an object from the source specified into X (see pp. 41 and 57ff).

R↓ Rolls the *stack* contents one *level* down (see p. 42).

CC Composes, cuts, or converts complex numbers (hence this key should actually read CCCC; see pp. 166ff).

Prefix to access golden labels. Press twice to cancel it.

Prefix to access blue labels. Press twice to cancel it.

ENTER↑ Context sensitive key, cf. pp. 25f and see pp. 333ff.

x⇄y Swaps the contents of the bottom two rows (see p. 42).

+/- If pressed during input of a mantissa or exponent then changes its sign (cf. pp. 25f). Else multiplies x times **−1**.

E Allows entering an exponent of ten for convenient input of very big or very small numbers (cf. pp. 25f).

← Context sensitive key, cf. pp. 25f and see pp. 333ff.

/ If there is an open question like `Are you sure?`, enters N for denial. Else divides y by x (cf. f. 23 on p. 28).

7, 8, 9, 4, 5, 6, 1, 2, 0 Enter the respective digit.

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[XEQ] If there is an open question like `Are you sure?`, confirms it;
else – if in *RUM* – calls the routine carrying the label specified and starts executing it (see pp. 214ff);
else (in *PEM*) inserts a call to the subroutine carrying the label specified (see pp. 227f).

[×] Multiplies y times x.

[▲] Context sensitive key, see pp. 32f and 333ff.

[−] Subtracts x from y.

[3] If there is an open question like `Are you sure?`, enters **Y** for 'yes'. Else enters the digit **3**.

[▼] Context sensitive key, see pp. 32f and 333ff.

[+] Adds x to y.

[.] Enters a decimal radix mark in numeric input (cf. p. 25).
If pressed twice in numeric input, allows for entering a fraction (see pp. 73f and 136ff).
In *register* or *flag* addressing, [.] heads a local address (see pp. 60ff).

[R/S] Context sensitive key, see *Section 3* and pp. 333ff.

[EXIT]/[ON] Context sensitive key, cf. pp. 25f; see also pp. 32f and 333ff.

Menus – Items à la carte

Your *WP43* features 786 operations, far too many for its keyboard. Hence, most of them live in *menus*. Beyond commands, also applications, browsers, constants, conversions, routines, *submenus,* digits, symbols (a.k.a. characters), *system flags*, or *variables* defined may be stored in *menus*: collectively, we call them *menu items* or simply *items*. *Menus* keep the keyboard relatively tidy.

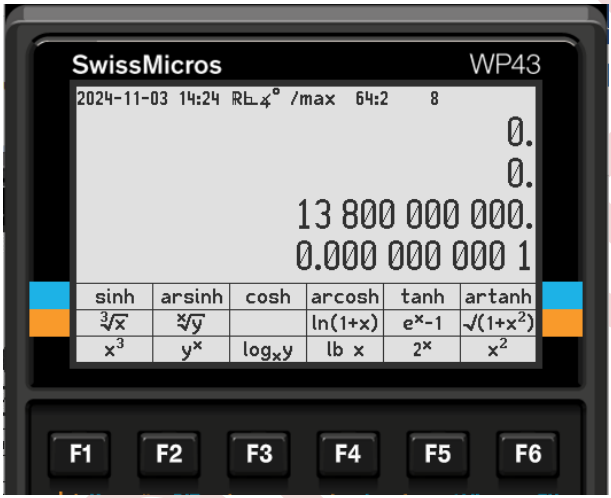
You find 35 *menus* on the keyboard.[24] Open any of them by accessing its label (cf. p. 20) – this will cause the lower third of the *WP43* screen (called the *menu section*) displaying the respective *menu view*. Whenever

a *menu* is called the first time, its top *view* will appear, comprising up to 18 *items*:

- up to six assigned to the unshifted top row of *WP43* keys,
- up to six to the 🟨-shifted, and
- up to six to the 🟦-shifted, all readily accessible for you.

Example:

Press [EXP] and EXP will open showing the functions stored therein as pictured. As long as this *menu view* is displayed, simply press, e.g., ...



- [F2] for [yˣ],
- 🟨 [F2] for [ˣ√y] (note the golden stripes framing this row in display),
- 🟦 [F3] for [cosh], i.e. the hyperbolic cosine (note the blue stripes).
- If you press 🟨 [F3], nothing will happen since no label is displayed there – no operation is linked to it.

Saving print space in the manual, we may, e.g., write cosh for a function in the blue row, ˣ√y for a function in the golden row, and yˣ for a function in the unshifted row.

A predefined *menu* may comprise more *views* – this will be indicated by a dashed line above the *menu section*. Whenever such a *multi-view menu* shows up, [▲] will advance to its next *view* and [▼] will return to the previous one, changing the labels displayed. Because *multi-view menus* are circular, also pressing [▲] (or [▼]) repeatedly will return to the 1st *view* after all other *views* were displayed (thus, for a *menu* containing two *views*, both [▲] and [▼] will show the next *view*).

[24] Just [EXP] and [TRI] are not underlined for space reasons. Note, however, that all *menu names* are stored without underlines, hence they are also displayed this way.

Every *menu view* will stay on screen – granting direct access to all its *items* displayed – **until you leave it** (via ▼ or ▲, if applicable, via EXIT, or by calling another *menu*).

Catalogs are special *menus*: their *items* are sorted alphabetically and may be searched this way (see *ReMS 2*). Thus, there are various possibilities to call a *cataloged item*. We will hence refer to such an *item* using the notation 〔item〕. Picking an *item* from a *catalog* (e.g., CONST) will EXIT this *catalog* automatically.[25]

We will print the same notation referring to an arbitrary 〔item〕 whose precise location in its *menu* is irrelevant at the moment. In all other cases the background colors introduced above will be used from here on to indicate the access path to a specific *menu item*. Note labels of (*sub-) menus* are displayed inverted in *menu views* (cf. f. 24).

Pressing EXIT in a *menu* will return to the *menu* you called before, if applicable. The last four *menus* you called are preserved on a *menu stack* therefore.[26]

Before demonstrating the operation of your *WP43* in detail, let's return to our introductory problem solving examples for four general remarks:

1. We presume you have graduated from an US *High School* at minimum, passed *Abitur*, *Matura*, or an equivalent graduation. So we will not explain basic mathematical rules and concepts within these manuals. Please turn to respective textbooks.
2. As long as you stay with one coherent set of units while calculating reasonably you will get meaningful results within this set. Thus, it were a waste of time and energy to enter any units and pull them through your calculations. Should you need to convert special inputs into common units or require results expressed in particular or local units, however, U→ and →| will help (see pp. 299ff).[27]

[25] … with one exception – see *Section 2*.

[26] Holding EXIT depressed for longer than 1 *second* will call EXITALL, exiting and closing all open predefined *menus*, so MyMENU will appear usually (see *Section 6*).

[27] A quick unit analysis (a.k.a. dimension check) is recommended before starting a calculation of a formula you have derived yourself, or doing exploratory math (see *Part 2* of this *Section*). *SI* is the largest coherent set of units available on this planet.

3. Although you entered just *integer numbers* for both edges of your little patch of land on pp. 21f, your *WP43* calculated the diagonal using *real numbers* (see next page). This also allows for decimals in input and output. Alternatively, you may enter fractions such as e.g., 6 ¼ if this carries a benefit for you.[28]

4. In five decades of scientific pocket computing, a wealth of examples has been published – more and better than we can ever invent. We did not intend to copy all of them; instead, we recommend the media mentioned in footnote 29. All computations described there for any scientific calculator can be done on your *WP43* – distinctly faster and often in a more elegant way. Nevertheless, we included over 200 new and vintage examples in this manual to support you learning and getting accustomed to your new tool.

Problem Solving, Part 2: Elementary Stack Mechanics

Most commands of your *WP43* are mathematical functions or operations taking and returning *real numbers* (a.k.a. *reals*) like 3.141 592 or 0.125 or -5.67×10^{-8} (note *integers* like 3 or 12 345 678 or −12 321, as well as *fractions* like $^{4}/_{5}$ or $^{137}/_{7}$ are mere subsets of *reals*). – Any command you choose may operate on one, two, three, or four numbers at once to generate a result. In spite of the hundreds of functions available, you will find them simple to operate by a single, all-encompassing rule:

Working with different sets of units carries risks – see this expensive experiment: *https://www.latimes.com/archives/la-xpm-1999-oct-01-mn-17288-story.html*. Sure it was not nice, but there were many people chuckling about that regard from the Dark Ages.

[28] Your *WP43* features many *data types* more – we will introduce them to you in *Section 2*.

[29] Almost all user guides, handbooks, and manuals printed for *HP* calculators in two decades, beginning with the *HP-9100A* of 1968 (*HP*'s 1st desktop calculator, without any *IC* – but with a small cathode ray tube built in for display) are still available at low cost (together with almost complete information about those *devices* built until 1990) in a 14 GB media package distributed by *David Hicks'* online *Museum of HP Calculators* (see *http://www.hpmuseum.org/cd/cddesc.htm*).

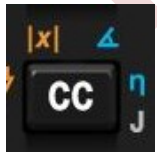
Compare with *Eric Rechlin's* free online offer (at *https://literature.hpcalc.org/*) covering pocket calculators between 1972 and 2023 as mentioned in f. 18 on p. 23.

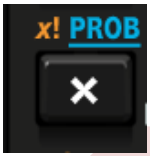
When you press a function key, your *WP43* will execute the operation assigned to it immediately.[30]

***Monadic* functions** operate on <u>one</u> number only. Find on the keyboard: the reciprocal [1/x], the logarithms [ln] and [lg], the exponentials [e^x] and [10^x], and the square root [$\sqrt{x}$]; furthermore [|x|] (always returning a positive number), [+/−] (cf. p. 30), and the factorial [x!].

Examples to be executed consecutively:

[6] [4] [1/x]	0.015 625
[ln]	−4.158 883 083 359 672 ,
[|x|]	4.158 883 083 359 672 ,
[lg]	0.618 976 711 428 782 ,
[10^x]	4.158 883 083 359 672 ,
[e^x]	64. ,
[$\sqrt{x}$]	8. ,





and [x!] returns 40 320.

Each *monadic* function replaces x by the respective $f(x)$ (= $x!$ in last case). Everything else will stay as it was.

Please check the *IOI* for the many more *monadic* functions provided on your *WP43*.

***Dyadic* functions**, on the other hand, operate on <u>two</u> numbers and return one. Think of the four basic arithmetic operators **+** and **−**, **×** and **/**.

Example:

Assume owning an account of 1234.00 US$ and withdrawing 56.70 US$ from it. What will remain?

Solving such a problem easily may go like this:

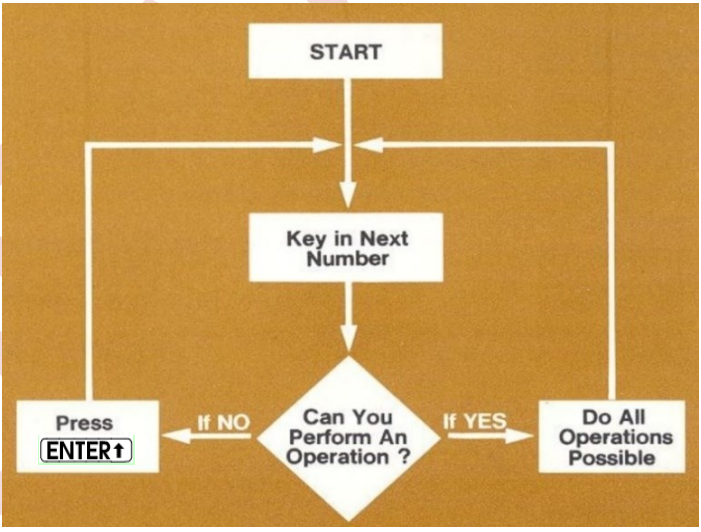
[30] If it requires trailing parameters it will execute as soon as parameter input is completed.

On a piece of paper:		→	On your *WP43*:		
Write down the 1st number:	1234.00		Key in the 1st number:	1 2 3 4	1 234
Start a new line and write down the 2nd number:	56.70		Close 1st input:	ENTER↑	
Draw line below.			Key in the 2nd number:	5 6 . 7	56.7
Subtract:	1177.30		Subtract:	−	1 177.3

This is the essence of *RPN*:

Provide the required operands,
then execute the requested operation
by accessing the corresponding label.

HP explained *RPN* using this scheme. And a major advantage of *RPN* compared to all other calculator operating systems we know is it sticks to this basic rule – always.[31]



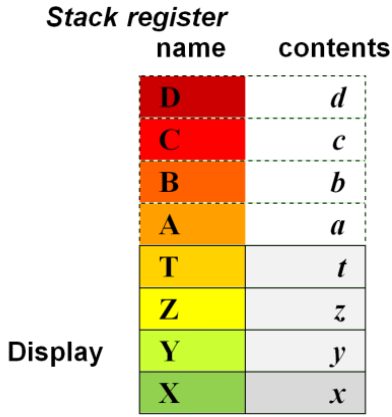
[31] This rule applies to all functions regardless of the kind of objects they operate on. This way of writing operations is called *postfix* notation since the operator is entered after the operands (cf. f. 1). It suits electronic calculating very well (and eases work for human brains) – see further below.

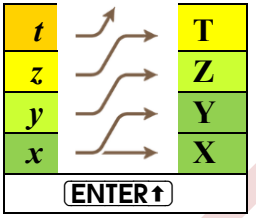
The picture overleaf is copied from *HP's* brochure 'ENTER↑ *vs.* =' of 1974.

Some people might claim said global rule strictly holds for *RPL* only. *RPL* is a programming language and notation developed from *RPN* in the 1980's. Maybe they are even right. In my opinion, however, *RPL* strains the *postfix* principle beyond the pain

And as paper holds your operands while you are calculating manually with paper and pencil, a space holding them is also required on your *WP43*.

The *stack* does this job, and this is its *raison d'être*: It is a pile of linked *registers*, workspace for your calculations. Bottom up, these *stack registers* are traditionally called **X**, **Y**, **Z**, and **T**, optionally followed by **A**, **B**, **C**, and **D** on your *WP43*.[32] Your input is entered in **X** always, and at least ***x*** is always displayed in *RUM* – ***y***, ***z***, and ***t*** <u>may</u> be, so you can see the actual contents of the bottom four *stack registers* at a glance.





ENTER↑ separates two input numbers by closing ***x*** and copying it into **Y** (cf. above). The contents of the upper *stack registers* are lifted out of the way before. For a *4-level stack* (*4LS*), ***z*** is moved into **T**, then ***y*** into **Z**, then ***x*** copied into **Y**. Thereafter, **X** can take new input without loss of information.

This is the classical functionality of ENTER↑, beginning with the *HP-35* of 1972 until the *HP-42S* ceased production in 1995. ENTER↑ affects all *stack registers*, and the previous content of the <u>top</u> *register* is lost. It is often said ENTER↑ 'pushes ***x*** on the *stack*'.[33]

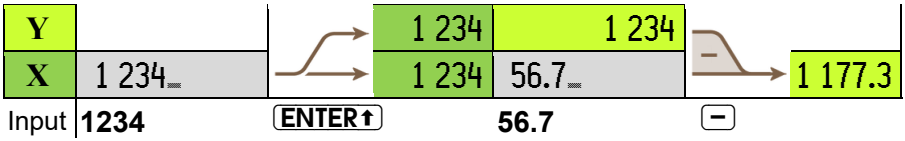
Let's look at our account example again, putting it in a *stack diagram*:[34]

barrier, exceeding the limit where it becomes annoying for humans. Not for everybody, of course, but also for many scientists and engineers. Thus, we stick to classic *RP<u>N</u>* on *WP43* as we did on *WP 34S* and *31S*.

[32] This optional *8-level stack* (*8LS*) was invented by *Paul* and me and launched with *WP 34S* in 2011. *WP 31S* features it as well. See the further text for its advantages.

[33] Actually, it pushes ***x*** under it. There are other dialects of *RPN* but this is the main stream.

[34] At begin, arbitrary data from earlier work may be present in the upper *stack registers* **Y**, **Z**, etc. These data are irrelevant for the current calculation, so we left them aside here (actually, they are pushed out of the way – compare the picture above). In further



After entering the 2nd number (**56.7** = x), pressing [−] subtracts x from the 1st number (**1234** = y) and puts $f(x, y) = y - x =$ **1177.3** into X. This simple method applies to all *dyadic* functions on your *WP43*:

Put the operands on the *stack*,
then call the operation $f(x, y)$,
and its result will be displayed.[35]

Also a few ***triadic* functions** are featured. Executing such a function replaces x by $f(x, y, z)$ – see p. 43 for an instance. All *triadic* functions provided on your *WP43* are found in *menus*.

SLVC and ↭ are the only functions operating on all bottom **four** stack registers at once. See *Section 4* for more about SLVC and the *IOI* for ↭.

Furthermore, there are some functions operating on one number but returning two (like DECOMP) or three (like DATE→). Other operations don't consume any *stack* input at all but may return one, two, or three objects (like RCL, SUM, or L.R.); then these objects will be pushed on the *stack*, taking one *register* each (cf. p. 41).

Further ways of parameter handling will be shown in *Section 2*.

Problem Solving without any Formula

The following little real-life problem shows how *RPN* is useful and fast in solving step-by-step problems where there is no formula, and where the next calculation depends on the result of the previous one.

---

stack diagrams we will omit entirely all *stack* contents irrelevant for the calculation under investigation and use plain bold black text denoting numeric input from here on, for space reasons.

[35] This eliminates the need for a key [=] – also beyond dyadic operations, since this rule applies by analogy there as well.

Example (with *SDC*): [36]

You need to fix a tile that has fallen off your roof. The roof is 28 *feet* up, and you have a stepladder that is 29 *feet* long. You could also borrow your neighbor's 38 *foot* ladder. Is either ladder good for the job? You could try leaning each one in turn against the roof and seeing which is better, but it's raining heavily, so why not work it out first on your *WP43* by seeing what angle each ladder will make with the vertical when you lean it against the roof?

Solution:

We don't need more than two decimals for the following calculation, so enter

DISP FIX 2 .

First try it for your own ladder. Enter its length:

29 ENTER↑ .

Then enter the height and divide so you can get the angle,

28 / .

You get 1.04. That can't be right; *sines* and *cosines* should be smaller than 1.

Use ↶ to get the numbers back, then

x≷y to swap them. Now press

/ to divide them again, in the right order now, and see 0.97.
(Actually, simply pressing 1/x would get the same result, making the correction more quickly.)

Anyway, this is the sine of the angle between the ladder and the vertical. Or is it the *cosine*? Press

TRI **arcsin** and see the answer, 74.91 *degrees*. No, that is not right, it must be the *arc cosine* that you need. Press

↶ to undo the *arc sine*, then

arccos returning 15.09 .

Your ladder would be only 15 *degrees* away from the vertical. That is uncomfortably steep (and it may even tip over as you stand on it). So try the same calculation for your neighbor's 38 *foot* ladder. Type

29 ENTER↑

38 /

arccos.

[36] This is courtesy of *Gene Wright*. It is set in the *USA* – see the case and units.

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This time the answer is 40.26 *degrees*. The ladder would be at a rather shallow angle and might slip away as you stand on it.[37]

Neither ladder is really suitable. Maybe you should ask some other neighbors if they would lend you a ladder with a better length. What would be a good length? 30 *degrees* would probably still be too much; about 20 *degrees* would be about right. So type the height again and divide by the *cosine* of 20 *degrees*.

28 [ENTER↑].

20 **cos**.

[/].

That gives 29.80. A 30 *foot* long ladder would be almost ideal – if a neighbor has one.

The argument in school calculators is "*Algebraic lets you enter the equation just as you see it on paper!*" Well, for one thing, a lot of programming work falls outside of actual equations. For another, even in the field of calculators, in real life we don't usually *have* an equation in front of us. I think through it, "Let's see – I need **A**, and then square that; now take **B**, multiply it by **C** and add that result to the earlier one. Now I need **D** raised to the power of **E**, and divide the earlier result by that. Done. Oops, no, I still need to take the log of that..." That's the way much of real engineering is.

It's only in school that you get canned problems and have the equation in the book and you're supposed to just drop the numbers in the chute and turn the crank – then we wonder why we get graduates who passed all their classes and yet show a disconnect between that knowledge and any understanding of what the problem is in their circuit on the workbench! I've hired a lot of electronics technicians and a few engineers, and I gave all the applicants a circuit-analysis test with a dozen *simple* problems. *Not one* of them ever got it all correct. It was kind of frustrating.

Algebraic tells you what you get. *RPN* tells you how you get there. (*Garth Wilson* of *WilsonMinesCo.com*)

[37] No, don't move this ladder closer to the wall! Seems it must not lean against the roof – does any *US* house-owner know why? And turning the stepladder by 90° isn't allowed either. Riddles over riddles! Anyway, such manoeuvres would have ruined this example.

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Looking Closer at the Automatic Stack

Beyond [ENTER↑], there are [x⇄y], [STO], [RCL], [R↓], [R↑], [x⇄], and [VIEW] dealing with *registers* (not only those in the *stack*).[38] You find these eight – together with the new [RBR], [FILL], and [DROP↓], as well as the *menu* [STK] – all within the small area of the keyboard shown here.

Your *WP43* features even more commands of this kind: CLX, CLSTK, CLREGS, STOSTK and RCLSTK, STOCFG and RCLCFG, STOIJ and RCLIJ, and more flavors of STO and RCL, as well as DROPy, y⇄, z⇄, t⇄, and ⇄. The first three are found in CLR (on 🟦-shifted ⬅), the last five in [STK].

And your *WP43* allows you to expand the traditional *4LS* of 1972 to 8 *levels* (as the *WP 34S* and *WP 31S* do since 2011). Look at the *status bar* at top of your display: it indicates an **8** at its right side – cf. p. 20.

Thus, the fate of certain *stack* contents will depend on the operation executed as well as on the *stack* size at time of execution. Operations on the *4LS* work as known from *HP's RPN* calculators since the *HP-45*. On the *8LS* of your *WP43*, everything works in analogy – just granting you more workspace for your intermediate results. Benefits will come for free. They will become evident in the following.[39]

Find on the next two pages what [ENTER↑], [FILL], [DROP↓], [DROPy], [x⇄y], [R↓], [R↑], and further representative functions do in detail on *stacks* of either size. Then you will also know why [ENTER↑] and [R↑] show arrows pointing up while [R↓] and [DROP↓] point down.

- [0] [FILL] will clear the entire *stack* most easily, regardless of its size. Though also CLSTK is provided (in CLR) for backward compatibility, program space economy, and speed of execution.

[38] These are known from previous *RPN* calculators for decades, the first five since 1972.

[39] Should you ever want to go back to the old *4LS*, [CF] [SYS.FL] [SSIZE8] will clear this *system flag*.

	Stack register	Assumed initial contents	*Stack* contents after executing … ENTER↑	FILL	DROP↓	DROPy	x⇄y	R↓	R↑
With 8 *stack registers*	D	$d = 8.$	7.	1.1	8.	8.	8.	1.1	7.
	C	$c = 7.$	6.	1.1	8.	8.	7.	8.	6.
	B	$b = 6.$	5.	1.1	7.	7.	6.	7.	5.
	A	$a = 5.$	4.	1.1	6.	6.	5.	6.	4.
	T	$t = 4.$	3.	1.1	5.	5.	4.	5.	3.
	Z	$z = 3.$	2.	1.1	4.	4.	3.	4.	2.
	Y	$y = 2.$	1.1	1.1	3.	3.	1.1	3.	1.1
	X	$x = 1.1$	1.1	1.1	2.	1.1	2.	2.	8.
With 4 only	T	$t = 4.$	3.	1.1	4.	4.	4.	1.1	3.
	Z	$z = 3.$	2.	1.1	4.	4.	3.	4.	2.
	Y	$y = 2.$	1.1	1.1	3.	3.	1.1	3.	1.1
	X	$x = 1.1$	1.1	1.1	2.	1.1	2.	2.	4.

- Depending on the problems you solve and the way you proceed, you may find that x and y should be swapped before executing, e.g., [−], [/], or [y^x] sometimes – [x⇄y] will do this for you.〔
- [R↓] (and [R↑]) once came handy for reviewing *stack levels* else unseen.[40] On your *WP43*, feel free to use [RBR] instead, displaying ten *registers* at once without disturbing the *stack* at all (see pp. 283ff).

[40] Note most pocket calculators display only x. Reviewing the other levels turns pointless, however, when all 4 *stack registers* are permanently displayed for a *4LS*. These two *roll commands* may be beneficial under particular conditions nevertheless – see below.

Stack register	Assumed initial contents	*Stack* contents after executing …					
		[RCL] [L] [41]	[s] [42]	[x²] [43]	[+] [44]	[→DATE] [45]	[DATE→] [46]
D	$d = 8.$	7.	6.	8.	8.	8.	6.
C	$c = 7.$	6.	5.	7.	8.	8.	5.
B	$b = 6.$	5.	4.	6.	7.	8.	4.
A	$a = 5.$	4.	3.	5.	6.	7.	3.
T	$t = 4.$	3.	2.	4.	5.	6.	2.
Z	$z = 3.$	2.	1.1	3.	4.	5.	11
Y	$y = 2.$	1.1	s_y	2.	3.	4.	12
X	$x = 1.1$	*last* **x**	s_x	1.21	3.1	0003-02-01	30
T	$t = 4.$	3.	2.	4.	4.	4.	2.
Z	$z = 3.$	2.	1.1	3.	4.	4.	11
Y	$y = 2.$	1.1	s_y	2.	3.	4.	12
X	$x = 1.1$	*last* **x**	s_x	1.21	3.1	0003-02-01	30

[41] [RCL] [L] represents an arbitrary function pushing one object on the *stack*.

[42] [s] represents an arbitrary function pushing two objects on the *stack*.

[43] [x²] represents an arbitrary *monadic* function.

[44] [+] represents an arbitrary *dyadic* function.

[45] Assume [→DATE] being called in *SDC* (i.e. YMD). [→DATE] represents an arbitrary *triadic* function here.

[46] Assume **11.123** or **11-12-30** in **X** initially and YMD here – see the last chapter of this *Section* as well as *Section 2*.

- For most functions, your WP43 copies x into the special *register* **L** automatically just before the new function is executed. RCL L as shown below recalls this ***last x*** – learn about its benefits on p. 52.

Note the previous content of the top *stack register* is lost when ENTER↑ or RCL is executed. Functions like S or DATE→ will even cost the contents of two *stack registers*. We recommend keeping SSIZE8 set mitigating the effects of such losses.

See the *IOI* for further information about the commands mentioned above.

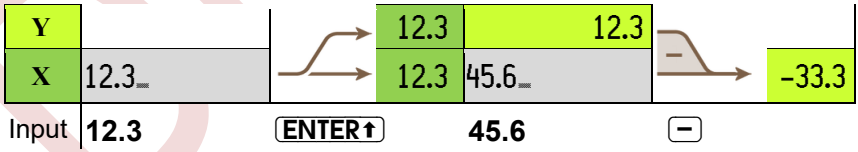
Problem Solving, Part 3: Advanced Stack Mechanics

Using the features and functions introduced above, *RPN* makes chain calculations as easy as can be:[47]

Example:

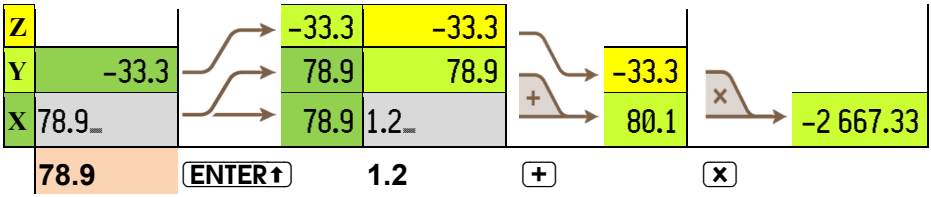
$$\frac{(12.3 - 45.6)\,(78.9 + 1.2)}{(3.4 - 5.6)^7} = ?$$

This is as a combination of 6 *dyadic* functions: two subtractions, one addition, one multiplication, one exponentiation, and one division. Starting top left in the formula, this is what will happen on the *stack* of your *WP43* while solving it:



This 1st parenthesis in the numerator is solved exactly as demonstrated in our little account problem above. Proceed to the 2nd parenthesis now:

[47] Side note: Above titles '*Elementary Stack Mechanics*' and '*Advanced Stack Mechanics*' go back to a textbook titled '*Elementary Quantum Mechanics*' I came across in my studies decades ago. It goes without saying that book was as far from being elementary as a theoretical physicist could be.



It is solved like the 1st. But in the 1st step of this sequence, the prior result of 1st parenthesis is lifted automatically into **Y** to avoid overwriting it with the new number keyed in now. This move is called *automatic stack lift* (*ASL*).

> Actually, such an *ASL* (originally called *automatic ENTER*) works as if ENTER↑ was pressed before **7** here. *ASL* is standard on *RPN* calculators since 1972, saving keystrokes.[48] Remember you need ENTER↑ just for separating two consecutive input numbers – cf. p. 36. In consequence, we need no (and can solve problems with a minimum of keystrokes.

After having solved the 2nd parenthesis by pressing +, the results of both upper parentheses were on the *stack* in **X** and **Y** – well prepared for the multiplication to complete the numerator. Thus, we just did it.

Now start calculating the denominator – once again the intermediate result is lifted automatically in the 1st step:

	-2 667.33	-2 667.33		-2 667.33		
-2 667.33	3.4	3.4	-2 667.33	-2.2	-2 667.33	
3.4	3.4	5.6	-2.2	7	-249.4…	10.693…
3.4	ENTER↑	**5.6**	−	**7**	EXP y^x	/

[48] Due to *ASL*, there is no need for clearing your *WP43* before starting a new calculation – old data are just lifted out of the way when new input is pushed on the *stack*. *ASL* affects all *stack registers* (as ENTER does) and the previous content of the top *register* gets lost again. See *ReMA H* for background information. Almost all commands enable *ASL* – with ten exceptions: CLX, ENTER, Σ+, Σ−, and six *matrix* editing commands. Some reasoning for the first four:

- CLX (called by CLX, or ⬅ for closed *x*) clears **X** to make room for a corrected value instead of the one deleted – and we don't want a useless zero on the *stack*.
- After ENTER↑, you usually want to key in the next input number.
- Σ+ and Σ− are designed for entry of measured, counted, or statistical data – they were not meant to be mixed with normal calculating operations. See the chapters about statistical functions in *Section 2*.

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Note the *ASL* when entering **7** affects two intermediate results here. Thus, everything is well prepared for the exponentiation in the penultimate step and the final division of the numerator y by the denominator x. Voilà!

Following this example, you have observed the five most popular *dyadic* functions in action: [+], [−], [×], [/], and [y^x]. Your *WP43* provides many more (please see the *IOI*).

Note the contents of the *stack* drop whenever a *dyadic* (or *triadic*) function is executed. Like the *ASL*, also this *automatic stack drop* affects all *stack registers* (cf. p. 43 and see pp. 53f for its benefits).

Using the *stack* as demonstrated above, *RPN* completely eliminates the need for any [(], [)], and [=] keys. See the following **example** with a slightly more elaborate formula than above. Find below a way for solving it step by step, and the corresponding *stack* diagrams.

$$2+\sqrt{\frac{1+\left|\left(\frac{30}{7}-7.6\times 0.8\right)^4-\left(\sqrt{5.1}-\frac{6}{5}\right)^2\right|^{0.3}}{\left\{sin\left[\pi\left(\frac{7}{4}-\frac{5}{6}\right)\right]+1.7(6.5+5.9)^{3/7}\right\}^2-3.5}}=\ ?$$

Set [MODE] RAD and start calculating at the red **7** ([ENT↑] stands for [ENTER↑]):

Z						1.75	1.75		
Y		7	7		1.75	5	5	1.75	
X	7_	7	4_	1.75	5_	5	6_	0.83...	0.91...
	7	[ENT↑]	**4**	[/]	**5**	[ENT↑]	**6**	[/]	[−]

								0.25...
				0.25...	0.25...		0.25...	12.4
0.91...			0.25...	6.5	6.5	0.25...	12.4	3
3.14...	2.87...	0.25...	6.5_	6.5	5.9_	12.4	3_	3
[π]	[×]	[TRI]	sin **6.5**	[ENT↑]	**5.9**	[+]	**3**	[ENT↑]

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0.25…								
12.4	0.25…		0.25…					
3	12.4	0.25…	2.94…	0.25…			27.6…	
7_	0.42…	2.94…	1.7_	5.00…	5.25…	27.6…	3.5_	24.1…
7	/	EXP	y^x 1.7	×	+	x^2	3.5	−

24.1… is the solution of the denominator. Let's continue with calculating the numerator now, basically following the same procedure, i.e. calculating inside out (as you would do with pencil and paper):

					24.1…	24.1…			
	24.1…	24.1…		24.1…	6.08	6.08	24.1…		24.1…
24.1…	7.6	7.6	24.1…	6.08	30	30	6.08	24.1…	1.79…
7.6_	7.6	.8_	6.08	30_	30	7_	4.28…	1.79…	4_
7.6	ENT↑	.8	×	30	ENT↑	7	/	−	4

		24.1…	24.1…		24.1…	24.1…			
	24.1…	10.3…	10.3…	24.1…	10.3…	10.3…	24.1…	24.1…	
24.1…	10.3…	6	6	10.3…	1.2	1.2	10.3…	10.3…	24.1…
10.3…	6_	6	5_	1.2	5.1_	2.25…	-1.05…	1.12…	9.24…
y^x	6	ENT↑	5	/	5.1	√x	−	x^2	−

	24.1…		24.1…						
24.1…	9.24…	24.1…	1.94…	24.1…	2.94…			0.34…	
9.24…	.3_	1.94…	1_	2.94…	24.1…	0.12…	0.34…	2_	2.349…
|x| [49]	.3	y^x	1	+ [50]	x≷y	/	√x	2	+

Even solving this formula required only four *stack registers*. There are **no pending operations** – each operation is executed individually, one at

[49] You will hardly execute this step manually since you realize immediately that x is positive. In an automatic evaluation of such a formula, however, this step may be essential.

[50] **X** contains the result of the numerator here. And the result of the denominator silently traveled into **Y** during all the calculations printed above. In the next step, we swap x and y to arrange the operands for dividing the numerator by the denominator.

a time, allowing perfect control of each and every intermediate result. [51]

> This is another characteristic advantage of *RPN*. In many real-life applications, intermediate results carry their own value, so further calculations may depend on the numbers you see there – this is called '*exploratory math*' and may well occur more frequently in your professional work than evaluating textbook formulas (cf. pp. 38ff).

Experienced *RPN* calculator users have determined that by starting every problem at its innermost number or parenthesis and working outwards (as you did with pencil and paper), you maximize the efficiency and power of your calculator.

> **Example** (continued):
> If, instead, you try solving the formula on p. 46 stubbornly from left to right, you will need more keystrokes and more *stack registers*. Try and count yourself!

There may be, however, some problems where four *stack registers* will simply not suffice, regardless of the way you tackle with them:

> **Example:**
>
> $$\frac{(1+2)\,(9+8)\;+\;(3+4)\,(11-6)}{(5-7)\,(10+12)\;-\;(13+14)\,(15+16)}\;=\;?$$
>
> This highly symmetric formula lacks an unambiguous 'inside', so let's begin with the numerator, assuming *SDC* of your *WP43*:

Z						3	3			
Y		1	1		3	9	9	3		51
X	1_	1	2_	3	9_	9	8_	17	51	3_
	1	ENT↑	2	+	9	ENT↑	8	+	×	3

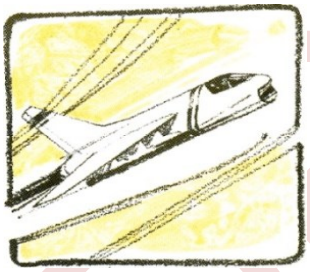
T					51	51				
Z	51	51		51	7	7	51			
Y	3	3	51	7	11	11	7	51		86
X	3	4_	7	11_	11	6_	5	35	86	5_
	ENT↑	4	+	11	ENT↑	6	−	×	+	5

[51] Thus, operator precedence is your job. Look up *App. 2* for confirmation or reminder.

T					86	86				86
Z	86	86		86	-2	-2	86		86	-44
Y	5	5	86	-2	10	10	-2	86	-44	13
X	5	7_	-2	10_	10	12_	22	-44	13_	13
	ENT↑	7	−	10	ENT↑	12	+	×	13	ENT↑

A				86	86				
T	86		86	-44	-44	86			
Z	-44	86	-44	27	27	-44	86		
Y	13	-44	27	15	15	27	-44	86	
X	14	27	15_	15	16_	31	837	-881	-0.097 61…
	14	+	15	ENT↑	16	+	×	−	/

Note that with pressing ENTER↑ in step 4 of last diagram, a *stack overflow* would have occurred in a traditional *4LS* leading to a wrong final result. You would hardly have noticed that.[52] Overflow probability is low but not negligible in such a *stack*.



We close with another real-life **example**:

For many years, solving the following formula for the *Mach number* of an airplane as a function of its *calibrated airspeed* (*CAS*) in *knots* (here 350) and *pressure altitude* (*PA*) in *feet* (here 25 500) was used for demonstrating the simplicity and coherence of *RPN*:[53]

$$\sqrt{5\left(\left[\left\{\left(1+0.2\left[\frac{CAS}{661.5}\right]^2\right)^{3.5}-1\right\}\{1-6.875\times10^{-6}\times PA\}^{-5.2656}+1\right]^{0.286}-1\right)}$$

[52] There is (and will be) no warning for stack overflow: even a watchdog on T will not help since some special advantages of *RPN* require the top *stack level* loaded (see the chapter after next). So you should rather keep SSIZE8 set.

[53] We quote this US-American *Mach-number* formula without having verified it.

The medieval *knot* and *feet* survived in aviation business and navigation, although made obsolete by *SI* for two centuries (see *Unit Conversions* in *Section 5*).

TN for Europeans: *CAS* does not mean *C·A·S*, nor does *PA* mean *P·A* in that formula.

Solve it like this with DISP FIX 3:

	350	350			0.28		0.056	
350	350	661.5	0.529	0.280	0.2	0.056	1	1.056
350	**ENT↑**	**661.5**	**/**	**EXP x²**	**.2**	**×**	**1**	**+**

					0.210	0.210	
1.056		1.210		0.210	6.8…	6.8…	0.210
3.5	1.210	1	0.210	6.875×10^{-6}	6.8…	25 500	0.175
3.5	**y^x**	**1**	**−**	**6.875×10^{-6}**	**ENT↑**	**25500**	**×**

	0.210		0.210					
0.210	−0.175	0.210	0.825	0.210		0.580		
−0.175	1	0.825	−5.265 6	2.759	0.580	1	1.580	
+/−	**1**	**+**	**−5.2656**	**y^x**	**×**	**1**	**+**	

1.580		1.140		0.140			
0.286	1.140	1	0.140	5	0.698	0.836	
.286	**y^x**	**1**	**−**	**5**	**×**	**√x**	

i.e. 84% of the speed of sound.

As you have seen, the way to solve a problem using *RPN* stays the same regardless of its size and complexity. You are always in control.

While a *stack overflow* may (seldom but silently) happen on a *4LS*, you will be obviously on the safe side with an *8LS* (as it is *startup default* on your *WP43*), even dealing with the most advanced mathematical expressions you will meet in your professional life as a scientist or engineer. Promised. The *8LS* warrants absolutely worry-free calculations.[54]

Let's quote a part of the *HP-25 OH* once more, just replacing all strings '*HP-25*' by '*WP43*':

[54] Contriving a case leading to *stack overflow* even for an *8LS* is trivial; though that will be just a contrived case – no real-world formula. This is why there is no need for more than 8 *stack levels*. A so-called 'infinite' *stack* (as featured in *RPL* and on the *HP Prime*) would prohibit some nice shortcuts – see pp. 54f.

Now that you've learned how to use the calculator, you can begin to fully appreciate the benefits of the *Hewlett-Packard* logic system. With this system, you enter numbers using a parenthesis-free, unambiguous method called *RPN* (…).

It is this unique system that gives you all these calculating advantages whether you're writing keystrokes for a *WP43* program or using the *WP43* under manual control:

- *You never have to work with more than one function at a time.* The *WP43* cuts problems down to size instead of making them more complex.
- *Pressing a function key immediately executes the function.* You work naturally through complicated problems, with fewer keystrokes and less time spent.
- *Intermediate results appear as they are calculated.* There are no "hidden" calculations, and you can check each step as you go.
- *Intermediate results are automatically handled.* You don't have to write down long intermediate answers when you work a problem.
- *Intermediate answers are automatically inserted into the problem on a last-in, first-out basis.* You don't have to remember where they are and then summon them.
- *You can calculate in the same order you do with pencil and paper.* You don't have to think the problem through ahead of time.

RPN takes a few minutes to learn. But you'll be amply rewarded by the ease with which the *WP43* solves the longest, most complex equations. With *RPN*, the investment of a few moments of learning yields a lifetime of mathematical bliss.

And calculations with other *data types* (to be introduced in *Section 2*) follow the same simple rules. Thus, at the bottom line, we recommend:

Keep SSIZE8 set and let your *WP43* care for the arithmetic while you care for the mathematics! [55]

[55] With the opportunity of an *8LS*, why are there only four *stack levels* displayed?
Simple reason: Once accustomed to *RPN*, you know how your data move on the *stack*.

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LAST*x* for Reusing Numbers

Your *WP43* saves ***x*** in the special *register* **L** (for LAST***x***) automatically just before the new command is executed.[56] What is your benefit?

Example (from the *HP-15C OH*):

Two close stellar neighbors of Earth are *Rigel Centaurus*[57] (4.3 *light-years* away) and *Sirius* (8.7 *light-years* away). Use the speed of light, c (2.997 925 ×10^{8} m/s, or 9.460 536 ×10^{15} m/*year*), to figure the distances to these stars in *meters*.

Solution (with DISP SCI 01):

4.3	4.3
ENTER↑	4.3
9.460536 E **15** [58]	9.460 536×10^{15}
×	4.1×10^{16}
8.7	8.7

Watching *stack* mechanics reliably working exactly as expected carries no valuable information and will become boring or even distracting very soon.

If, however, you feel annoyed by a cluttered screen displaying more than you really need, use DSTACK (on second view of DISP) to reduce the number of *stack levels* displayed to 3, 2, or even just 1, letting your brains compete with your fellow *RPN* users since 1972.

Free space will flow in top down – ***x*** will always be shown directly above the *menu section*. See here an exemplary display for DSTACK **2**.

And don't be afraid: multi-line output of your *WP43* will be displayed entirely always, regardless of the DSTACK settings you choose.

[56] … as all *RPN* calculators did since the *HP-45*. The few commands operating on ***x*** without loading **L** are mentioned explicitly in the *IOI*.

[57] This is identical to *Alpha Centauri*. *Rigel* usually means a star in the constellation *Orion*.

[58] Noting the precision of the other inputs here, three digits had sufficed (i.e. 9.46 ×10^{15}). But we wanted to demonstrate the advantages of LAST***x***.

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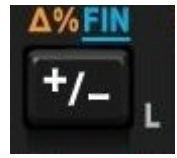
RCL L 9.5×10^{15}

× 8.2×10^{16}



RCL L is reached by pressing RCL, then +/− – note L printed bottom right of +/−.[59]

Result: *Rigel Centaurus* is 4.1×10^{16} m away from us, *Sirius* 8.2×10^{16} m.

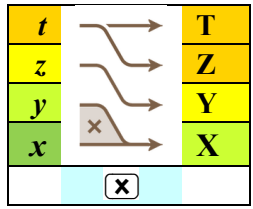
RCL L will often allow reusing lengthy numbers (no need for a second input, avoiding input errors). It may also allow for recalling intermediate results without the need for storing them explicitly elsewhere.

Top Stack Register Repetition

Whenever a *dyadic* or *triadic* function is executed, the *stack* will drop and the content of its top *register* will be repeated – cf. p. 43. You may use this *top stack register repetition* for some nice tricks. See the following compound interest calculation on a *4*LS:[60]

Example:

Assume your bank pays you 3.25% p.a. on an amount of 15 000 US$. What would be your status after 2, 3, 5, and 8 *years*? [61]



Solution:

Here, you are interested in currency values only, so set the display format to DISP FIX 2, causing the output being rounded to cents:[62]

[59] Allocating a dedicated keyboard label to LAST*x* (like on *HP-42S* or *DM42*) doesn't pay on your *WP43* since no keystroke will be saved.

[60] *TNG*: *Compound interest* = Zinseszins.

[61] Those were the days, my friend…! Inflation was balancing those interest rates but saving was definitely more fun then, nevertheless.

[62] Internally, numbers are kept and processed with far higher precision.

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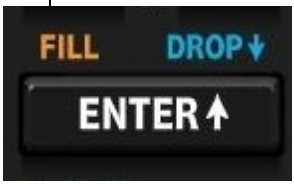
T		1.03	1.03	1.03	1.03	1.03	1.03
Z		1.03	1.03	1.03	1.03	1.03	1.03
Y		1.03	1.03	1.03	1.03	1.03	1.03
X	1.032 5_	1.03	15 000_	15 990.84	16 510.55	17 601.17	19 373.66

1.0325 FILL **15000** [×] [×] [×] [×] [×] [×] [×] [×]

after 2 *years* 3 *years* 5 *years* 8 *years*



Each multiplication consumes x and y for the product $x \times y$ put into **X**, followed by z copied into **Y**, then t into **Z**. Hence the interest rate is kept as a constant on the *stack*, so the computation of the accumulated capital value becomes a simple series of [×] strokes.[63]

Another application benefitting from *top stack register repetition* is the *Horner scheme* for evaluating polynomials. It tells us:

$$p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n$$

$$= \Big(\big((\ldots (a_n x + a_{n-1})\, x + \cdots + a_3 \big)\, x + a_2 \big)\, x + a_1 \Big)\, x + a_0$$

Example:

Solve $7 + 6.4x - 2.1x^2 + 5.2x^3 - 3x^4$ for $x = 0.908$.

Solution:

This problem can be rewritten $\{ [(-3x + 5.2)\, x - 2.1]\, x + 6.4 \}\, x + 7$ and is easily solved this way (set [DISP] FIX [0][1]):

	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9	0.9
	0.9	0.9	0.9	-2.7	0.9	0.9	2.2	0.9	0.1	0.9	0.9
.908_	0.9	-3_	-2.7	5.2_	2.5	2.2	2.1_	0.1	6.4_	5.9	12.9

.908 FILL **3** [+/−] [×] **5.2** [+] [×] **2.1** [−] [×] **6.4** [+] [×] **7** [+]

[63] Debt calculations are significantly more complicated – avoid debts as long as possible, in your very own interest! In the long run, it is better for you and your economy. When debts become inevitably necessary, however, you and your *WP43* can cope with them as well (see *TVM* in *Section 5*).

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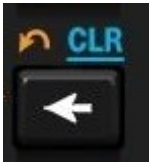
Note how the values float automatically down the *stack* to be used in the multiplications.[64]

Both examples work with the *8LS* as well – just with ***d*** repeated instead of ***t***. [FILL] loads the entire *stack* with ***x*** (regardless of its depth), far easier and more reliable than hitting [ENTER↑] repeatedly.

Error Recovery: ⬅, [EXIT], and [↶]

Nobody is perfect – errors will happen although you are equipped with such a powerful tool. Stay cool – your *WP43* allows undoing the last command executed, restoring the calculator state as it was before that error occurred.

1. If you get an **error message** in response to a function call, read it, and when you know what went wrong, press ⬅ or [EXIT]; this will erase the message and return to the state before that error happened (see pp. 72f and 334f).



2. If you have erroneously called a **wrong function**, just press [↶] to UNDO it immediately. [↶] recalls the state as it was before that wrong operation was executed. [↶] operates on the entire *stack* (be it a *4LS* or *8LS*), statistical storage, and *system flags*. Since your *WP43* features *RPN*, this is all you need for resuming your work as if nothing had happened at all.[65]

Example:
Assume you pressed – while you were watching an attractive fellow student or collaborator – [×] inadvertently instead of [/] in the fourth last step solving the lengthy formula on p. 46. Shhh… !! Murphy's Law!

[64] Try solving such a problem with an 'algebraic' calculator (made by, e.g., *Casio*, *Sharp*, or *Texas Instruments*) for comparison – it will cost you many more keystrokes

[65] [↶] will undo just the very last operation before [↶], nothing more – [↶] [↶] = [↶]. And operations explicitly confirmed can't be undone (see the last chapter of this *Section*).
Previous *RPN* calculators used LAST***x*** for error correction – [↶] works easier and more comprehensive. – Operations on *general purpose registers* or *variables* (see next chapter) may need manual undoing.

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Luckily, however, there is absolutely no need to start calculating that formula all over again – that user error is easily undone as follows:

A	...	789	210	789	210	456	789	210
T	...	456	789	456	789	789	456	789
Z	...	123	456	123	456	456	123	456
Y	...	2.94...	123	2.94...	123	123	0.34...	123
X	...	24.1...	70.9...	24.1...	0.12...	0.34...	2_	2.349...
		x⇄y	⌫	↶	/	√x	2	+
		Fine so far	Oops!	Undo	Resume			Correct result

The entire stack is restored by ↶ as it was before ⌫.

So don't worry – be happy!

That's all you have to know about calculations with *reals* on the *stack*. Such capabilities did suffice for high flying applications already – see below (and note the *HP-35* was absolutely revolutionary at its time – killing the slide rule – but is a primitive scientific pocket calculator from today's point of view, 53 years later; learn more about it in *ReMA H*).

THE HP-35, THE WORLD'S FIRST HANDHELD SCIENTIFIC CALCULATOR, orbited in space aboard three manned *Skylab* missions (1973-74). Solar research figured prominently among the wide assortment of experimental research conducted. Used as a backup to on-board computers, the HP-35 calculated predocking rocket burns necessary to align the Apollo Command Module with *Skylab*. In addition, the HP-35 helped *Skylab* crews aim their telescopes at stars in attempts to measure ultraviolet radiation.

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There are, however, far more places than just the *stack* in your *WP43* where you may store your data and results.

Addressing and Manipulating Objects in *RAM*

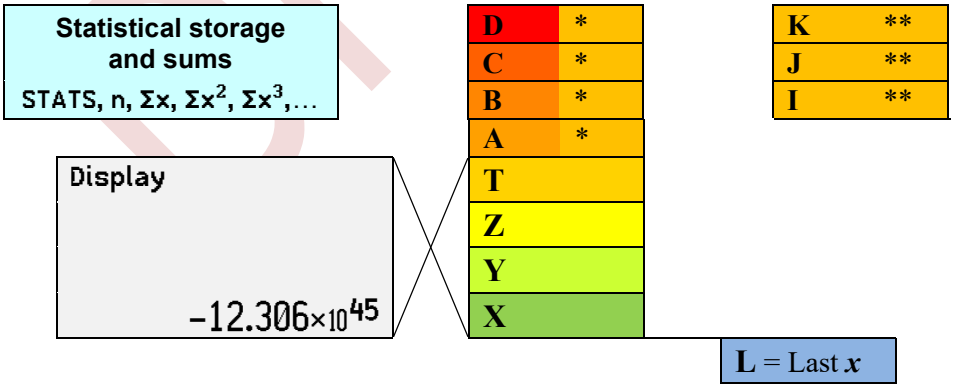
The *stack* provides workspace and temporary storage for your calculations. For longer term storage, feel free to employ additional *registers*, *variables*, and *flags*. The remaining chapters of *Section 1* will tell you how to use them.

Sketches below and on the page after next show the address space of your *WP43*. Depending on the way you configure and use its memory, a subset of all these addresses will be accessible for you.

Depending on the ***stack*** size you choose, either **T** or **D** will be the top *stack register*. **A** - **D** (*) will be allocated to a big *stack*, if applicable.

I, **J**, and **K** (**) may carry parameters of probability distributions; **I** and **J** will also serve as *index pointers* in *matrix* operations (see *Section 2*). **K** may be used as default *alpha register* in some special cases as well (see *Section 3*).

Special *registers* and *stack*



Unless required for said purposes, you may use **A**, **B**, **C**, **D**, **I**, **J**, and **K** as additional global ***general purpose (GP) registers***.

Statistical data can be stored value by value, point by point, or *en masse* in a *matrix* called STATS. The statistical sums will be accumulated automatically in a set of dedicated *summation registers*, separated from your other data (see *Section 2*).

See the sketch overleaf:

Each ***GP register*** (like each *stack register*) can hold any object you store therein (see *Section 2*). *GP registers* are beneficial, e.g., keeping intermediate results for repeated use or serving as records.

Example (assuming *SDC*):

$$\sqrt{3+\left(\frac{1.09}{1.78}\right)^2}\times\frac{ln\left[3+\left(\frac{1.09}{1.78}\right)^2\right]}{4\,cos\left[3+\left(\frac{1.09}{1.78}\right)^2\right]}=\ ?$$

Solution:

First, calculate the repeating term $3+\left(\frac{1.09}{1.78}\right)^2$ and store it:

1.09 ENTER↑ **1.78** /	6.123 595 505 617 978×10⁻¹
EXP x^2 **3** + STO K	3.374 984 219 164 247

Then solve the entire expression, e.g., like this:

$\sqrt{x}$	solves the 1st factor of the expression,
RCL K ln	solves the numerator,
×	2.234 647 088 154 349
RCL K TRI cos	solves the 2nd part of the denominator,
/	2.238 529 534 683 649
4 /	5.596 323 836 709 123×10⁻¹

That's it – solving this expression has become really easy this way.

A ***flag*** is an *item* featuring only two states, *set* and *clear* (like a bit). Employ *user flags* for signaling whatever you want.[66] *Flags* are most useful for program control – learn more about them in *Section 3*.

[66] *System flags,* on the other hand, carry *names* and reflect or control specific system states – see *ReMS 1*. Single-letter *flags* serve as shortcuts to some *system flags* as specified in the picture overleaf.

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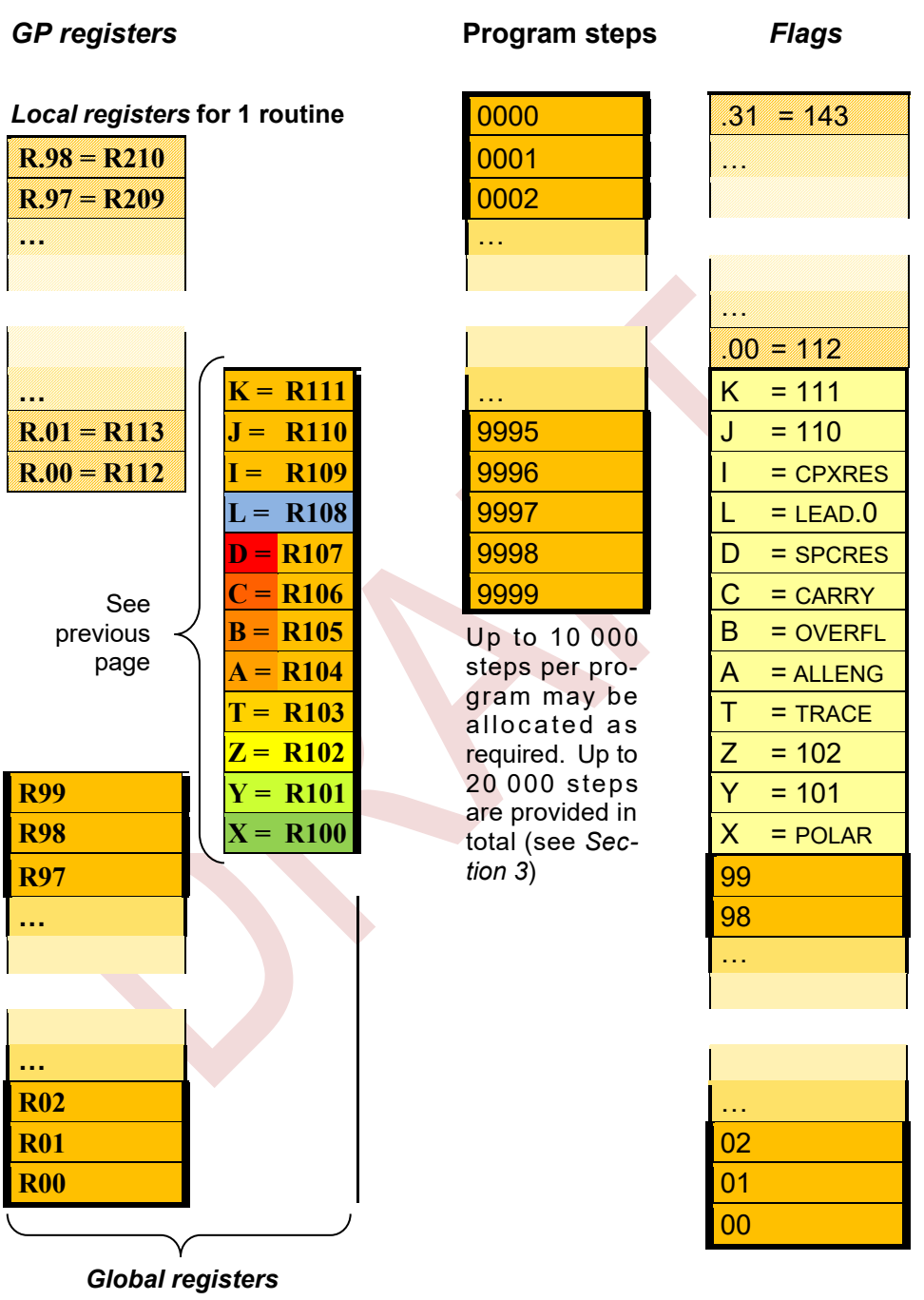
GP registers
Program steps
Flags
Local registers for 1 routine
R.98 = R210
R.97 = R209
…
…
R.01 = R113
R.00 = R112
K = R111
J = R110
I = R109
L = R108
D = R107
C = R106
B = R105
A = R104
T = R103
Z = R102
Y = R101
X = R100
See previous page
R99
R98
R97
…
…
R02
R01
R00
Global registers
0000
0001
0002
…
…
9995
9996
9997
9998
9999
Up to 10 000 steps per program may be allocated as required. Up to 20 000 steps are provided in total (see Section 3)
.31 = 143
…
…
.00 = 112
K = 111
J = 110
I = CPXRES
L = LEAD.0
D = SPCRES
C = CARRY
B = OVERFL
A = ALLENG
T = TRACE
Z = 102
Y = 101
X = POLAR
99
98
…
…
02
01
00

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Variables are named global storage locations. Such *names* shall begin with a letter and may consist of up to 7 characters. Creating a new *name*, feel free to use all characters except **+**, **−**, **×**, **/**, **^**, **(**, **)**, **=**, **|**, **!**, **:**, **;**, comma, and space. Note all ***variables* (and *menus*) must carry unique *names***. Your *WP43* decodes *names* as follows:

- Upper and lower case letters are interpreted differently (e.g., **X** ≠ **x**, **CHECK** ≠ **Check** ≠ **check**).
- Superscript and subscript characters are read as normal characters (e.g., **Var**^{1} = **Var**$_{1}$ = **Var1**); nevertheless, *names* will be always displayed as created, for easier reading).

Thus, check the *item names* provided already (compiled in *ReMA N*) for conflicts. In particular, do not attempt to call your *variables* **X**, **Y**, **Z**, or **T** since these *names* are reserved for the bottom four *stack registers* – but feel free to create *variables* named **x**, **y**, **z**, **t**, **XX**, etc. And since *items* may appear in *menus*, we recommend that no *names* share their first six characters.

Like each *register*, also each *variable* can hold any type of data. Explicitly clearing a *RoV* is often dispensable because you can just overwrite its content.

While processing input for memory addressing (i.e. entering parameters for, e.g., comparing, storing, recalling, or copying), you will only need a subset of 30 labels plus 🟨 and 🟦. The calculator mode supporting just these 30 is called *transient alpha mode* (*TAM*). It is automatically set in memory addressing. Within *TAM*, the operational keyboard of your WP43 is temporarily reassigned as shown overleaf.

This kind of a picture is called a *virtual keyboard.* Dark red background highlights modified active functions in *TAM.* White print denotes *primary functions* here as well, such as the 1st key in row 2 directly addressing (possibly *stack*) *register* **A**.[67] Also all the other lettered *registers* can be called directly in *TAM* – *stack registers* **X**, **Y**, **Z**, and **T** via unshifted *softkeys*.

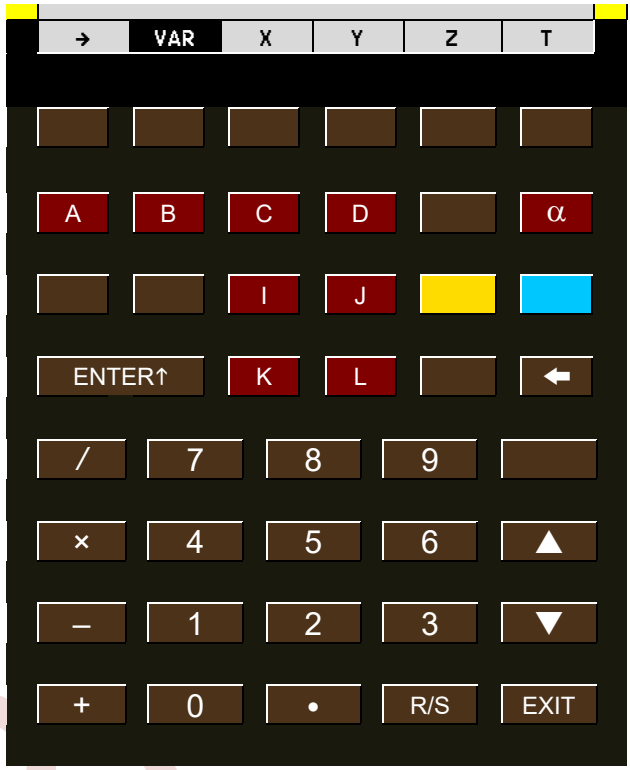
Access to numbered *registers* stays as easy as can be. [→] is for indirect addressing (see pp. 63ff), [.] for local addresses (see p. 254). What

[67] What is printed in white on your physical calculator is called a *default primary function*.

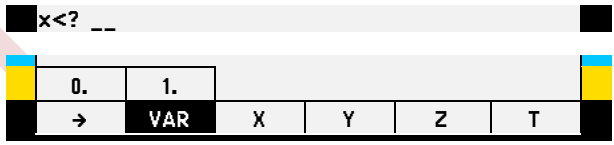
is not printed on this *virtual keyboard* cannot be called in *TAM*.

Variables defined at execution time show up in VAR in alphabetical order, so you can easily select the *variable* of your choice by pressing the respective *softkey*. Alternatively, you can address it via [α] (or create a new *variable* this way): see pp. 63f for how to do this.

🟨 and 🟦 are required just for *softkeys* in *TAM*.

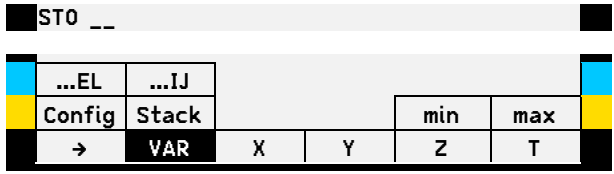


Menus are context sensitive. If a **comparison** is called, e.g., x<?, the *menu* will look like this:



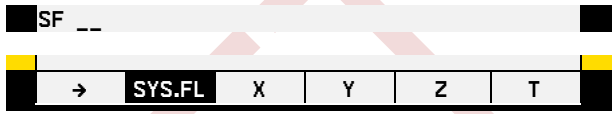
This allows for directly comparing x with **0.** or **1.** as well as with the content of any *RoV* (the latter via VAR).

If [STO] **or** [RCL] are called, on the other hand, the *menu* will look like this instead:



Save all your specific calculator settings (a.k.a. its *configuration*) easily in a *RoV* via STO **Config** and recall them via RCL **Config** (see p. 86). Use STO **Stack** to store the entire *stack* in a block of 4 or 8 *registers* (depending on *stack* size at execution time); RCL **Stack** will recall it when you need it. And **max** (or **min**) lets you work with the maximum (or minimum) of x and the source content – you may use ▲ as shortcut for **max** and ▼ for **min** here.[68]

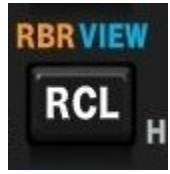
For commands operating on ***flags***, **SYS.FL** will grant access to the *system flags* provided:



For commands operating on ***labels***, see *Section 3*. For all other operations asking for a trailing parameter, the *menu view* will stay as pictured on previous page.

TAM will be terminated as soon as sufficient characters are entered for the parameter(s) of the respective operation. You may undo pending input keystroke by keystroke using ← and correct it – or just abort the pending command by EXIT; the latter will leave *TAM* immediately, returning to the mode set and the *menu* displayed before, if applicable.

For just inspecting any *RoV* without disturbing the *stack*, use VIEW.



Example:

DISP FIX 5

VIEW K returns

```
K= 3.374 98
                0.000 00
                0.000 00
                0.559 63
```

… as expected from previous problem.

Note the view into *register* **K** is displayed adjusted to the left just below the *status bar*. This view will vanish with your next keystroke.

[68] For **...EL** and **...IJ** see *Section 2*.

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For inspecting a set of *registers*, use RBR instead. Call FLAG STATUS for checking the status of all *flags.* See pp. 283ff for more.

Note you are granted unlimited access to all the *registers* and *user flags* allocated; there is nothing like '*memory protection*' on your *WP43*. You are the sole and undisputed master of its memory. Thus, taking care of it is your responsibility as well – keep suitable records to avoid inadvertently overwriting or deleting your precious data.[69]

You will not, however, get 20 000 program steps, access to 211 *registers*, and 144 *user flags* all at the same time – see *ReMA B*.

Addressing Tables

Parameterized Comparisons:

1	User input Echo	TEST x<?, x≤?, x=?, x≈?, x≠?, x≥?, x>? OP? _ (with *TAM* set), e.g., x<? __				
2	User input Echo	0. or 1. OP *n*? e.g., x=0.?	*Stack* or *lettered register*[70] *or variable defined*[71] OP? *x* e.g., x≥? Y	*Register number* (range as specified on p. 66) OP? *rnn* e.g., x≠? r23	→ opens indirect addressing OP? → _	α turns on *AIM* for typing a (new) *variable name* OP? '_
3	User input Echo	Compares *x* with the real number **0**.	Compares *x* with *y*.	Compares *x* with the content of *register* **R23**.	See pp. 66ff for more about indirect addressing.	*Var. name*[72] OP? '*xx*' e.g., x>? 'ST1'

[69] You will learn about *local data*, preventing your programs from interfering with data of other programs, in *Section 3*. This will ease your administration.

[70] *SOLR*, i.e. Y, Z, T, A, B, C, D, L, I, J, or K.

[71] We recommend calling an existing *variable* via VAR instead of keying in its name.

[72] Cf. p. 60. The 7th input character will terminate entry and close *AIM* – shorter *names*

x>? 'ST1' compares *x* with the content of the *variable* '**ST1**'.
Press [TEST] x>? [α] [S] [T] [🟧] [1] [ENTER↑] for this.

Register operations requiring just one *RoV* trailing:

1 User input	[RCL], [STO], [VIEW], [x≷], [y≷], [z≷], [t≷], [DEC], [DSE], [DSL], [DSZ], [INC], etc.			
Echo	OP _ (with *TAM* set), e.g., RCL __ [73]			
2 User input	*SOLR* [74] or *variable defined* [71]	*Register number* (range as specified on p. 66)	[→] opens indirect addressing where applicable (s. pp. 66f)	[α] turns on *AIM* for typing a (new) *variable name*
Echo	OP *x* e.g., DEC K	OP *nn* e.g., VIEW 10	OP → _	OP '_
	Decrements *k*.			
3 User input		*Register #* (range as specified on p. 66)	*SOLR* [74] *or variable defined* [71]	*(New) variable name* [72] (cf. p. 60)
Echo		OP → *nn* e.g., STO →45	OP → *x* e.g., x≷ →Z	OP '*xx*' e.g., INC 'Zähler1'
	Stores *x* in the location where *r45* is pointing to.	**		***

need a closing [ENTER↑].

Attempts to write in a *variable* undefined at execution time will create it automatically, containing zero initially. Attempts to read from an undefined *variable* will throw an error.

Examples: Let **UNDEF** be an undefined variable. Then RCL **UNDEF** will throw an error while STO **UNDEF** or INPUT **UNDEF** will create **UNDEF**.

[73] For [RCL] and [STO] exclusively, any of [+], [−], [×], [/], [▲], or [▼] may precede step 2. Entering such a key twice will cancel it (e.g., [RCL] [×] [×] = [RCL]). See the chapter after next for this *store and recall arithmetic*. Note such operators are disallowed in [RCL] Config (= RCLCFG), [RCL] Stack (= RCLSTK), RCLEL, RCLIJ, and the corresponding store operations.

[74] i.e. [X], [Y], [Z], [T], [A], [B], [C], [D], [L], [I], [J], or [K].

** Swaps *x* and the content of the register where *z* is pointing to.

*** Increments the *variable* called **Zähler1**.

Clearing a single *variable* or *register* is most easily done by storing zero in it. For deleting a *user variable* from memory and freeing the space allocated to it use DELITM instead (see p. 316).

Other operations requiring one trailing parameter:

1	User input	[ALL], [FIX], [SCI], [ENG], [ASR], [CB], [FB], [SB], [BestF], [CF], [FF], [SF], [DSTACK], [ERR], [LocR], [PAUSE], [RM], [SIM_EQ], [TDISP], [TONE], [WSIZE], [→INT], bit and *flag* tests, etc. (see the *IOI* for a complete list)		
	Echo	OP _ (with *TAM* set), e.g., FIX __		
2	User input	*Lettered flag*[74] or *system flag*[75] or *variable defined*,[71] if applicable	*Flag or bit number (#)* or *# of decimals* or any other # applicable (see p. 66 for valid ranges)	[→] opens indirect addressing where applicable (see pp. 66f and the *IOI*)
	Echo	OP *x* e.g., SF I	OP *nn* e.g., SCI 12	OP → _
		Sets *flag* 109.		
3	User input		*Register number* (range as specified on p. 66)	*SOLR*[74] or *variable defined* [71]
	Echo		OP → *nn* e.g., DSTACK →12	OP → *x* e.g., FIX →A
			Shows as many *stack registers* as specified in **R12** (see pp. 66f).	Sets fix point format with the number of decimals stored in **A**.

[75] See *ReMS 1*. We recommend calling *system flags* via their shortcuts or via SYS.FL.

For…	…the valid number range is…	
registers	0 - 99	for direct addressing of global *registers*
	.0 - .98	for direct addressing of local *registers*
	0 - 210	for indirect addressing (≥ 112 with local *registers* only)
		upper limits depend on current allocation
user flags	0 - 99	for direct addressing of global numbered *flags*
	.0 - .31	for direct addressing of local numbered *flags* if at least one local *register* is allocated
	0 - 143	for indirect addressing (≥ 112 with local *flags* only)
decimals	0 - 15	(entering any digit except 0 or 1 will terminate input and close it)
integer bases	2 - 16	
bit numbers	0 - 63	
word size	1 - 64 *bits*	

Note you may key in 5 ENTER↑ instead of 0 0 5, for instance.

Registers and *flags* 100 - 111 can be directly addressed by single letters.

System flags can be called by their *names*; one-letter shortcuts exist for some of them (cf. p. 59).

Variables are called by their *names* always.

Please see *ReMS 1* for all other command parameters and their valid ranges, as well as for a compilation of all *system flags*.

Indirect Addressing – Working with Pointers

Parameters for many functions can also be specified by supplying a *RoV* pointing to the respective parameter.

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Example (assuming *SDC*):

Assume x = **12.34**, j = **45.67**, and $r12$ = **7.891**. Then…

STO J

RCL → J will return 7.891 since **J** is containing **12.34** at execution time and thus is pointing to **R12**. And then…

SF → X will set *flag* 7, and...

DISP FIX → X will display 7.891 000 0 showing 7 decimals.

Since the content of the *register* specified is not used directly but as a pointer to the *register* wherefrom we want to read (or whereto we want to write), this method is called indirect addressing.

Each and every *register* of your *WP43* can be used for indirect addressing.[76] And each and every *register* can be accessed this way. This applies to *local registers* as well.

Indirect addressing is most beneficial in programs with the parameter for a function calculated automatically (see examples in *Section 3*). It may be even combined with *store and recall arithmetic* as explained below, and become mightier yet.

Store and Recall Arithmetic

As mentioned in f. 73 on p. 64, arithmetic operations (and two conditional picks, *max* and *min*) can be performed upon the contents of *registers* or *variables* by pressing STO or RCL followed by the respective operator (+, −, ×, /, ▲, or ▼), trailed by the address or *name* of the storage space.

Example for *store arithmetic*:

123.4

STO − K closes input and subtracts **123.4** from k. The difference is stored in **K**. The *stack* and **L** remain unchanged here.

[76] Several vintage calculators, on the opposite, featured just one dedicated *register* for indirect addressing, if at all. See the *HP-34C* or *HP-15C*, for instance. Even the *HP-15C CE* of 2024 still features only one register for this purpose.

The same result could be achieved by the keystroke sequence

123.4
[RCL] [K]
[x≷y]
[−]
[STO] [K]
[RCL] [L] but this is far clumsier (replacing one step by five) and would cost one *stack register* in addition.

General rule for *store arithmetic*:

$$\text{new content of the } RoV \text{ specified} = \text{old content of this } RoV \begin{Bmatrix} + \\ - \\ \times \\ / \\ max \\ min \end{Bmatrix} \text{current } x$$

Example (from the *HP-67 OHPG*):
During harvest, farmer *Flem Snopes* trucks tomatoes to the cannery for three days. On Monday and Tuesday he hauls loads of 25 *tons*, 27 *tons*, 19 *tons*, and 23 *tons*, for which the cannery pays him $55 per *ton*. On Wednesday the price rises to $57.50 per *ton*, and *Snopes* ships loads of 26 *tons* and 28 *tons*. If the cannery deducts 2% of the price on Monday and Tuesday because of blight on the tomatoes, and 3% of the price on Wednesday, what is *Snopes'* total net income? (Note the truck used by *Flem* in 1976 looks like *Ford's Model T* of 1910. A frugal farmer.)

Solution:

[DISP] FIX [2]		Standard in financial calculations
25. [ENTER↑] **27** [+]		Total of Monday's and Tuesday's
19 [+] **23** [+]	94.00	tonnage
55 [x]	5 170	Gross amount for these days
[STO] [J]	5 170	Take **J** for accounting
2 [FIN] %	103.40	Deduction for these two days
[STO] [−] [J]	103.40	Subtracted from the total in **J**

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26. ENTER↑ **28** +	54.00	Wednesday's tonnage
57.5 ×	3 105.00	Gross amount for Wednesday
STO + J	3 105.00	Added to the total in **J**
3 %	93.15	Deduction for Wednesday
STO − J	93.15	Subtracted from the total in **J**
RCL J	8078.45	Snopes' total net income from his tomatoes

In analogy to *store arithmetic*, *recall arithmetic* is provided as well.

Example for *recall arithmetic*:

78.91

RCL / 23 closes numeric input and divides **78.91** by ***r23***. This operation is performed in **X** alone. **L** is loaded with **78.91**. All other *stack* contents and ***r23*** stay unchanged.

Alternatively, the same result could be achieved by the sequence

78.91
ENTER↑
ENTER↑
RCL 23
/
STK y≷ L
DROPy but that would replace one step by six and cost two additional *stack registers*.

General rule for *recall arithmetic*:

$$\text{new } x = \text{old } x \left\{ \begin{matrix} + \\ - \\ \times \\ / \\ max \\ min \end{matrix} \right\} \text{ content of the } RoV \text{ specified}$$

Stack-wise, both store and recall arithmetic work like *monadic* functions. They may operate on each and every *RoV* provided, also on the *stack* and even on **L**.

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Example:

What does RCL − Y ?

L	old value	1.0

Z	3.2	3.2
Y	2.1	2.1
X	1.0	−1.1
		RCL − Y

Register arithmetic is most beneficial in automatic routines. *Indirect addressing* may be combined with it, then looking like STO × → K, for instance.

Although both these techniques have been more important in times when *program memory* was scarce and processor speed was low, they may be still beneficial today, in particular when you are out to create compact routines executing quickly. See *Section 3*, pp. 230ff, for examples and advantages, and look up the *IOI* for more information.

Clearing and Resetting

Besides ← and ↶, various other ways are feasible to remove obsolete, erroneous, or unwanted information from your *WP43*. CLR contains the following:

CLX	Clears the *stack register* X (i.e. resets it to zero)[77]	CLSTK	Clears all *stack registers* allocated (but not L)
CLΣ	Clears all statistical data and returns freed space	CLREGS	Clears all global and local *GP registers*
CF	Clears the *flag* specified	CLFall	Clears all numbered *flags*
CLP	Clears the *program* specified	CLPall	Clears all programs[78]

[77] You can as well use DROP↓ to get rid of x. See the *IOI* for the subtle differences.

[78] CLKEYS, CLREGS, CLFall, CLPall, CLMall, CLVall, CLall, DelVall, DelMall, and RESET ask for your confirmation. They can't be undone by ↶. – Note the *stack* and *variables* are not touched by CLREGS. Find more about *GP registers* in next chapter.

CLCVAR	Clears all user *variables* of the *current program*	**CLVall**	Clears all user *variables*
CLMyM	Clears MyMENU	**CLMall**	Clears all *user menus*
CLMyP, **CLMyα**	Resets MyPFN or Myα to their contents in *SDC*	**DelVall**, **DelMall**	Deletes all *user variables* or *menus* from memory

CLall Clears all programs and data in *RAM* (*variables*, numbered *user flags*, assignments, and all *registers* incl. the *stack*)[79]

CLLCD Clears the entire screen NE of the coordinates specified

RESET Resets your *WP43* to *SDC* (just the contents of *flash memory* will stay untouched)[80]

CLKEYS Clears all key assignments

DelITM Deletes a user-defined *item*

CLMENU Clears the *programmable menu*

For your reference, the *startup default* settings of your *WP43* are:

> 2COMPL, ALL **00**, DEG, DENMAX **9999**, DSTACK **4**, GAP **3**, HIDE **0**, J/G **1752.0914**, LinF, LocR **0**, RM **0**, TDISP **0**, and WSIZE **64**. RANGE is set to 999.
>
> The *system flags* AUTOFF, DECIM., DENANY, ENDPMT, MULT×, PROPFR, SSIZE8, TDM24, YMD, and αCAP are set, all others are clear.

Turn to *ReMS 1* for more information about the *items* mentioned above.

Up to here you have learned how to address *registers*, *flags*, and *variables* in your *WP43*. Various methods for accessing these storage places have been shown. Such calculatory capabilities did suffice for orbiting the Earth and controlling spacecraft docking maneuvers.

There are, however, far more objects than just *reals* you can manipulate in your *WP43* – let's introduce them to you.

[79] Display formats as well as other user settings will remain unchanged. Only RESET clears everything except the contents of *flash memory* (see *Sections 3* and *6*).

[80] If you can't execute the command RESET for any reason whatsoever, a hard reset will do almost the same: use the **RESET** hole on the calculator backside if necessary.

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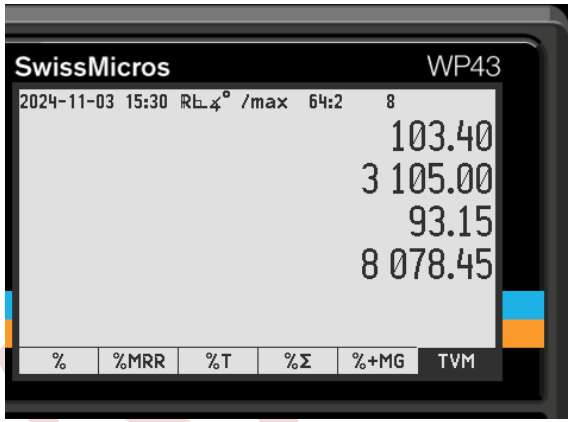
SECTION 2: DEALING WITH VARIOUS OBJECTS AND DATA TYPES

Some Display Basics

The screen is your window to your *WP43* – there you see the latest results and what is going on. Top down, you find ...

- the *status bar*,
- up to 4 rows of *standard numeric output* (and more), and
- the *menu section* displaying up to 3 rows of *items* (cf. pp. 31ff).



Let's look at the four numeric rows top down first:

1. For *startup default* DSTACK **4**, the **top (T) numeric row shows** *t*. It will display the output of SHOW (see p. 150) instead, if applicable.

   Its left side is for VIEW (cf. p. 62) and for echoing command input until it can be executed (with all required parameters entered); cf. p. 27. You may edit a pending command keystroke by keystroke using ⬅, or cancel it by EXIT (cf. p. 62).

2. For DSTACK **4** or **3**, the **2nd row from top shows** *z*. It may also be used by SHOW if the top row doesn't suffice.

   Its left side is for any error message or output of a binary test, or for ***temporary information*** **(*TI*)** heading *z*, if applicable.

   TI is any displayed information exceeding the plain contents of **X**, **Y**, and **Z** in *RUM*. *TI* will vanish with next keystroke:

   - If it is an error message, ⬅ or EXIT will just clear it (cf. p. 55) – any other keystrokes will be executed in addition; else also ⬅ and EXIT will execute.

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- [f][f] and [g][g] will remove any *TI* from the screen most easily and free of side effects. [81]

3. For DSTACK **4**, **3**, or **2**, the **3rd row from top shows y**. It may also be used by SHOW if the top rows don't suffice.

   Its left side is for *TI* heading y, if applicable.

4. The **bottom numeric row (X) shows x**. It may also be used by SHOW if the top rows don't suffice.

   Its left side is for...

   a. *TI* heading x, if applicable,

   b. echoing numeric or text input until it will be closed (cf. p. 25 and see pp. 205ff). X can take up to 42 digits, a sign, and a radix mark in *SDC* or some 40 alphanumeric characters depending on their widths. You may edit pending input keystroke by keystroke using ←. Numeric input will be checked and interpreted according to the calculator settings as soon as it is closed.

Data Types Supported

Your *WP43* can do more for you than just handling *reals*: it can also deal with *integers*, *fractions*, *complex numbers*, *angles*, *times*, and *dates* in various formats. And it can handle *real* and *complex vectors, matrices,* and *text strings* as well. All these *data types* are covered in dedicated chapters below (all the *data types* available are compiled in *ReMA B*).

But how shall your *WP43* learn about the specific meaning of your input? Some input **examples** will show (each presuming *SDC*):

[81] Also if you choose displaying less than 4 *stack levels* (see DSTACK in the *IOI*), echoes and *temporary information* will nevertheless appear at the positions specified, whenever applicable. And operations resulting in multi-row output will always display their entire output regardless of the DSTACK setting.

Input	Display	Meaning
12345.678901 EXIT	12 345.678 901	*Reals* (∈ ℝ), see pp. 85ff
12.3 E **45** ENTER↑	12.3×10^{45}	
901.23.4567 ENTER↑	> 901 $^{23}/_{4}$ 567	*Fraction* (∈ ℚ), see pp. 136ff
270 (→∠) →MULπ	1.5π	*Angle* shown in *multiples of* π
123.45678901 d.ms	123°46' 7.89"	*Angle* entered in *sexagesimal degrees*. See pp. 139ff for further angular displays also supported.
12345678901 ENTER↑	12 345 678 901	*Integers* (∈ ℤ) of various bases and lengths, see pp. 148ff
1234567890 # H	$12\ 34\ 56\ 78\ 90_{16}$	
10100110111 # 2	$101\ 0011\ 0111_{2}$	
12.3 CC **4.56**+/- EXIT	12.3−i×4.56	*Complex numbers* (∈ ℂ) in rectangular or polar notation; mantissa plus exponent format is settable as well; see pp. 166ff
	12.3∠−4.56°	
1.020314235 .d	0001-02-03 14:23:50	*Extended date*, see pp. 199ff
13.345678901 h.ms	13:34:56.789 01	Sexagesimal *time*, see pp. 202ff

Some of these inputs may be displayed differently depending on particular *system flag* or mode settings. Displays in *SDC* are printed below in light blue, further widespread formats in light grey fields:

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	DECIM. set	~~DECIM.~~
12345.678901 EXIT		
GAP 4	1 2345.6789 01	1 2345,6789 01
GAP 3	12 345.678 901	12 345,678 901
GAP 2, 1, or 0	12345.678901	12345,678901

12.3 E **45** ENTER↑		
MULT×	12.3×10^{45}	$12{,}3\times10^{45}$
~~MULT×~~	$12.3\cdot10^{45}$	$12{,}3\cdot10^{45}$

12.3 CC **4.56** +/- EXIT		
MULT×	12.3−i×4.56	12,3−i×4,56
~~MULT×~~	12.3−i·4.56	12,3−i·4,56

13.345678901 h.ms		
TDM24	13:34:56.789 01	13:34:56,789 01
~~TDM24~~	1:34:56.789 01pm	1:34:56,789 01pm

	Y.MD	D.MY	M.DY
1.020314235 .d	0001-0…	01.02.0314 23:50:00	01/02/0314 11:50:00pm

Obviously, your *WP43* reacts to your input very flexibly. And you will recognize the various *data types* and formats easily.

Knowing these now, how can you mix data of various types in calculations? Ten (not only numeric) *data types* of *WP43* are listed in column 1 of the matrix below. It shows what will happen when you combine them: an object of the *DT* as indicated in one of the narrow columns at right (y) plus or minus an object of the *DT* in column 1 (x) will return an object y +/− x of the *DT* printed at the intersection (thus, wherever a *DT* is shown there, the corresponding combination is legal for addition or subtraction – else **Illegal input data type for this operation** will be thrown). Grey fields point to restrictions, yellow fields mark asymmetries.

x \ *y*	1	2	3	4	5	6	7	8	9	10
1 ℤ *Long integer* [82]	**1**	2	3	4	5	6	7	8	9	10
2 ℝ *Real number*	2	**2**	3	4	5	6	7	8	9	2
3 ℂ *Complex number*	3	3	**3**	—	—	—	7	9	9	3
4 *Angle* (in various formats) [83]	4	4	—	**4**	—	—	7	—	—	4
5 *Time* interval (in H.MS)	5	5	—	—	**5**	6[84]	7	—	—	5
6 *Date* (in various formats)	6[85]	6[85]	—	—	6[84]	**2**[86]	7	—	—	6[85]
7 *Text string* [87]	7	7	7	7	7	7	**7**	7	7	7
8 *Real matrix* or *vector*	8	8	9	—	—	—	7	**8**[88]	9[88]	—
9 *Complex matrix* or *vector*	9	9	9	—	—	—	7	9[88]	**9**[88]	—
10 *Short integer* [82]	1	2	3	4	5	6	7	—	—	**10**

Examples:

A *complex number* (*DT* 3) plus or minus a *real* (*DT* 2) will result in a *complex number*. And a *text string* (*DT* 7) plus a *time* (*DT* 5) will append the time string as displayed to said *text string* in small font. See the respective chapters below.

The following matrix shows the resulting *DT* of products and ratios in analogy (note that neither *dates* nor *text strings* can be multiplied or

[82] If *integers* of different types (*DT* 1 and 10) or different bases are combined, output will be an *integer* of the type and base given in **Y**.

[83] Angular output (e.g., of ARCSIN) will be tagged according to the current *ADM*.

[84] A *time* plus a *date* returns an *extended date* (see below). Minus is not supported.

[85] A *date* ***y*** plus (minus) a number ***x*** adds (subtracts) the respective number of decimal *days* to (from) said *date*. The result will be an *extended date* – it may contain a time of day. A number ***y*** plus a *date* ***x*** works alike. A number minus a *date* is not supported. See the chapters about *dates* and *times* below. Consult *ReMA B* for *date* limits.

[86] A *date* minus a *date* returns a number of decimal *days*. Plus is not supported.

[87] Only additions are supported: First, any numeric ***x*** or ***y*** will be converted to a *text string* according to the display format at execution time; for a *matrix*, its descriptor will be used (e.g., **[3×4 ℂ Matrix]**, see further below). String ***x*** will be appended to string ***y*** then.

[88] Matrix dimensions must match (see *Vectors & Matrices: Calculating* further below).

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divided, and actually some combinations are physically forbidden – cf. f. 27 on p. 33):

… **times** an object x of the *DT* below returns a **product** of the *DT* printed at the intersection.	An object y of *DT* … 1	2	3	4	5	8	9	10
1 ℤ *Long integer* [82]	**1**	2	3	4	5	8	9	10
2 ℝ *Real number*	2	**2**	3	4	5	8	9	2
3 ℂ *Complex number*	3	3	**3**	3 [89]	—	9	9	3
4 *Angle* [89]	4	4	3	**2**	—	8	9	4
5 *Time* interval	5	5	—	—	—	—	—	5
8 *Real matrix* or *vector*	8	8	9	8	—	**8**[88]	9[88]	8
9 *Complex matrix* or *vector*	9	9	9	9	—	9[88]	**9**[88]	9
10 *Short integer* [82]	1	2	3	4	5	8	9	**10**

… **divided** by an object x of the *DT* below returns a **ratio** of the *DT* printed at the intersection.								
1 ℤ *Long integer* [82]	**1/2**[90]	2	3	4	5	8	9	10
2 ℝ *Real number*	2	**2**	3	4	5	8	9	2
3 ℂ *Complex number*	3	3	**3**	3	—	9	9	3
4 *Angle* [89]	2	2	3	**2**	—	8	9	2
5 *Time* interval [91]	2	2	—	4	**2**	—	—	2
8 *Real matrix* [92]	8	8	9	8	—	**8**[88]	9[88]	8
9 *Complex matrix* [92]	9	9	9	9	—	9[88]	**9**[88]	9
10 *Short integer* [82]	1	2	3	4	5	8	9	**10**

[89] A red number indicates a physically illegal combination, frowningly allowed here for those who can't get accustomed to strong *data types*. Convert using [.d] where necessary (see three pages below).

[90] E.g., **15** [ENTER↑] **3** [/] returns integer 5 while **14** [ENTER↑] **5** [/] will return 2.8.

[91] The time interval will be converted to decimal *hours* before division.

[92] The *matrix* x must be invertible. Dividing by x is equivalent to multiplying times the inverted *matrix* x^{-1} (see *Vectors & Matrices: Calculating* further below).

Following tables are for powers and roots:

… raised to a power of $x \geq 0$ of the *DT* below returns a result of the *DT* printed at the intersection.	A number $y > 0$ of *DT* … 1	2	10
1 or 10 (i.e. *long* or *short integer*)[82]	1	2	10
2 ℝ *Real number*	2		
… raised to the power of $x < 0$ (with x of *DT* 1, 2, or 10[82]) returns a *real*.	2 (1 / 10 for 1^{-1} only)		
The xth root ($x > 0$ of the *DT* below) of y returns a result of the *DT* printed at the intersection.			
1 or 10 (i.e. *long* or *short integer*)[82]	1[93] / 2	2	10[93]
2 ℝ *Real number*	2		

… raised to a power of $x \geq 0$ of the *DT* below returns a result of the *DT* printed at the intersection.	A number $y < 0$ of *DT* … 1	2	10
1 or 10 (i.e. *long* or *short integer*)[82]	1	2	10
2 ℝ *Real number*	2 / 3[94]		
… raised to the power of $x < 0$ (with x of *DT* 1, 2, or 10[82]) returns a result of *DT*:	2 / 3[94] (1 / 10 for $(-1)^{-1}$ only)		
The xth root ($x > 0$ of the *DT* below) of y returns a result of the *DT* printed at the intersection.			
1 or 10 (i.e. *long* or *short integer*)[82] x odd	1[93] / 2	2	10[93]
2 (or 1 or 10 with even x)	3[94]		

Any number of *DT* 1 or 2 raised to *complex* power will return a *complex number*, as well as any *complex number* raised to arbitrary power. Raising a *short integer* to *complex* power is not supported, nor are powers

[93] Square roots of squares, cube roots of cubes, logarithms of powers, etc. stay in the *DT* they were. For other *integers* $y > 0$, results will become *real* (for *DT* 1) or truncated *short integers* (for *DT* 10); for $y < 0$, results will become *complex* instead of *real*.

[94] Complex results require CPXRES set. Else an error message will be thrown.

involving *DT* 4 … 9. – This is the matrix for logarithms:

… combined in $\log_x y$ with a number $x > 0$ of the *DT* below returns a result of the *DT* printed at the intersection.	A number $y > 0$ of *DT* … 1	2	10
1 or 10 (i.e. *long* or *short integer*) [82]	1^{93} / 2	2	10^{93}
2 ℝ *Real number*	2	2	10^{93}

And for $y < 0$ of *DT* 1 or 2, logarithms will return a *complex* result if this is allowed (see the chapters about *complex numbers* below); logarithms of negative *short integers* are not supported.

Here is the matrix for some popular *monadic* functions:

… processed by an operation printed below returns a result of the *DT* printed at the intersection.	A number $x > 0$ of *DT* 1	2	10
lg , lb x , $\sqrt{x}$, $\sqrt[3]{x}$ [95]	1^{93} / 2	2	10^{93}
ln , e^x	2	2	10^{93}
10^x	1	2	10

The following is for integer divisions:

… IDIVR-divided by a number x of the *DT* below returns an **integer ratio** in X and a remainder in Y of the *DTs* printed at the intersection.	A number y of *DT* … 1	2	10
1 or 10 (i.e. *long* or *short integer*) [82]	**1**; 1	**1**; 2	**10**; 10
2 ℝ *Real number*	**1**; 2	**1**; 2	**10**; 2

[95] For roots of negative operands see previous page

Additionally, explicit *DT* conversions are available where necessary:

Any closed object x of *DT* …						… will be converted into an object x of the *DT* below by the keystrokes printed at the intersection.
1 *long integer*	2 *real*	4 *angle*	5 *time*	6 *date*	10 *short integer*	
[#] [I], [ROUNDI]		—	—	—	[#] [I]	1 ℤ *Long integer*
[.d]						2 ℝ *Real number*
[∠→] …			—	—	—	4 *Angle*
[h.ms]		—	—	—	—	5 *Time*
[#] ***n***		—	—	—	[#] ***n***	10 *Short integ.* of base ***n***

Recognizing Calculator Settings and Status

Radix marks and digit group gaps are obvious in the numeric display; so are date and time display modes (Y.MD / D.MY / M.DY and TDM24) in the date-and-time string in the *status bar*. Also *program entry mode* is easily recognized (see pp. 214ff).

The following symbols may appear trailing said string in the *status bar*, listed below from left to right in various sets – indicators shown in *SDC* are printed in a light blue field again:

Indicator	Set by	Deleted by	Explanation, remarks
ℂ	CPXRES	~~CPXRES~~	With CPXRES, *complex* results of operations on *reals* are allowed, like $\sqrt{-1}$. Else a domain error will be thrown in such a case.
ℝ	~~CPXRES~~	CPXRES	

⌞	~~POLAR~~	POLAR	Rectangular or polar notation chosen for displaying *complex numbers*.
⊙	POLAR	~~POLAR~~	

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Indicator	Set by	Deleted by	Explanation, remarks
∠°	DEG	setting any other *angular display mode* (*ADM*)	Current *ADM* setting: decimal *degrees*, *grads* or *gon*, *radians*, *multiples of π*, *mils*, or sexagesimal *degrees*.
∠g	GRAD		
∠r	RAD		
∠π	MULπ		
∠‾	MIL		
∠"	d.ms		
/max or /2345	DENANY [96]	Can only be modified by DENMAX	Fraction display settings. The current value of the max. displayable denominator D.MAX is shown behind the fraction bar (**/max** is *SDC* and means 9999). With ~~DENANY~~, DENFIX toggles a specific symbol trailing D.MAX in the *status bar*.
/2345f	~~DENANY~~ & DENFIX	DENANY	
/2345× or /2345·	~~DENANY~~ & ~~DENFIX~~	DENANY	
64:1	1COMPL	setting any other *integer sign mode* (*ISM*)	*Short integer* settings. First two digits tell the *word* size, the symbol after the colon tells the *ISM*. *Startup default* is 64 *bits* (the max.) and 2's complement. CARRY and OVERFLow may follow the *ISM* but are only lit if set.
64:2	2COMPL		
64:u	UNSIGN		
64:s	SIGNMT		
wrap	M.WRAP	M.GROW	*Matrix grow mode*, displayed instead of ***ws***:*ISM* as long as a *matrix* is open for editing.
grow	M.GROW	M.WRAP	
END	ENDP	BEGINP	Payment mode in the *Time Value of Money* application, displayed instead of ***ws***:*ISM* as long as TVM is open (see *Section 5*).
BEG	BEGINP	ENDP	

[96] The symbol "&" denotes a logical "and" and a comma a logical "or" in this table.

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Indicator	Set by	Deleted by	Explanation, remarks
A	▲ if α is set	ENTER↑ in *AIM*, EXIT in Myα, ~~ALPHA~~	α or SF ALPHA will set *AIM*. For A (α), upper (lower) case letters can be entered then. *AIM* will start with A except for editing equations or creating *variables*.
α	▼ if A is set		
8	SSIZE8	~~SSIZE8~~	*Stack* size.
4	~~SSIZE8~~	SSIZE8	
	program waiting for user input	program running	Will also be lit if a program is stopped by EXIT or R/S – then will be cleared by next keystroke.
	see remarks	*WP43* idling	Operation executing in *RUM*. Not lit with a program running.
	see remarks		Program running (see *Section 3*).
	timer running in background	idle timer	See the TIMER (a.k.a. stopwatch) application in *Section 5*.
	serial I/O in progress	idle communication line	
U	USER	USER	Toggles *user mode* (see *Section 6*).
	see remarks		Lit if power is supplied through the *USB* cable, else clear.
	low battery	supply voltage > 2.5 V	A low battery will reduce processor speed. Your *WP43* may shut off when voltage drops below 2.0 V.

The *SDC* is indicated in a *status bar* like this:

With M.DY and 12h time format, CPXRES, POLAR, MULπ, DENFIX and a 4-digit D.MAX, unsigned *short integers*, CARRY and OVERFLOW, in *alpha input mode*, with ~~SSIZE8~~, a program waiting for input, the timer running in background, in *user mode*, and with a low battery, the *status bar* will look like

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See below and look up *ReMS 1* for these *system flags* and calculator modes. Note also or , and might appear at right end.

Special Displays (Returned by RBR, STATUS, VERS, etc.)

Some applications using the screen in special ways are listed here:

1. RBR allows browsing all *registers* currently allocated (see pp. 283f).
2. FLAG STATUS returns free space available, all *user* and *system flags* set, and more (see pp. 285f).
3. The *Matrix Editor* for filling *matrices* and modifying their *elements* is described comprehensively on pp. 176ff.
4. calls the timer or stopwatch application (see pp. 286f).
5. α.FN FBR browses all characters defined in both fonts provided.

Further commands throw *temporary information* (as defined on p. 72):

a. ERR and MSG display the corresponding error message. See the *IOI* and *ReMA C*. Tests show their results (see pp. 226ff).

b. cov, r, s_{xy}, VIEW, WDAY, $\hat{x}$, and $\hat{y}$ return results with headers.

c. Commands returning two or three values at once (like →P, R←, DATE→, DECOMP, $\bar{x}$, s, L.R., SUM, M.DIM?, RCLIJ, and Σ+) tag their output (see, e.g., pp. 21 and 107ff).

d. VERS? displays a message in X showing the firmware version running on your *WP43* (WHO works in a similar way):

```
WP43 v0.23.1 2022-10-14
```

A few far-reaching commands (like CLall or RESET) will ask you for confirmation before executing. Then answer either

- Yes (by pressing 3, ENTER↑, XEQ, or YES) or
- No (by pressing /, EXIT, ←, or NO);

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any other input will be ignored. Note that such actions explicitly confirmed can't be undone by ↶.

Localizing Numeric Output

You can summon display preferences for *reals*, *times*, and *dates* all at once according to your region's customs using dedicated commands (all contained in the 2nd view of DISP). In the table overleaf, …

- radix mark denotes the decimal separator; [97]
- GAP states the digit group interval – after ***n*** digits a narrow blank is displayed (cf. examples on p. 75); this follows *ISO 80000-1*.[98]
- date stands for the date format (Y.MD, D.MY, or M.DY).[99]
- *JG* states the year the *Gregorian calendar* was introduced in the particular region, typically replacing the *Julian calendar* (or national calendars in East Asia); check the dates in *Wikipedia*.[100]
- background colors are printed according to p. 74.

[97] See *https://en.wikipedia.org/wiki/Decimal_separator* for a world map of radix mark use. Looks like an even score in this matter. Thus, the international standard *ISO 80000-1* allows either a decimal point or a comma as radix mark and requires a narrow blank as unambiguous separator of digit groups (it explicitly states that points or commas shall not be used as group separators to avoid ambiguity).

Note most people using radix commas employ multiplication dots (while those using radix points need crosses to avoid misunderstandings, causing further ambiguities in *vector* multiplication, however).

[98] As far as we know, *WP43* is the very 1st pocket calculator worldwide displaying numbers the way internationally agreed on. Previous calculators had to use, e.g., points or commas as crutches since they could not display narrow blanks.

[99] See *https://en.wikipedia.org/wiki/Date_format_by_country* for a world map of date formats use. The international standard *ISO 8601* states Y.MD for dates and 24h for times. This combination carries some inherent logic and is common in East Asia (see SETCHN and SETJPN).

[100] Officially, the *Gregorian* calendar became effective at 1582-10-15 in the catholic world. Many states and territories switched later for various reasons. You can enter the date applicable at your location using J/G (see the *IOI* for this command). Note there are still other calendars widespread – see the chapter about *Dates* below.

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Com-mand	GAP	Radix mark	Time	Date	*JG*	Remarks
SETCHN **CHINA**	4 [101]	point	24h	Y.MD	1949	
SETJPN **JAPAN**	3	point	24h	Y.MD	1873	
SETIND **INDIA**	3 [102]	point	24h	D.MY	1752	Applies to the area of former British India (India, Pakistan, Nepal, Bhutan, Bangladesh, Myanmar, and Sri Lanka).
SETEUR **EUROPE**	3	comma	24h	D.MY	1582	Also applies to Latin America and – with other *JG*s – to Indonesia, South Africa, Viet-nam, and the area of the late Soviet Union.
SETUK **UK**	3	point	12h	D.MY	1752	Also applies to Australia and New Zealand.[103]
SETUSA **USA**	3	point	12h	M.DY	1752	

You can save above settings for time and date, as well as your choices of

[101] Chinese counting and traditional mathematics work with powers of 10 000 while (originally Indian, then Persian, then) European counting and mathematics work with powers of 1000. Thus, Chinese count using intervals

一 (1), 十 (10), 百 (100), 千 (1000),
万 (10^4), 十万 (10 万), 百万 (100 万), 千万 (1000 万),
亿 (10^8), 十亿, 百亿, 千亿,
兆 (10^{12}), … Compare details of a Chinese calculator keyboard pictured overleaf (note the reverse sequence). GAP **4** takes care of this notation. GAP **3** formats the European way.

[102] Proper South Asian (a.k.a. Indian) formatting nowadays requires separators every two digits for numbers > 1000. Think of *lakh* = 10^5, *crore* = 10^7, *arab* = 10^9, and, e.g., *lakh crore* = 10^{12}. Actually, an amount of 50 cr. Rupees reads 50,00,00,000 Rs in Indian newspapers. Reading greater numbers aloud may be challenging.

Sri Lanka and Nepal turned to GAP **3** recently.

[103] 24h (a.k.a. Indian '*military time*') is taking over in the *UK*, so SETIND will work there as well then – but set GAP **3** thereafter.

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formats for *reals*, *fractions*, *angles*, and *integers* (see respective chapters below), and all *system flags* collectively at one single location. STOCFG will store this *configuration* in the *RoV* you specify.[104] Use RCLCFG to recall it and set (or reset) your *WP43* accordingly.

Real Numbers: Changing the Display Format

Any number you enter using one . or E is read as a *real* by your *WP43* unless there is additional information given (cf. pp. 73f). The majority of functions provided operate on *reals*.

A closed *real* will be displayed right adjusted (cf. pp. 25f). All its digits will be shown in *SDC* if 16 are sufficient for doing so – your *WP43* will automatically turn to mantissa plus exponent format (*MEF*) if a number is too big or too small; this prevents missing unexpected big or small results.[105] There are two flavors of *MEF*:

SCI is called *scientific notation*. E.g., SCI **2** will show the age of the universe in *years* like 1.38×10^{10}.

ENG displays like SCI but the exponent will always be a multiple of three, corresponding to the *SI* unit prefixes – it is called the *engineer's notation*. ENG **2** will show 13.8×10^{9} for said age.[106]

Furthermore, FIX fixes the radix mark on the screen (thus called *fixed point notation*) while it floats in the other formats – see below.

[104] Actually, it stores even more – see *Section 6.*

[105] Regardless of the format or notation you select, these options affect the display only. Internally, your *WP43* continues using its full precision. Use SHOW to display x with this precision until next keystroke.

[106] Seems real engineers prefer reading *SI* prefixes instead of exponents of 10, but these prefixes don't apply everywhere – e.g., not here. Exponents of 10 are also beneficial to mitigate confusion due to identical terms used for different quantities, e.g., 1000 billion (*USA*) = 1 billion (Continental Europe), 1 quadrillion (*USA*) = 1 billiard (*CE*).

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You can specify the number of decimals you want to see in FIX or SCI. For ENG, the mantissa will show one digit more than you specified. Check the following **examples**:

Input \ Format	*SDC* (ALL **0**, GAP **3**, ~~ALLENG~~)	FIX **2**	SCI **4**	ENG **4**
71.2345678				
EXIT	71.234 567 8	71.23	$7.123\,5\times10^{1}$	71.235
1/x	$1.403\,812\,826\,951\,692\times10^{-2}$	0.01	$1.403\,8\times10^{-2}$	14.038×10^{-3}

Now let's continue with the last number divided by **1000**, then multiply it times 10 in every subsequent row, and look at the displays for FIX **4**, SCI **4**, and ENG **4**:

FIX **4**	SCI **4**	ENG **4**
$1.403\,8\times10^{-5}$	$1.403\,8\times10^{-5}$	14.038×10^{-6}
0.000 1	$1.403\,8\times10^{-4}$	140.38×10^{-6}
0.001 4	$1.403\,8\times10^{-3}$	$1.403\,8\times10^{-3}$
0.014 0	$1.403\,8\times10^{-2}$	14.038×10^{-3}
0.140 4	$1.403\,8\times10^{-1}$	140.38×10^{-3}
1.403 8	1.403 8	1.403 8
14.038 1	$1.403\,8\times10^{1}$	14.038
140.381 3	$1.403\,8\times10^{2}$	140.38
1 403.812 8	$1.403\,8\times10^{3}$	$1.403\,8\times10^{3}$
14 038.128 3	$1.403\,8\times10^{4}$	14.038×10^{3}
140 381.282 7	$1.403\,8\times10^{5}$	140.38×10^{3}
1 403 812.827 0	$1.403\,8\times10^{6}$	$1.403\,8\times10^{6}$

Compare p. 75 and note that formatting choices may lead to different screen space requirements.[107]

SCI and ENG are boring simple – they display a constant number of digits for each number, while FIX shows each number sharp to the last decimal specified. Thus, FIX **2** is the favorite format of financial people.

[107] Deviating from previous calculators, $\times10^{0}$ is suppressed for SCI / ENG on your *WP43*.

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For ALL, its parameter tells your *WP43* how many decimal zeros you allow before the display shall overflow either to SCI or ENG, determined by the *system flag* ALLENG (or **A**). See the case below.

Once you have chosen a display format, DSP lets you change its parameter (without the need to open DISP and call FIX, SCI, ENG, or ALL again), as demonstrated in this **example** starting in *SDC*:

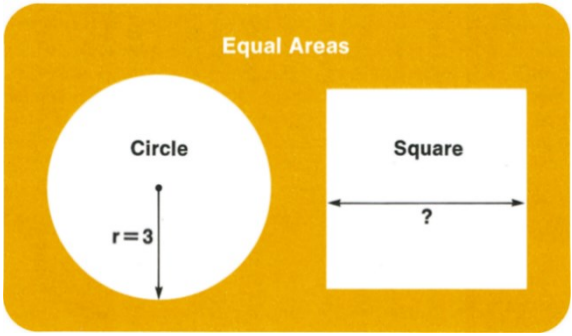
–700	–700_	
1/x	$-1.428\,571\,428\,571\,429\times10^{-3}$	turns to SCI.
DSP 2	$-1.428\,571\,428\,571\,429\times10^{-3}$	allowing max. 2 zeros is not sufficient get a fixed point notation again.
DSP 3	$-0.001\,428\,571\,428\,571$	but 3 zeros suffice.
10 /	$-1.428\,571\,428\,571\,429\times10^{-4}$	this result is too small again.
SF A	$-142.857\,142\,857\,142\,9\times10^{-6}$	turns to ENG.
DSP 4	$-0.000\,142\,857\,142\,857$	4 zeros suffice here.
10 /	$-14.285\,714\,285\,714\,29\times10^{-6}$	this is too small again.
CF A	$-1.428\,571\,428\,571\,429\times10^{-5}$	returns to SCI.
1000 ×	$-0.014\,285\,742\,857\,143$	

Usually, you will be happy with far less than 16 digits shown: just focus on the important (and reliably significant) ones. – FIX, SCI, ENG, ALL, and almost all other functions controlling *real* display format are found in DISP (see the *ReM*).

Reals: Squares and Cubes and Their Roots

√x is on the keyboard of your *WP43*, while x^2, x^3, $\sqrt{(1+x^2)}$, and $\sqrt[3]{x}$ live in EXP (cf. p. 32).[108] The following **example** (triggered by the *HP-65 OH*) solves some of the most 'popular' problems of ancient mathematics:

[108] Note you can produce a square using ENTER↑ × as well without calling EXP.



What size square has the same area as a circle whose radius is **3**? And what size cube has the same volume as a sphere whose radius is **3** again? And what can we tell about their surface areas?

Solutions (with FIX **3**):

The area of a circle is $A_C = \pi r^2$. The area of a square is $A_{sq} = a^2$. The volume of a sphere is $V_S = \frac{4}{3}\pi r^3$, while its surface is $A_S = 4\pi r^2$. The volume of a cube is $V_{cu} = a^3$, while its surface is $A_{cu} = 6a^2$. Thus,

3 EXP x^2 π ×	returns	28.274 for the area of the circle. Then
√x	returns	5.317 for the edge length of the square.

Furthermore, 3 x^3

π × 4 × 3 /	returns	113.097 for the volume of the sphere.
Then $\sqrt[3]{x}$	returns	4.836 for the edge length of a cube with same volume.
Thus, x^2 6 ×	returns	140.320 for the cube's surface. Finally,
3 x^2 π × 4 ×	returns	113.097 for the surface of the sphere.

Here is a little winter sports problem of our time (as long as cold suffices):

Example:

Chuck Carver swings down a ski run with moderate 30 km/h. The shape of his skis allows for turns with 12 m radius. He claims having carved this way without any sliding on a short flat part of the run. If true then how many *g* he had to withstand there? Can we believe his story?

Solution (with FIX **1**):

Centrifugal acceleration is $a_c = 2\pi\, {v^2}/{r}$. So the total acceleration along *Chuck's* body axis is $a_T = \sqrt{g^2 + a_c^2}$. This means in multiples of g

$$ {a_T}/{g} = \sqrt{1 + \left({a_c}/{g}\right)^2}. $$

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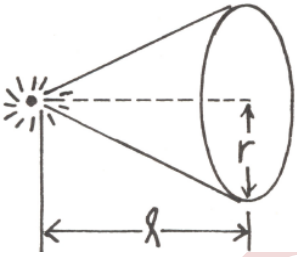
30 ENTER↑ **3.6** /	8.3	Chuck's speed in $^{m}/_{s}$
EXP x^2	69.4	
2 × π × **12** /	36.4	his centrifugal acceleration
CONST $g_\oplus$ /	3.7	
$\sqrt{(1+x^2)}$	3.8	meaning 3.8 times *g*.

This might be possible to stand shortly for a young sportsman; but the snow under him can't bear the corresponding forces – it will break, so *Chuck* will inevitably slide in a greater radius meaning less acceleration.

Another **example** from an early brochure about the *HP-35*:



The solid angle a circular detector covers of a point source is given by … :

$$\Omega = 2\pi \left[1 - \sqrt{\frac{1}{1 + \left({}^{r}/_{l}\right)^2}} \right]$$

How big is it for ***r*** = 2.5 cm and ***l*** = 9.3 cm?

Solution (with FIX **4**):

2.5 ENTER↑ **9.3** / EXP $\sqrt{(1+x^2)}$ 1/x +/- **1** + **2** × π ×

returns Ω = 0.215 4. You can solve for annular detectors by analogy.

A technical application found in an *HP* manual of 1988:

When an incompressible fluid with negligible viscosity flows steadily along a pipe, *Bernoulli*'s equation holds at any point ***i*** along the pipe. It's given by

$$p_i + \frac{1}{2}\,\rho\, v_i^2 + \rho\, g\, H_i = K$$

where ***p*** is the pressure, ***ρ*** is the fluid density, ***v*** is the fluid velocity, ***g*** is the acceleration due to gravity, ***H*** is the height of the fluid above some arbitrary reference point, and ***K*** is a constant. … The continuity equation says that the mass of fluid in a pipe is conserved (none enters or exits the pipe). The continuity equation for an incompressible fluid with steady flow is

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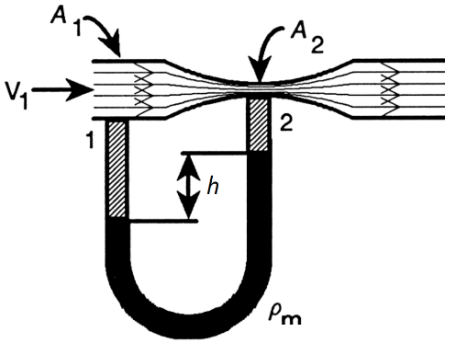

$$A_1 v_1 = A_2 v_2$$

where ***A*** is the pipe's cross sectional area and ***v*** is the fluid's velocity at points 1 and 2, respectively. Informally, the equation states that the flow rate in volume per time is constant at any point in the pipe.

Example:

A *Venturi tube* shall be used to measure speed of fluid flow.

The U-shaped tube in the figure is called a manometer and is used to measure pressure differences. The manometer equation for pressure difference is $p_2 - p_1 = -\rho_m\, g\, h$, where ***p*** is the pressure at points 1 and 2, $\boldsymbol{\rho_m}$ is the density of fluid in the manometer, ***g*** is the acceleration due to gravity, and ***h*** is the difference in fluid levels in the manometer. When the continuity equation, *Bernoulli's* equation, and the manometer equation are combined, the speed of fluid flow at point 1 in the figure is found to be

$$v_1 = A_2 \sqrt{\frac{2\,(\rho_m - \rho)\, g\, h}{\rho\,(A_1^2 - A_2^2)}}$$

If the manometer is filled with mercury ($\boldsymbol{\rho_m}$ = 13 595 $\mathrm{kg/m^3}$) and the fluid in the pipe is water ($\boldsymbol{\rho}$ = 1000 $\mathrm{kg/m^3}$), find the velocity of fluid flow at point 1. Assume that the cross sectional area of the pipe at point 1 is 0.073 $\mathrm{m^2}$ and at point 2 is 0.0507 $\mathrm{m^2}$. The difference in mercury levels in the manometer is 9.0 cm. Follow the keystrokes below to find $\boldsymbol{v_1}$.

DISP FIX 2	since the least precise input carries 2 digits.
13595 ENTER↑ E 3 +/−	12 595.00
2 × CONST $g_\oplus$ × .09 ×	22 232.66
.073 EXP x^2 .0507 STO K	0.51
x^2 +/− E 3 ×	2.76
/ $\sqrt{x}$ RCL K ×	4.55

The fluid velocity at point 1 is 4.6 $\mathrm{m/s}$.[109]

[109] We actually helped this problem by assuming the manometer was read with 1 mm precision. The original text just stated 9 cm but printed a 5-digit result.

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Here is a high flying problem excavated in an earlier calculator manual:

Example:

Finding himself floating dangerously close to the jagged peaks of the Canadian Rockies, intrepid balloonist *Chauncy Donn* frantically cranks open the helium valve on his spherical balloon. Gas from the helium tank increases the balloon's radius from 7.5 *meters* to 8.25 *meters*. *Donn* clears the mountain tops safely. How much did the volume of the balloon increase?

Solution: [110]

We know the volume of a sphere from p. 88, thus the difference of two such volumes is $\Delta V = \frac{4}{3}\pi(r_2^3 - r_1^3)$. One decimal shall do.

8.25 EXP x^3		561.5
7.5 x^3 −		139.6
π ×		438.7
4 × **3** /	returns	584.9 m^3 for the volume increase.

Reals: Percent Change

Δ% calculates the percentage of change from y to x.

Example (continued from previous page):
This is a volume increase of how many percent?

Solution:

7.5 x^3		421.9
8.25 x^3		561.5
Δ%	returns	33.1 % increase of the balloon volume.

[110] In the *HP-21 OH* of 1975, the balloonist *Ike Daedalus* had to increase the radius from 25 to 27 *feet*. 1976 brought some progress at least.

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Another **example**, more down-to-earth:

Pia Power strives for designing an optimum bicycle gearing for her home in a hilly area. She is free to choose sprockets and gear clusters to her liking.

Solution:

As long as drag can be neglected, an optimum bike gearing will feature constant velocity ratios between subsequent gears (or uniform percental increase of distances per crank revolution). There are several ways *Pia* can approximate this, depending on the number of sprockets chosen at front and rear. One inexpensive and solid way is taking 3 front sprockets of 48, 36, and 24 teeth and a standard 7-gear cluster featuring 13 through 30 teeth (in 2013). This will result in the following distances per crank revolution (d/r_c in *meter*) for a 26" MTB:

Gear	1	2	3	4	5	6	7	8	9	10	11	12
Front	24				36			48				
Rear	30	26	23	20	26	23	20	23	20	17	15	13
d / r_c	1.660	1.915	2.165	2.490	2.873	3.247	3.734	4.330	4.979	5.858	6.639	7.660
Δ%		15.4	13.0	15.0	15.4	13.0	15.0	16.0	15.0	17.7	13.3	15.4

Assuming *Pia* pedals steadily with a frequency of 60 rpm, such a gearing will allow for conveniently riding with velocities between 6 and 28 km/h. After some months of bike training, Pia pedals with 80 rpm, reaching velocities up to 37 km/h this way.

Using three functions provided in STAT ([Σ+], [x̄], and [s] as explained on pp. 107ff), you can determine an average speed increment of (14.9 ± 1,4)% per gear step here. This gearing turns out being quite uniform and convenient for town and country.[111]

[111] You get 12 out of 21 theoretically possible gears here only. This is due to gear overlaps; and you will want to avoid extreme chain skew for sake of components life. Conditions change if you plan for a recumbent bike with a long chain: Then you may think about three sprockets with a 7-gear cluster leading to 17 different, usable gears following a scheme called '*half-step-plus-granny*'. Speed increase in *half-step* range will be 9% per gear step. This gearing will cover velocities from 5 to over 40 km/h for 60 rpm pedaling frequency.

During the *Tour de France* in 2022, 36 vs. 30 teeth was the *granny* gear for mountain roads in the Alps. Some training was required obviously.

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Feel free to check and try other configurations with your *WP43*. You can obtain up to 13-gear clusters and 14-gear hubs nowadays, so there is a wide field for considerations and designing.[112]

Reals: Logarithms and Powers (a.k.a. Antilogs)

Your *WP43* features four logarithmic functions: two on its keyboard and two more in EXP (cf. p. 32):

〔ln〕 computes the *natural logarithm* of x, i.e. the logarithm of x to base of Euler's e. 〔ln〕 inverts 〔e^x〕.

〔lg〕 returns the *decadic* (a.k.a. *common*) *logarithm*, i.e. the logarithm to base 10. 〔lg〕 inverts 〔10^x〕.[113]

EXP lb x computes the *binary logarithm*, i.e. the logarithm to base 2. 〔lb x〕 inverts 〔2^x〕.

EXP $\log_x y$ is the most general of these logarithmic functions, returning the logarithm of y to base x. 〔$\log_x y$〕 inverts 〔y^x〕.

The operating manual of the world's very first electronic pocket calculator featuring transcendental functions, the *HP-35* (cf. p. 56), presented just one single **example** for this then new class of pocket-able functions:

> Although logarithms were originally used to speed multiplication and division, they have particular significance in scientific and engineering problems. There is, for example, a logarithmic relationship between altitude and barometric pressure. Suppose you wish to use an ordinary barometer as an altimeter. After measuring the sea level pressure (30 inches of mercury) you climb until

[112] Not every offer will turn out being sufficiently wear-resistant for your application though. Also a bottom bracket spur gear is available featuring 18 steps. The website *http://ritzelrechner.de/* may support you selecting your optimum bicycle gearing.

[113] Labels **LOG** or **log** are often printed on calculators for the decadic logarithm. Though this is a mathematically ambiguous notation, so we avoided it (cf. *ISO 80 000*, 2-12.5 and ...6; see also *https://physics.nist.gov/cuu/pdf/sp811.pdf*, p. 33, 10.1.2).

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the barometer indicates 9.4 inches of mercury. How high are you? Although the exact relationship of pressure and altitude is a function of many factors, **a reasonable approximation** is given by: [114]

$$\frac{altitude}{[feet]} = 25\,000 \times ln\left(\frac{30}{pressure/[inches\ of\ Hg]}\right)$$

Solution:

DISP FIX 0 0 should suffice here.

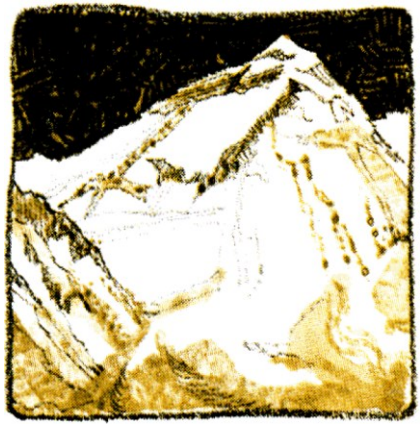
30 ENTER↑ 9.4 / ln 25 E 3 × returns 29 012.

[We suspect that you may be on *Mt. Everest* (29 028 *feet* in 1972).]

Note the concise and factual style of this problem. The *HP-35* was a calculator made by engineers for engineers, and its manual was alike. Above case was reprinted in the *HP-45 OH* one year later. Thereafter, it underwent slight modifications:

Example (from the *HP-21 OH*):

Having lost most of his equipment in a blinding snowstorm, ace explorer *Buford Eugobanks* is using an ordinary barometer as an altimeter. After measuring the sea level pressure (30 *inches of mercury*) he climbs until the barometer indicates 9.4 *inches of mercury*.[115] Although the exact relationship of pressure and altitude is a function of many factors, *Eugobanks* knows that **an approximation** is given by the formula …



This problem remained in subsequent manuals, just the explorers changed. A picture of the scenery was added in 1976, and not every snowstorm was worth mentioning anymore. But in 1978, more coherent units reached even expeditions in the *Himalayas* – and also climate and methods changed:

[114] Emphases in these quoted examples were added by me.

[115] He lost his equipment, went to the seashore, and then climbed?!? The story became more plausible in 1978.

Example (from *Solving Problems with Your HP Calculator*):

With most of his equipment lost in an avalanche, mountaineer *Wallace Quagmire* must use an ordinary barometer as an altimeter. Knowing the pressure at sea level is 760 *mm of mercury*, *Quagmire* continues his ascent until the barometer indicates 238 *mm of mercury*. Although the exact relationship of pressure and altitude is a function of many factors, *Quagmire* knows that **an approximation** is given by the formula:

$$\frac{altitude}{[\mathrm{m}]} = 7\,620 \times ln\left(\frac{760}{p/_{[mm\ of\ Hg]}}\right)$$

Where is *Wallace Quagmire*?

Solution:

760 [ENTER↑] **238** [/]

[ln] **7620** [×] returns 8 847.

Quagmire appears to be near the summit of *Mt. Everest* (8 848 m). – And it seems neither he nor his barometer returned from this expedition since this case did show up neither in the *HP-41C OHPG* nor later anymore. Perhaps something went wrong with the recalibration of his instrument?

For standard *SI* units, the altitude approximation formula reads:

$$\frac{altitude}{[\mathrm{m}]} = 7\,620\ ln\left(\frac{1\,013}{p/_{[\mathrm{mbar}]}}\right) = 7\,620\ ln\left(\frac{101\,300}{p/_{[\mathrm{Pa}]}}\right).$$

Beyond the barometric scale, there are more logarithmic scales used in science and engineering, e.g.,

- in astronomy for assessing the brightness of stars or
- in chemistry for the power of acids (pH); most popular may be
- the *decibel* (dB) in acoustics and electronics (see U→ on pp. 299f) and

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- the so-called *upwardly unlimited Richter scale* for magnitudes of earthquakes.[116]

Example:

One of the strongest earthquakes observed in last decades was the one causing the devastating tsunami in the Indian Ocean near Indonesia in 2004-12. It had a magnitude of 9.1. Another one near Japan in 2011-03 – with a magnitude of 9.0 – led to another tsunami and the '*Fukushima nuclear accident*'. Compare them with the '*great San Francisco earthquake*' of 1906 with a magnitude of 7.9.

Solution (with FIX **0**):

The formula for comparing the energies released in two different earthquakes (with their magnitudes known) reads

$$E_2/E_1 = 10^{1.5(M_2-M_1)}.$$

Again, no decimals are needed here – we can continue with the display settings as they are:

9.1 [ENTER↑] **7.9** [−] **1.5** [×] [10^x] returns 63. and

9 [ENTER↑] **7.9** [−] 1.5 [×] [10^x] returns 45. .

So the energy released in said Japanese earthquake in 2011 was 45 times greater than the '*great San Francisco earthquake*'. And said earthquake in the Indian Ocean was even 63 times more intense.[117]

Even small numeric differences will gain significance when raised to powers. Human brains are not well equipped for such operations, so we recommend taking good care (and your *WP43*) in such cases:

Example:

What difference in magnitude will cause double destruction?

[116] The actual moment magnitude scale for earthquakes differs but is logarithmic as well.

[117] Taking into account that published magnitudes of earthquakes never show more than one decimal, we did not lose anything real setting *WP43* to FIX **0** here.

For comparison: The asteroid impact at Chicxulub some 66 million years ago leading to extinction of the dinosaurs is estimated having caused an earthquake of magnitude 11.

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Solution:

Rewriting the formula of previous problem results in $\Delta M = \frac{2}{3}\, lg\left({}^{E_2}/_{E_1}\right)$

Thus, for double destruction we need a magnitude difference of

DSP 0 1

2 lg 2 × 3 / equaling 0.2 only.

But there are also friendlier applications of logarithms, e.g., in electronics:

Example:

How many *bits* are required if the *unsigned integer* 3.75×10^9 shall be the maximum to be handled by a microprocessor?

Solution:

3.75 E 9 EXP lb x returns 31.8 , so 32 *bits* will suffice.

If we had a tri-state logic, however,

RCL L 3 $\log_x y$ returns 20.1 , so 21 cells would do.

Using the inverse of the general logarithm ($\log_x y$), i.e. (y^x), your *WP43* allows for raising any positive *real* base to an arbitrary *real* power, as well as any negative *real* base to an arbitrary *integer* power, all returning *real* results (cf. the *Mach number* formula on p. 49).

Let's return to Fukushima once more for a really down-to-earth **example** applying logarithmic functions:

Locations in a distance of 30 km (!) to the nuclear power plant *Fukushima Dai-ichi* being devastated by the tsunami in 2011-03 showed radioactivity in the soil of some 1...3 *megabecquerel* / m^2 corresponding to an annual radiation dose of 4 *millisievert* (mSv) in 2013 (see the map overleaf for details[119]). This was mainly caused by ^{137}Cs then, having a half-life of 30.2 *years*.[118]

To the best of our knowledge today, an unborn child shall not receive a dose of more than 1 mSv before birth. So when will it be reasonably safe to let the evacuated inhabitants (some 150 000) of the villages in that area return to their homes finally?

[118] In 2011-12, the radioactive iodine blown out of the reactor in March had become irrelevant already (^{131}I has a half-life of only 8 days). The caesium remained.

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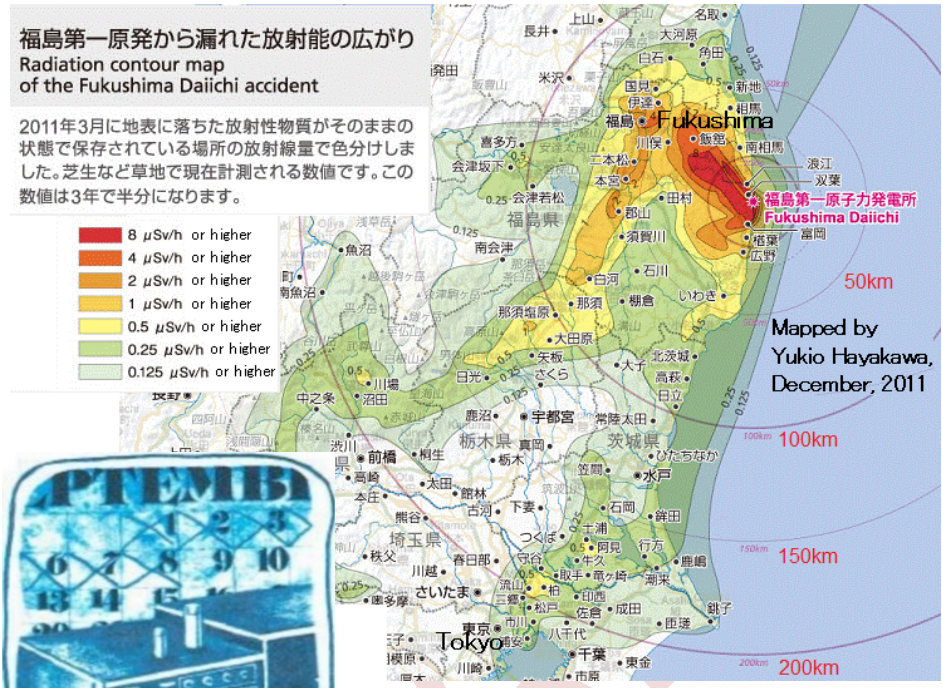

Solution (with FIX **0**):

Assuming there will be no further nuclear accident there, the caesium set free in 2011 will stubbornly decay following the laws of nature (= physics). With a radio-activity a_0 at time zero, the activity a at a later time t will be

$$a = a_0 \times 2^{-(t/T_{1/2})} \;\Rightarrow\; t = T_{1/2} \times lb\left(\frac{a_0}{a}\right).$$

1 mSv in 9 months corresponds to an annual dose of $^4/_3$ mSv. The 2013 annual dose of 4 divided by $^4/_3$ equals 3, and

3 [EXP] **lb** × **30.2** [x] returns **48.** *years*.

So you can recommend reproductive people shall rather not become pregnant living in that area before 2061. Senior inhabitants may return far sooner.[120]

With a probability of 94.6%, ^{137}Cs decays emitting an electron with kinetic energy of up to 512 keV plus a γ-ray of 662 keV. This is for your information only, it doesn't affect the calculation here. – Natural radioactive background is (0.1 ± 0.08) μSv/h on average, strongly depending on location.

[119] Source of the map (though it disappeared):
http://www.odonata.jp/ico2012/radioactivity/radiation_contour_large_map.gif.

Above mathematically simple law links science and society quite closely.[121] – Similar considerations apply to the waste of nuclear power plants: while it is true they don't emit any carbon dioxide in operation (but the CO_2 emitted building necessary disposal sites should be taken into account), they produce many *kilotons* of radioactive material decaying with half-lifes exceeding thousand years (about 400 *kilotons* are present on our planet already, adding some 12 *kilotons* per year). This means you have to put them 'away' safely for really long times – a task kept under wraps

[120] Note that different limits are considered 'reasonably safe' for the public by authorities of different nations. By nature, all such limits are arbitrary to some extent since we talk about probabilities here, and there are very few step functions in probability but smooth transitions (see the chapter after next). Furthermore, a major fraction of today's world-wide knowledge about radiation damage in human bodies in the long range is still based on extrapolation of experiences collected since 1945 following two large-scale events in Japan (and 67 more near Bikini until 1958). Another experiment well known by the public was started in the USSR in 1986 – Belarus and the Ukraine have to bear the consequences until today (and for centuries to come – see p. 307). Mankind knows of the physics of radioactivity for some 120 years only so far, that's not long (just 4 half-lifes of ^{137}Cs).

There are further risks linked to agriculture in the Fukushima area – they are beyond the scope of this simple problem setup though.

Also note the setting of this problem covers a worst case scenario. Actually, radioactivity is washed to deeper layers of soil with rain and time, reducing the activity seen at the surface. And there are mitigation efforts in that area (many km^2): at some places the contamination was washed off houses and trees, and the top layers of contaminated soil were removed, storing them in big black plastic bags 'elsewhere'. 2.3 million m^3 are deposited there already, 12 million more are expected by the Japanese authorities – an area of 1.6 km^2 is provided for 'interim storage'.

Cost of disposal were estimated to be 1.9×10^{12} ¥ by the Japanese administration in 2019 (cf. *Frankfurter Allgemeine Zeitung of 2019-03-10* – this article is behind a paywall now). Furthermore, 1.3 million m^3 of tritium-contaminated water are stored separately (equivalent to a cube of about 110 m edge length). These waters are mixed with sea water and flow slowly into the Pacific since September 2023 – and they will continue to do so for the next 30 years. Tritium is radioactive hydrogen with a half-life of 12.3 *years* and emitting short range radiation (α-particles). Thus, you should rather not drink or inhale it. Check locally applicable radiation limits.

Success of the mitigation efforts mentioned may reduce the waiting time calculated above; failure will not extend it at least. Today is too early for a definitive assessment – we still know too little about long term effects of radiation in biology.

[121] In consequence of this law of nature, radioactivity (once set free) decreases slowly: after 10 half-lifes you will still see $1/1024$ of the original activity (take 137 *gram* of ^{137}Cs, for instance, being 6.022×10^{23} atoms; even after 302 *years*, 5.881×10^{20} atoms will be present still). Although radioactivity decreases continuously, it will actually never vanish – it will just become less relevant with time.

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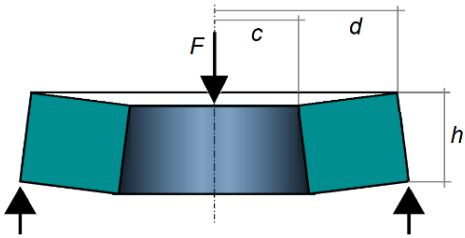
for decades but not solved by waiting so far.[122]

Another industrial **example**, dealing with mechanical transducers:

To measure loads, you can use a bending steel ring supported at its outer radius ***d*** and loaded at its inner radius ***c*** with a force ***F*** (deformation is widely overstated here). The stress on its bottom face is

$$\sigma = \frac{3}{\pi}\ F\ \frac{d-c}{h^2\ a\ ln\left(d/c\right)} \quad \text{with} \quad a = \frac{d+c}{2}\ .$$



A reasonable stress level for precise measuring is 200 $\mathrm{N/mm^2}$ under full load. Assume you want to design a compact load transducer for 5 000 kg featuring an outer radius ***d*** = 25 mm and choose ***c*** = 15 mm, how big shall be ***h***?

[122] Mankind has absolutely no experience with locking something away reliably for many millennia (look at the *Giza* pyramids, for instance). Note the final repository sites and the material stored there must also be tagged properly (☠ ☢ **KEEP OFF !!** ☢ ☠) in a way staying comprehensible for millennia – no experience either.

Sad real-life case: A huge concrete coffin holds almost 90 million *litre* (i.e. 90 000 $\mathrm{m^3}$ or a cube of 45 m edge length) of US nuclear waste on the Marshall Islands (remember Bikini). Now sea-level rise (caused by *anthropogenic global warming* which is evident today but very few were aware of 60 *years* ago) is eating away at the dome, and the *USA* are not interested in helping the tiny Pacific Ocean republic to do anything about it (see the *Los Angeles Times of 2019-11-10*). So much about taking responsibility. And 60 are a lot less than 5000 *years*, so you can guess (and look forward to) the number of surprises in forthcoming centuries.

You might meet people talking about 'transmuting' the entire long-living radioactive waste to isotopes with shorter half-lifes by some nuclear reactions. Please ask them if this is physically possible for all that waste, and how much energy is needed for that transmuting process. Compare with the energy 'produced' by nuclear power plants before.

As a matter of fact, the companies which made profits with those power plants for decades are very reluctant definitely solving the waste problem they created so far. They would not, certainly, if they expected making money from doing that. – Note that no insurance company on this planet offers any policy covering nuclear power plants.

Now you have got all the information you need to assess nuclear fission power plants. Consider what you hand down to your descendants.

Nuclear fusion power plants differ. Therein, processes generating the solar fire shall be mimicked. Such plants are predicted being operational in some 40 years (but the same was claimed 50 years ago already). As far as I know today, they will not produce any long-lived radioactive material. I will be most happy when this turns out being true.

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Solution: [123]

$$h = \sqrt{\frac{3}{\pi} \times \frac{F}{\sigma} \times \frac{2\,(d-c)}{(d+c)\ ln\left(d/c\right)}}$$

You can solve it like this:

DISP FIX 0 1	
25 ENTER↑ 15 / ln	
25 ENTER↑ 15 + × 1/x	this is the divisor of the last fraction.
25 ENTER↑ 15 − × 2 ×	this is the entire rightmost factor.
5 E 3 × CONST $g_{\oplus}$ × 200 /	
3 × π / √x	returns 15.1 mm for *h*.

Reals: Hyperbolic Functions

Hyperbolic functions tell us something about free hanging ropes, cables, chains, and the like. Your *WP43* provides three hyperbolic functions and their inverses in the f-shifted rows of EXP and TRI:

sinh	*Hyperbolic sine*	arsinh	Inverse *hyperbolic sine*
cosh	*Hyperbolic cosine*	arcosh	Inverse *hyperbolic cosine*
tanh	*Hyperbolic tangent*	artanh	Inverse *hyperbolic tangent*

We found the following **example** for applying these functions in the *HP-32 OH* though we modified it a bit:[124]

[123] The formula on previous page was derived by the author developing load cells for industrial scales in 1987.

Also see here a cut through the body of an industrial load cell with a bending ring of hexagonal cross section. Properly treated and equipped, such steel springs allow for measuring 5 000 kg with a display resolution better than 1 kg, legal for trade.

[124] The *HP-32* was *HP*'s first pocket calculator featuring hyperbolic functions. It was launched in 1978 – while the *SR50* of *Texas Instruments* (*HP's* arch rival in those years of the so-called '*calculator wars*') provided hyperbolic functions on its *SR50* four years earlier already.

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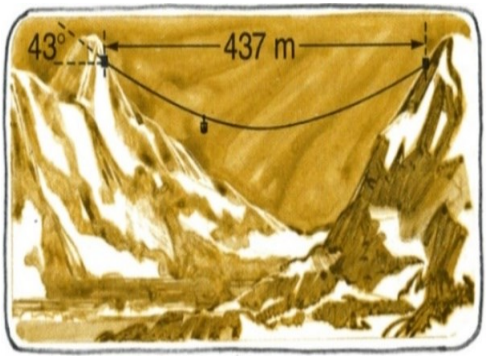

In *Upper Lagunia*, a tram[125] carries tourists between two peaks in the *Baruvian Alps* that are the same height and 437 *meters* apart. How long does it take the tram to travel from one peak to the other if it moves along its cable at 135 *meters per minute*? Before the tram latches onto the cable, the angle from the horizontal to the cable at its point of attachment is found to be 43°.[126]

Solution (assuming *SDC*):
The travel time is given by $t = \left({}^{d}/_{v}\right)\left[{}^{\tan\alpha}/_{arsinh(\tan\alpha)}\right]$

DISP FIX 2	will suffice here. Then …
43 TRI tan	is a result we need twice.
ENTER↑	duplicates it on the *stack* for numerator and denominator.
arsinh /	
437 ×	489.30 m is the length of the cable.
135 /	3.62, i.e. a bit more than 3 ½ *minutes*.

Reals: Probabilities – Factorials, Combinations, Permutations, and Distributions

Beyond Δ% and x!, you find a lot more operations for probabilities in your *WP43*, going far beyond the *Gaussian* distribution (it features all the preprogrammed functions implemented in *WP 34S* – and more). These operations are stored in the three adjacent *menus* PROB, STAT and FIT.

[125] *TN*: British readers might frown here at least.

[126] The picture will hardly look as painted with a real tram on the cable.

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PROB includes the functions C_{yx} and P_{yx} for *combinations* & *permutations*.

	NBin:	Geom:	Hyper:	Binom:	Poiss:
LgNrm:	Cauch:		Expon:	Logis:	Weibl:
Norml:	t:	C_{yx}	P_{yx}	F:	χ^2:

Example (from the *HP-32 OH*):

Willie's Widget Works wants to take photographs of its product line for advertising. How many different ways can the photographer arrange their eight widget models?

Solution:

The total number of possible arrangements is given by the *factorial* $8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

8 [x!] returns 40 320 for this number.

Example (continued):

The photographer looks through his viewfinder and decides that he can show only five widgets if his camera is to capture the intricate details of the widgets … How many different sets of five widgets can he select from the eight?

Solution:

The number of sets equals the number of possible *combinations* (i.e. the number of possible different sets of y different objects taken in quantities of x objects at a time; no object appears more than once in a set, and different orders of the same x objects are not counted separately here):

8 [ENTER↑] 5 [PROB] C_{yx} returns 56 for this number.

Example (continued):

Again, there are different arrangements feasible. How many pictures of different widget arrangements are possible within these limits?

Solution:

The number of possible arrangements follows the statement above. Thus,

5 [x!] returns 120 for this number.

Then [×] returns 6 720 for the number of significantly different pictures.

This is the number of possible *permutations* of 5 items out of 8 (i.e. the number of possible different arrangements of y different objects taken in quantities of x objects at a time; no object appears more than once in an arrangement, and

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different orders of the same x objects are counted separately here). It can be obtained in one step by keying in [127]

8 ENTER↑ **5** P_{yx} returning 6 720 .

Furthermore, PROB contains 9 continuous and 5 discrete distributions for calculating probabilities, confidence intervals, etc. [128] These functions share a few features:

- Discrete distributions (e.g., *Poisson*) are confined to integers. Summing up a *probability mass function* (*PMF*) $p(n)$ to get a *cumulated distribution function* (*CDF*), $P(m)$ starts at $n = 0$. Thus,

$$P(m) = \sum_{n=0}^{m} p(n) .$$

- Continuous distributions (e.g., *normal*) operate on *reals*. Integrating such a continuous *probability density function* (*PDF*) $f(x)$ to get a *CDF* works as

$$P(x) = \int_{-\infty}^{x} f(\xi) d\xi .$$

- Many frequently used continuous *PDF*s look more or less like the ones plotted in the upper diagram here. The lower diagram shows their

[127] These challenging tasks changed the photographer significantly within one year.

[128] In a nutshell, discrete statistical distributions deal with (an integer number of) "events" governed by a known mathematical model. Such statistical events may be persons entering a shop, radioactive nuclei decaying, faulty parts appearing, etc. The *PMF* then tells you the probability to observe a certain number of such events, e.g., 7. And the *CDF* gives the probability to observe up to 7 such events then, but not more.

For doing statistics with continuous statistical variables – e.g., the heights of three-year-old toddlers – similar rules apply: Assume we know the applicable mathematical model; then the respective *CDF* returns the probability for their heights being less than an arbitrary limit, e.g., <1 m. And the corresponding *PDF* tells you how these heights are distributed in a sample of let's say 1000 kids of this age.

BEWARE: This is a very rudimentary sketch of this topic only – turn to a good textbook to learn dealing with statistics properly.

TNG: *PMF* und *PDF* = *Wahrscheinlichkeitsdichte*, *CDF* = *Verteilungsfunktion* bzw. *Wahrscheinlichkeitsverteilung*.

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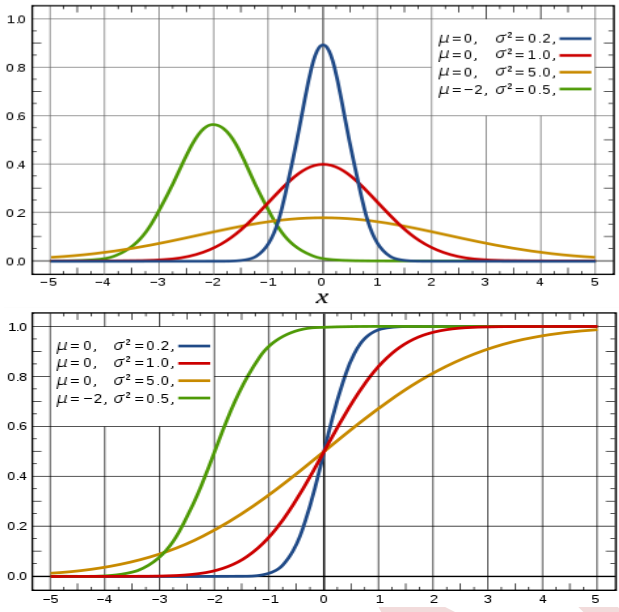
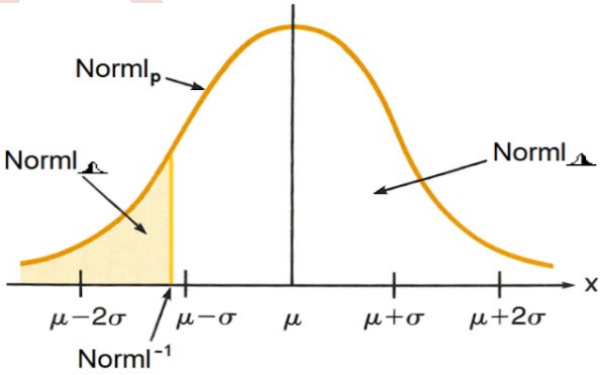


corresponding *CDF*s, using the same scale and colors.

Typically, any *CDF* will start at 0 with a slope of almost 0, become steeper then, and run out at 1 with its slope returning to 0. This holds even if the respective *PDF* don't look as nice and symmetric as the *normal distributions* plotted here.

Thus, you will get the most precise results for the *CDF* on its left side using P. On its right side, however, where P approaches **1**, the *error probability* $Q = 1 - P$ will be more precise. Thus, also Q is computed in your *WP43* for each distribution, independently of P.



For an arbitrary *PDF* (e.g., NORML$_P$), you will find the corresponding *CDF,* named NORML$_\Delta$ (returning P), and NORML$_\Delta$ (returning Q), as well as the inverse of the *CDF* (the so-called *quantile function*) called NORML^{-1} on your *WP43*. This naming convention also applies to the *binomial*, *Cauchy* (a. k.a. *Lorentz* or *Breit-Wigner*), *exponential*, *Fisher's F*, *geometric*, *hypergeometric*, *logistic*, *log-normal*, *negative binomial*, *Poisson*, *Student's t*, and *Weibull distributions*. Just the *chi-square distribution* is denoted differently following mathematical tradition. Cf. PROB on p. 123 or see *ReMS 2*.

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Find applications of the normal distribution in the next five chapters, of *chi-square* on pp. 110f and 118f, of *Student's t* on pp. 121ff, of the *hypergeometric distribution* on pp. 127f, and of the *Weibull distribution* on pp. 127f.

Reals: *1D* Statistics – Sampling Data, Calculating Means, Standard Deviations, and Confidence Limits

A wealth of commands for sample and population statistics is found in STAT, applicable in *1D* or *2D*. After initial clearing all statistical records by [CLΣ], use [Σ+] to accumulate your experimental data. Weighted data require the data value in **X** and its weight in **Y**, pairs of data or coordinates of data points shall be entered in **X** and **Y**. [Σ-] is provided for easy data correction – it will remove the last data point entered.

Data analysis functions are contained in STAT as well: e.g., the *arithmetic mean* $\bar{x}$, *sample* and *population standard deviations (SD)* s and σ, and the *standard error* s_m (a.k.a. *SD of the mean*).

Example (from the *HP-32 OH*):

Norman Numbercruncher, a rising young math professor at *Mammoth University* (note the dress code of 1978), has developed a new test for measuring the mathematical abilities of college freshmen. To evaluate its effectiveness, he administers the test to the 746 students in *Calculus I*. Exhausted after grading the tests, *Numbercruncher* decides to randomly select 8 of the 746 tests and estimate the *SD* of all the scores from the sample of 8. The scores on the tests selected were 79, 94, 68, 86, 82, 78, 83, and 89. What *SD* does *Numbercruncher* calculate?

Solution:

[STAT] [CLΣ]

	CLΣ	$\bar{x}_G$	ε	ε_p	ε_m	FIT
	Σ-	$\bar{x}_w$	s_w	σ_w	s_{mw}	HISTOG
	Σ+	$\bar{x}$	s	σ	s_m	SUM

0 [ENTER↑] 79 [Σ+] returns

001 data point 0.
79.

Σ+ takes ***x***, stores it in STATS, displays feedback incl. *temporary information*, and disables *ASL* (cf. f. 48). – Continue, overwriting ***x*** with each new entry:

94 Σ+ **68 Σ+** **86 Σ+** **82 Σ+** **78 Σ+** **83 Σ+**

89 Σ+ returns

```
008 data points      0.
                    89.
```

DISP **FIX** **3**

s returns s_x = 7.836, the *SD* estimated for the 746 students based on a sample of 8.[129]

When your data constitutes not just a sample of a population but rather all of the population, the *SD* of the data is the true population *SD* (denoted σ) … The difference between the values of s and σ is small, and for most applications can be ignored. Nevertheless, if you want to calculate the exact value of the population *SD* for an entire population, you can easily do so with just one keystroke on your *WP43*.

Example (continued):

Suppose the data from the previous example represented all the final exam scores from *Numbercruncher's* seminar on transcendental functions. Since this is the first time *Numbercruncher* has given this seminar, he wants to calculate the *SD* of the test scores to determine how good his exam was. *Numbercruncher* takes his calculator in hand, enters the data, then proceeds as follows

σ returns σ_x = 7.330 for the *SD* for all scores on final exam. [129]

This is actually a no-brainer with your *WP43*. Thus, let's turn to something completely different:

Example:

Archibald is the undisputed champion of the *Golden Bow*, his archers club. In his standard exercise, aiming at a target disk of 1.5 m diameter at a distance of 50 m, his arrows scatter symmetrically around its center showing quite a small variance. Actually, *Archibald's*

[129] Actually, Σ+ stores also ***y***. Hence, your *WP43* returns *SD*s for both ***x*** and ***y*** data – but the latter is irrelevant in this case.

records tell his arrows scatter with a long-term *SD* of 1 *foot* at that distance. Assuming his shots are distributed *normally* around the center of the disk, how often must he walk to collect an arrow in the green? [130]

Solution (with FIX **3**):

0 STO I		*Archibald's* mean = center of the disk.
1 U→ x: m ← foot	0.305	= 1 *foot* in *meters*, *Archibald's SD*.
STO J STO K		store this *SD* for later.
1.5 ENTER↑ **2** /	0.750	m = radius of the target disk.
PROB Norml: Norml$_{\blacktriangle}$	0.007	= the error probability.
2 x 1/x	72.102	so *Archibald* has to collect an arrow in the green only once in 72 shots on long-term average.

Example (continued):

One of his buddies and competitors, *Bill*, also sends his arrows to the same target disk, with his hits scattering symmetrically around the center of said disk, too. He, however, has to pick up about one out of 15 arrows in the green on average. What is his *SD* in the target plane?

Solution:

15 1/x	0.067	, i.e. about 7% of *Bill's* arrows miss the target disk.
2 /	0.033	~3% misses on either side.
1. STO J x<>y		to get the *standardized normal* distribution.
Norml^{-1}	−1.834	, its corresponding lower limit.
+/− **.75** x<>y /	0.409	m = *Bill's SD*.
STO **0** **0**		Just store it since we will need it again.
RCL K		Note that *Archibald's SD* is only ...
Δ%	−25.470	% narrower than *Bill's*, but his rate of misses is more than 5 times less.

There are applications of this methodology in industry, where the

[130] Many of our customers live in a country where long range weapons play a significantly greater role in public than in (almost all) civilized societies (feel free to check and assess irregular real-life consequences, e.g., in schools). Hence this shooting case was chosen, though refraining from using firearms.

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scattering (a.k.a. variation, *variance*) of a production process is compared with its tolerance limits. This may result in so-called *capability indices*, directly linked to the amount of scrap to be expected in the process investigated. Please consult applicable literature and standards – look for *process capability*.

Let's continue our **example**, guiding to more advanced statistics:

Bill quietly practiced in a *Zen* cloister during his summer vacation. Returning, he went to the *Golden Bow* immediately on next weekend and sent 50 arrows to his club's standard target disk. Only two missed, with one of them scratching the very edge of the disk. Cheers! But is this just a lucky chance success (within the usual statistical scattering of results to be expected)? Or is it probably related to his training efforts?

Solution:

Calculate *Bill's* new *SD*:

1.5 ENTER↑ **50** /	0.030	
2 /	0.015	= 1.5% misses on either side.
Norml^{-1}	−2.170	, the corresponding lower limit of the *standardized normal* distribution.
+/− **.75** x≷y /	0.346 m	= *Bill's* new *SD*.

Now, is this *significantly* better than his old *SD* (s_o)? Statisticians have found it is better (with a *confidence level* of 95%) if it is lower than the 95% *confidence limit* of s_o. Assume s_o was based on 60 shots, then the formula for its *single-sided lower 95% confidence limit* reads:

$$\sigma_L = s_o \times \sqrt{\frac{59}{\left(\chi^2_{59;\,0.95}\right)^{-1}}}$$

The denominator is the *inverse chi-square* for 95% probability and 59 *degrees of freedom (dof)*. Calculate inside out as usual:

59 STO I		the *dof* must be stored in **I**.
.95 χ^2: $(\chi^2)^{-1}$	77.931	compute the *inverse chi-square*.
/	0.757	
√x	0.870	
RCL x **0** **0**	0.356 m	for σ_L.

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Looks like *Bill's* training made a difference! … Wait, … with 95% confidence only … If we had required 99% instead, the lower *confidence limit* had been 0.337 m (you can verify this easily now); and then *Bill's* new weekend result had been an insufficient indicator for a *significant* improvement.[131]

Reals: Frequencies and Histograms

For the evaluation of *1D* (or two independent *2D*) samples comprising 25 data points at least, you can categorize these data in bins and analyze their frequencies within these bins sensibly.

How to proceed for new data:

1. Press STAT **CLΣ** to clear all prior statistical data.
2. Enter your sample data into STATS using **Σ+** as usual.
3. Open **HISTOG**.
4. Within this *submenu*, choose **HISTOX** if the relevant data are in the 1st column of STATS (e.g., if you entered only x data) or **HISTOY** if they are in the 2nd column. The data will be copied into a new *matrix* called HISTO. Your *WP43* will propose values for the lower limit of the lowest bin, the number of bins, and the upper limit of the top bin.
5. Modify the values for **↓BIN**, **nBINS**, and/or **↑BIN**, if you want.[132]
6. Open **HPLOT** for plotting the histogram.
7. Optionally, you may fit a 'normal' *Gauss* curve to your histogram by calling **HNORM**.

---

[131] Applying statistics may cause you to have more doubts than without – but such is life: Doubts increase with knowledge (only braindead people have never any doubts and therefore may feel great most easily).

Note that standard *confidence limits* and *levels* (also for indicating *significant differences*) may depend on the country, industry, or science you are working in. And "*significant"* is well defined in statistics – it may deviate from common language. See the chapter *Reals: Some Industrial Problems Solved* below.

But *significance* isn't everything, *standard errors* may be a better measure if applicable. Check the pertinent valid standards before blindly copying sample calculations.

[132] ***nBINS*** should be $\leq \sqrt{n}$, with ***n*** being the total number of your points. Default is $\sqrt{n}$.

Example (just for demonstration):

Plot a histogram for the following 16 data points $(x; y)$: 13, 13; 4, 5; 15, 12; 6, 4; 12, 13; 3, 2; 17, 15; 20, 17; 10, 10; 11, 10; 1, 1; 13, 11; 8, 8; 10, 8; 12, 10; 10, 9.

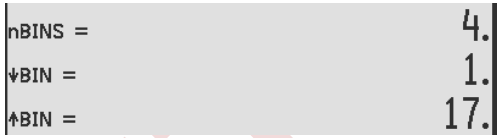
Solution:

STAT **CLΣ**

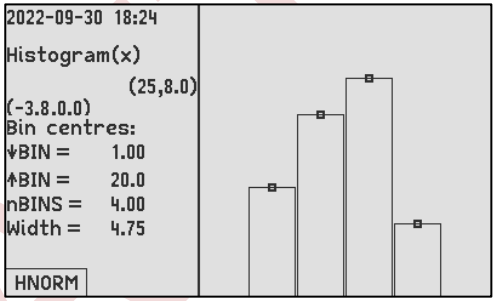
13 ENTER↑ **13** **Σ+** **5** ENTER↑ **4** **Σ+** … until **9** ENTER↑ **10** **Σ+**

▲ **HISTOG**

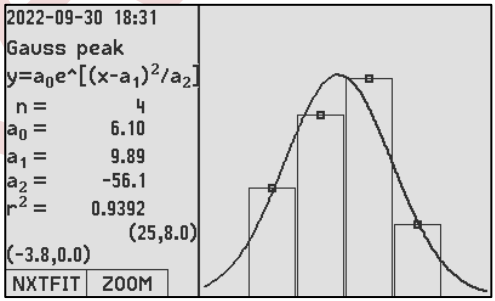
Choose **HISTOX** and you will get:



To continue with these automatic parameters, open **HPLOT** and you will see:

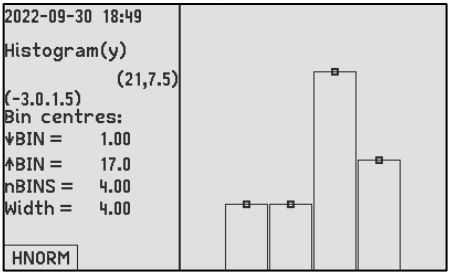


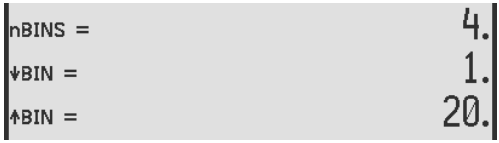
Let's check how this histogram compares with a *Gauss* curve. Just press **F1** (= HNORM) and you will get:



… looking quite well.

To do the same for the y column of your STATS data, **EXIT** this diagram and select **HISTOY**:





Continue with these automatic parameters – open **HPLOT**:

Obviously, a *Gauss* curve will not be a good fit model for these rather asymmetric data.

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Reals: *2D* Statistics – Curve Fitting, Interpolating, and Forecasting

FIT contains functions for curve fitting, featuring ten different regression models (linear, exponential, logarithmic, power, root, hyperbolic, and more – look up the *ReM*), forecasts [x̂] and [ŷ], and the *coefficient of correlation* [r]. The fit model applied will be displayed heading numeric output after any related command (like CORR, x̂, and ŷ). And after [L.R.], even the generic formula of the chosen regression model will be shown (see examples below).

The command BESTF tells your *WP43* to automatically select the regression model 'fitting your data best' (i.e. resulting in the biggest absolute *coefficient of correlation*). Then an elevated asterisk (*) will trail the name of the model chosen this way (please check the *IOI* for the nine models available for this and the parameter of BESTF). Like with all auto-functionality, you should know what you are doing here.

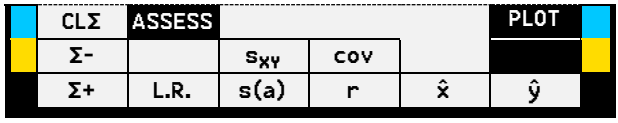
Example (from the *HP-27 OH*):

If *Galileo* had wished to investigate quantitatively the relationship between the time (***t***) for a falling object to hit the ground and the height (***h***) it has fallen, he might have released a rock[133] from various levels of the *Tower of Pisa* (which was leaning even then) and timed its descent by counting his pulse. The following data are measurements *Galileo* might have made:

t (*pulses*)	2	2.5	3.5	4	4.5 [134]
h (*Pisan feet*)	30	50	90	130	150

Unlike *Galileo*, you are equipped with a *WP43* – so what can you learn from this reported experiment? Let's look with FIX **3**:

[FIT] **CLΣ**



[133] Heaven beware! A big pebble would do as well. Just its impact must be heard – and it was more quiet there back then (far less tourists!).

[134] These raw data really don't look very plausible, and actually it is dubious whether *Galileo* made such experiments using the *Tower of Pisa* at all; but at least *HP* believed that its customers would believe that story in 1976.

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30 **ENTER↑** **2** **Σ+** returns

```
001 data point                30.000
                               2.000
```

Your next input after **Σ+** will overwrite x and shift previous y up:[135]

50 **ENTER↑** **2.5** **Σ+** **90** **ENTER↑** **3.5** **Σ+** **130** **ENTER↑** **4** **Σ+**
150 **ENTER↑** **4.5** **Σ+**

```
005 data points               150.000
                                4.500
```

▼

GaussF	CauchF		BestF		
ParabF	HypF	RootF			
LinF	ExpF	LogF	PowerF		OrthoF

BestF **448** instructs your *WP43* to choose – out of all 2-parameter fit models provided – the one suiting these recorded data best.[136]

▼

CLΣ	ASSESS				PLOT
Σ-		s_{xy}	cov		HISTOG
Σ+	L.R.	s(a)	r	$\hat{x}$	$\hat{y}$

L.R.

```
                               4.500
Power*          a1 =           1.994
y = a0 x^a1     a0 =           7.723
```

Your *WP43* chose power regression as the mathematical model fitting these experimental data best. Let's check the *correlation coefficient*:

r returns

```
Power* r =                     0.998
```

This is an almost perfect correlation. To be precise, what are the estimated errors of the curve parameters (we are doing physics, after all)?

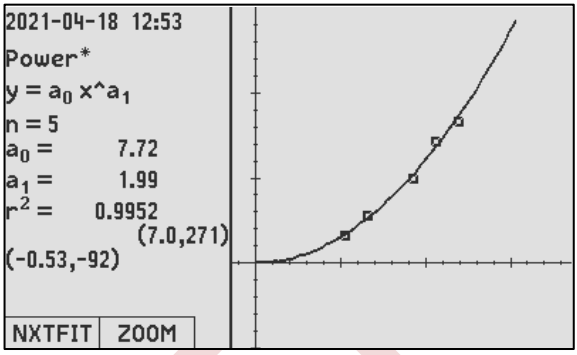
s(a)

```
s(a1) =                        0.080
s(a0) =                        0.095
```

[135] Thus, accumulating *2D* data will slowly overwrite the *stack* – if you want to save it, use STOSTK before and recall it by RCLSTK after accumulation. Note the mechanics of Σ+ were created for *1D* data on the *HP-80, HP-45, etc.* – they stayed this way also for *2D* data supported from the *HP-32* on. Cf. the first problem in previous chapter.

[136] See the *IOI* for more about BESTF.

Hence, the equation expressing the overall results of these experiments best is $h = (7.72 \pm 0.09) \times t^{1.99 \pm 0.08} = 7.72(9) \times t^{1.99(8)}$ with ***t*** measured in *pulses* and ***h*** in *Pisan feet*.[137] See the plot produced by ASSESS and ZOOM showing the data points and the fit curve. *Galileo* could not know around 1600 yet, but we know today that



$$h = \frac{1}{2}\, g\, t^2.$$

Determining the size of a *Pisan foot* and *Galileo's* heartbeat frequency during his reported arduous experimental work with rocks is left as an exercise for the reader.

In addition, we found the following **example** in various *HP* calculator manuals of 1976 - 78. It reads typical for the thinking at that time:

Big *Lyle Hephaestus,*[138] owner-operator of the *Hephaestus Oil Company*, wishes to know the slope and y-intercept of a least squares line for the consumption of motor fuel in the *United States of America*[139] against time since 1945 (!). He knows the data given in the table:

Motor fuel demand (millions of *barrels*)	696	994	1330	1512	1750	2162	2243	2382	2484
Year	1945	1950	1955	1960	1965	1970	1971	1972	1973

[137] This equation shows two alternative ways for specifying and printing uncertainties.

[138] Perhaps his ancestors emigrated from Greece: *Hephaistos* (Ἥφαιστος) was the ancient Greek god of fire, forging, and technology in general. Sources tell he even designed a rotating space station orbiting the Earth, looking like *Kubrick's* in '*2001*'.

[139] Compare *los Estados Unidos Mexicanos*, for instance. But you have guessed the location probably by the weird units used. Note a *barrel* of oil deviates from a barrel of whiskey and a barrel of whisky, of course. And don't forget the barrels of vine and beer. Also the chemical industry uses barrels. So many different volumes! Now you know why some people use *barrels* for measuring. Though help is on the way in *Section 5*.

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Solution (continuing with BESTF **448** but FIX **1**):

Hephaestus could draw a plot of motor fuel demand against time. However, with his *WP43*, *Hephaestus* has only to key the data into the calculator using the [Σ+] key, then press [L.R.].

[FIT] [CLΣ]

696 [ENTER↑] **1945** [Σ+] [140] **...** **...** **2484** [ENTER↑] **1973** [Σ+]

L.R.	Linear* a_1 =	61.2
	$y = a_0+a_1x$ a_0 =	−118 290.6

Your *WP43* selected linear regression as the mathematical model fitting these data best. Let's check the *correlation coefficient*:

r	returns	Linear* r =	1.0

Perfect! Based on this good correlation result, *Hephaestus* confirms the automatic choice and is even tempted to extrapolate the computed trend of motor fuel demand to (then) future years.

Example (continued):

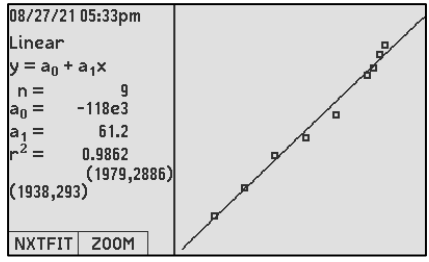
If *Hephaestus* wishes to predict the demand for motor fuel for the years 1980 and 2000, he keys in the new ***x*** values and presses [ŷ].

1980	ŷ	returns	Linear* ŷ =	2 808.6
2000	ŷ	returns	Linear* ŷ =	4 031.9

These were forecasts (i.e. extrapolations based on the fit model employed) of the demands in 1980 and 2000 at that time. Similarly, to determine the year that the demand for motor fuel is expected to pass 3 500 million *barrels*, *Hephaestus* keys in **3500** (the new value for ***y***) and presses [x̂].

3500	x̂	returns	Linear* x̂ =	1 991.3

– the demand was expected to pass 3.5 billion *barrels* in 1992.[141]



[140] The boss computes himself! And he is even able to do it properly! Looks like that was a time before general managers, CEO's, and big staffs became fashionable.

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Another **example** (from the *HP-15C OH*):

Agronomist *Silas Farmer* has developed a new variety of high-yield rice, and has measured the plant's yield as a function of fertilization. Use the [Σ+] function to accumulate the data below to find the values for Σx, Σx^2, Σy, Σy^2, and Σxy for nitrogen fertilizer application (x) versus grain yield (y).

x	Nitrogen applied (kg / ha)	0.0	20.0	40.0	60.0	80.0
y	Grain yield (t / ha)	4.63	5.78	6.61	7,21	7,78

Solution:

We are confident you can accumulate the data yourself using Σ+ and recall the sums from Σ now. Let's proceed to the regression *Silas* is really interested in:[142]

[DSP] [2]

[FIT] [▼] LinF returns LinF selected for L.R.

[▼] L.R. returns

Linear a_1 = 0.04

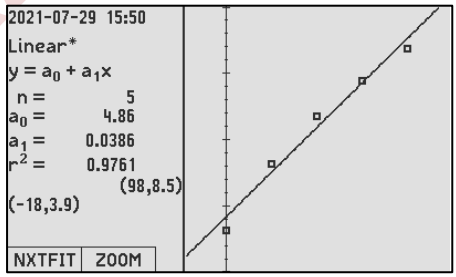
$y = a_0 + a_1 x$ a_0 = 4.86

Let's check the correlation coefficient:

r returns Linear r = 0.99

Plot the data using ASSESS again. – The plot looks good enough to work with the fit line in the range covered by data points; e.g., find how much grain is expected with 70 kg/ha fertilizer:[143]

70 [ŷ] returns 7.56 t/ha.



[141] If he had plotted his data, *Hephaestus* could have been warned looking at his last three data points. Frequently, plots carry extra information which may be lost when dealing with numbers only. *HP's* problem turns out being a suboptimal choice for extrapolating without plotting – actually it is an inadvertent warning of thoughtless number crunching.

[142] Hardly any real-world agronomist would be interested in the sums *HP* asked for, and also mean and *SD* were computed in that problem for demonstration purposes only.

[143] Extrapolations to higher nitrogen applications must be rated dubious since the plotted data show an evident curvature; so another fit model should be applied there.

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Reals: Chi-Square Statistic – Checking Expectations

The *chi-square statistic* measures the goodness of fit between two sets of frequencies.[144] It's used to test whether a set of observed frequencies differs from a set of expected ones sufficiently to reject the hypothesis under which the expected frequencies were obtained.

In other words, you are testing whether discrepancies between the observed frequencies (O_i) and the expected frequencies (E_i) are *significant*, or whether they may reasonably be attributed to chance. The formula generally used is

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

If there is a close agreement between the observed and expected frequencies, χ^2 will be small. If the agreement is poor, χ^2 will be large.[145]

Let's demonstrate the application using the following **example**:

A suspect dice from a *Las Vegas* casino is brought to an independent testing firm to determine its bias, if any. The dice is tossed 120 times and the following results obtained:

Number	1	2	3	4	5	6
Frequency	25	17	15	23	24	16

Solution (with FIX **0**):
The *expected* frequency is $120/6 = 20$ for each number here. For calculating χ^2, just enter:

25 ENTER↑ **20** − EXP x^2	25
17 ENTER↑ **20** − x^2 +	34
15 ENTER↑ **20** − x^2 +	59
23 ENTER↑ **20** − x^2 +	68

[144] *Goodness of fit* tells you how good both sets match, e.g., observed experimental data and a theoretical model. This and the following case are quoted from the *HP-27 OH*.

[145] Do not confuse this χ^2 defined here with the χ^2 distribution mentioned above. They are different! Unfortunately, however, both chi-squares are called and spelled equally. It looks like the mathematical naming commission was inattentive here at the crucial time, perhaps distracted by discussing the outcome of a recent casino visit (note probability theory and statistics were originally invented for comprehending gambling).

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24 ENTER↑ 20 − x^2 +	84
16 ENTER↑ 20 − x^2 +	100
20 /	5

Now, is this χ^2 large or small? Statisticians have found it is to be considered 'small' if χ^2 is less than the value of the inverse χ^2 *CDF* for the *degrees of freedom* (here $n - 1 = 5$) and the *confidence level* applicable (here 95%). As observed above already, also this χ^2 function is provided in your *WP43*. Simply key in:

5 STO I	5 for the *degrees of freedom*;
.95 PROB χ^2: $(\chi^2)^{-1}$	11.

Since 5 is less than 11, χ^2 is small enough to conclude that this dice is fair (with 95% confidence). [146]

Reals: Some Industrial Problems Solved

To get an idea of real-life opportunities covered by your *WP43* and of some constraints inherent to statistics, see the applications shown below. All of them are demonstrated here using the traditional *4LS* but work with the *8LS* as well.

Application 1 (quick and easy *MSA*):

Whenever you want to know something about specific properties of an object, you shall measure it. Therefore, knowing the properties of the measuring instrument or system you are going to use is essential.

Example:

Your colleagues in *R&D* have happily specified that particle accelerator beam pipes made of a special stainless steel shall have a magnetic susceptibility

[146] A *confidence level* of 95% corresponds to an *error probability* of 5%. Does this satisfy you? These 5% / 95% are typical settings for the *USA*, as in the vintage case quoted here. For comparison, e.g., in Germany 1% / 99% are standard. See examples below.

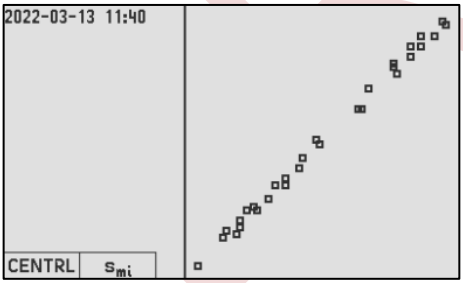
Such *chi-square tests* may be also applied to entire curves measured: then every data point of the curve shall be compared with the value predicted by some theoretical model at the same location.

TNG: *Confidence level = Vertrauensniveau*, *conf. limit = Vertrauensbereichsgrenze*.

≤ 0.01 also at their welding seams. How can you verify whether the susceptibility meter available in the lab of your organization is sufficiently precise (a.k.a. 'accurate enough') to control series production of such pipes? Or is it inevitable to invest in a better instrument? [147]

Solution (measure the precision of your measuring system):

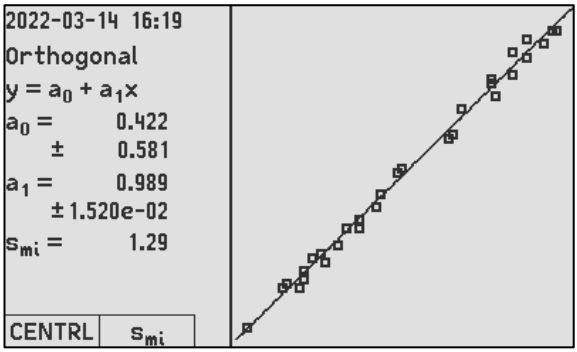
1. Collect ≥ 30 metal samples covering the susceptibility range you are interested in (this range could extend, e.g., from 0.00 to about 0.02 here).[148] Mark each sample unambiguously (e.g., by numbering it). Prepare a table with as many numbered rows as you have samples.
2. Use the measuring system under investigation to measure each sample carefully under controlled conditions. Write each measured value next to the respective sample number; record as many decimals as possible.
3. Measure all samples a 2nd time under the same conditions, but in another sequence (just shuffle the samples). Do not allow for looking at the 1st data measured (hiding these data will be essential if you acquire the values manually)! Open a 2nd column and record each 2nd measured value in the row carrying the respective sample number again.
4. Get your *WP43*. Press [FIT] [CLΣ] to clear all statistical data. Then enter all pairs of values using [Σ+]. The 1st measured value shall be y, the 2nd x – thus, input will be ***mv1*** [ENTER↑] ***mv2*** [Σ+] for each sample (instead, you can provide your data in a *matrix* – see pp. 194f).



5. Call [PLOT] to plot these points in a new window. The plot should look like an ant trail along the diagonal – see a real-world plot here.
6. Press [CENTRL] to fit an orthogonal regression line to your data points. Check if it deviates significantly from the diagonal $y = x$. Coarsely, this will

[147] Specifications and demands are written easily – actually verifying them in a production environment may become a real challenge, however. Check if your means suffice.

[148] Nature allows for positive susceptibilities only.
Note it is not required to know the exact susceptibilities of your samples beforehand: they shall just fall in said range, cover it fairly homogenously (cf. the plot below), and must stay constant in your measurements. No need for any investment in expensive gauges here – real life has proven various pieces of stainless steel scrap suffice (even in Novosibirsk). But 30 samples are the minimum required for this measurement.



apply (with a confidence of some 95%) if $a_0 \pm 2\,s(a_0)$ excludes **0** or $a_1 \pm 2\,s(a_1)$ excludes **1**.[149]

If it does then your measuring system may be running up still (wait some more time for warmup) or show an unstable zero (check and fix); then return to 2.

7. Else (i.e. if $a_0 \pm 2\,s(a_0)$ includes **0** and $a_1 \pm 2\,s(a_1)$ includes **1**) press [s_{mi}] to push the precision of your measuring instrument investigated on the *stack*. This measured precision applies under the boundary conditions prevalent during steps 2 and 3 of this test, and is definitely more realistic than most catalog values stated by manufacturers.[150]

   Then enter

   30 [×] [1/x] and multiply with the tolerance you have to control (**0.01** in this case). A result **≥ 1** will tell you this measuring instrument, device, machine, or system may be used for controlling series production with said tolerance zone (i.e. it is a *capable* instrument for this job) under said conditions; else you shall strive for a more precise device, better measuring conditions, or a wider tolerance. See *ReMA I* for more.

Application 2 (scrap rate, confidence limits):

Assume you own a little tool shop producing axle pins automatically in series, and want to know the quality of the parts produced after setup of the machine. You drew a *representative sample* of pins (all being nominally equal parts!) and measured their actual sizes precisely using a *capable* instrument (cf. Appl. 1). How do you know your batch will be ok?

Example:

A batch of turned pins is produced on a precision lathe. A sample of 10 pins

[149] If the gross shape of your scatter plot deviates fundamentally from the one shown here, this may point to surprises hidden in your experimental setup and/or data acquisition system. See *ReMA I*.

[150] Although displaying all ≥ 30 measured points clearly separated might exceed the resolution of a small pocket calculator screen by far, the calculations in steps 6 and 7 carry full precision, and the resulting s_{mi} will be correct.

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is drawn and their diameters measured with a precision gauge: 12.356, 12.362, 12.360, 12.364, 12.340, 12.345, 12.342, 12.344, 12.355, and 12.353 mm. From earlier large scale investigations, you know that diameters from this production process follow a *normal* distribution.[151]

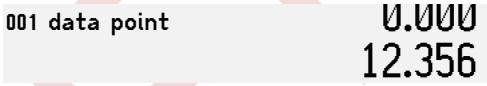
What pin diameters will you get in larger scale batch production? Statistics can't tell you about all of them but it will tell you where to expect almost all (e.g., 99%) of them.

Solution:

DISP FIX 3

STAT CLΣ

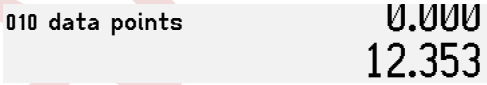
0 ENTER↑ 12.356 Σ+



Continue accumulating the remaining measured sample data:

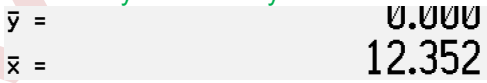
12.362 Σ+ …

12.353 Σ+ [152]



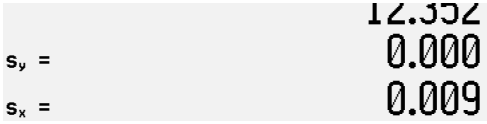
Knowing these pins are drawn from a *Gaussian* process, you get the best estimates for mean and standard deviation of your batch by

x̄



STO I

s



STO J We store x̄ and s_x here for the next steps already.

[151] This is a simplified case. In real life, also other errors may contribute.

[152] Some experimental data sets may consist of a series of ***x***-values (or ***y***-values) differing from each other by a comparatively small amount. Here you may save considerable input time by keying in only the differences between each value and a number ***m*** close to the average of the values. This ***m*** must be added to the calculated result of x̄, ŷ, x̂, and to a_0 returned by L.R. then.

Thus above, you could have simply entered **56** Σ+, **62** Σ+, **60** Σ+, etc. When calculating x̄ afterwards, divide the answer by **1000**, and add **12.3**. The computed *SD* had to be divided by **1000** then as well.

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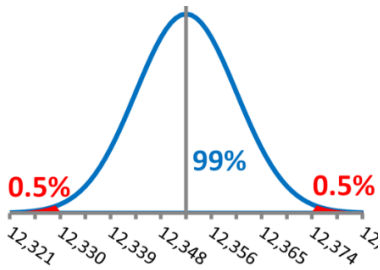
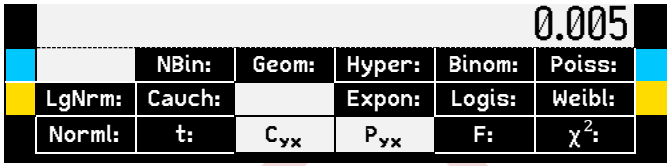
If 99% of such a batch is found inside some symmetric limits of a *Gaussian* process 0.5% will be out on either side.



Thus, based on the ten pins analyzed, you may expect 0.5% of all pins with diameters less than

.005 PROB



Norml:

$Norml^{-1}$	12.330

and another 0.5% with diameters greater than

.995 $Norml^{-1}$	12.375

If you should observe significantly more than 0.5% of your pins beyond either limit, this indicates your process may be running out of control.

Assume these pins shall have a nominal diameter of 12.35. Then – based on this sample analysis – you can safely commit holding a tolerance of ± 0.05 (you will hardly produce any scrap as long as your process continues running the way you found it). If your customer would try, however, forcing you to deliver a tolerance of ± 0.02, you must expect some losses:

12.35 ENTER↑ ENTER↑	12.350	
.02 −	12.330	= lower limit.
$Norml_{\triangle}$	0.006	= lower scrap = 0.6%.
x≷y	12.350	
.02 +	12.370	= upper limit.
$Norml_{\triangle}$	0.021	= upper scrap = 2.1%.
+	0.026	= total scrap.

What will hurt you even more than these 2.6% scrap you must expect now (i.e. more than 1 out of 40 pins) will be the inevitable necessity to establish a

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very precise and reliable sorting device to warrant only pins within 12.350 ± 0.020 will pass to your customer. Thus, stay firm (if you can afford it) and refuse that customer request to constrict your tolerance limits – it may well be you can't afford becoming weak here. Remember: the most cost effective parts filter in production is the one you don't need!

Are you interested in the mean pin diameter of your batch? So you know how much space you must provide to store a stack of, e.g., 50 pins? Determine the applicable mean and the size of its variation; then use them to find both upper and lower limit confining the mean with a probability of, e.g., 95%.

Example (continued):

Since we have got a sample drawn out of a *Gaussian* process, the *arithmetic mean* is applicable, the *standard error* indicates its variation, and *Student's* ***t*** is required for confidence limits. For the latter, we need its *degrees of freedom*:

Σ n	10.000	recall the number of data points.
1 − STO I	9.000	store the *degrees of freedom* (for computing t^{-1} below)
STAT s_m	0.003	is the standard error.

Having 95% inside means having 2.5% outside at either end (cf. previous diagram).[153] Thus, one must generally take 0.025 and 0.975 as arguments in two subsequent calculations using the *quantile function* to get both 95% limits below and above the sample average result:

.025 PROB t: $t^{-1}(p)$	−2.262
x	−0.006
STAT $\bar{x}$	12.352
x≷y R↓ [154]	12.352

[153] The value of 95% is called the *confidence level* of this calculation. In this case, you calculate the 95% *confidence limits* for the mean value.

Instead of 95%, also 99% are frequently applied. We recommend checking the applicable valid standards before blindly copying any calculations from here. And you are free to apply other *confidence levels* wherever they fit your needs and you can justify them.

[154] $\bar{x}$ returns both $\bar{x}$ and $\bar{y}$ (as shown above). Only $\bar{x}$ is interesting here, so pressing x≷y R↓ moves $\bar{y}$ quickly out of the way. In a program, STK DROPy will be a better

x≷y	-0.006	
+	12.346	= lower limit.
RCL L	-0.006	= last *x*.
2 x −	12.358	= upper limit.[155]

Now you know what to expect for the future average diameter of such batches. Hence a rail being

50 x	617.900	long inside will suffice for holding 50 pins in 97.5% of all cases.

12.346 and 12.358 are the *95% confidence limits* of the mean calculated above. So there is a chance of 2.5% that the mean will be < 12.346 (don't care here) and an equal chance that it will be > 12.358. These chances[156] are inevitable consequences of the fact you know something about a small *sample* only(drawn out of a large *population*), but want or have to tell something about said total *population*. If you can't live with these uncertainties or the widths of the confidence limits, don't blame statistics but draw a larger sample or collect more precise data instead. At the bottom line, improving precision and stability of production processes is the key.

Application 3 (significant change):

Assume you have drawn a sample out of an arbitrary industrial process at day 1. Then you have changed the process parameters, waited for stabilization, and have drawn another sample of same size at day 2 (there may well have been a longer time interval between both sampling days). Being serious, you have meticulously measured and recorded a critical quantity (e.g., a characteristic dimension) for each specimen investigated at both days. Now, is there any *significant difference*?

The following three-step test may save you unwanted embarrassment in your next presentation or after your next publication:

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alternative since it leaves the *stack* order as is (see *Section 3*).

[155] The upper *confidence limit* can be calculated so easily since $t^{-1}(p)$ is symmetric around the mean value. Else you had to repeat above calculation for an input of 0.975.

[156] Statisticians call them '*probabilities of a type I error*' or '*probabilities of an error of the 1st kind*'.

TNG: *Type I error = Fehler 1. Art.*

1. Accumulate your sample data. Then let your *WP43* compute the mean values and *standard errors* for both samples,[157] and their *normalized distance*

$$d = |\bar{x} - \bar{y}| \,/\, \sqrt{s_{mx}^2 + s_{my}^2}\,.$$ The calculation could look like this:

STAT s_m	returns both standard errors in **X** and **Y**.
EXP x^2 $x\leftrightarrow y$ x^2 + $\sqrt{x}$	so this is the entire denominator.
STAT $\bar{x}$	returns both $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$.
− |x|	thus, this is the numerator and …
$x\leftrightarrow y$ / STO D	this is d (choose another destination if SSIZE8 is set).

2. Let your *WP43* compute the critical limit t_{cr} of *Student's t* for f *degrees of freedom* and a probability of 97.5% now:

Σ n	recall the number of samples measured.
1 − STO I	store the *degrees of freedom* f for *Student's* t.
.975 PROB t: $t^{-1}(p)$	as mentioned above, the requested *quantile function* lives in PROB. It takes the *degrees of freedom* stored in **I** to get t_{cr}.

If $d < t_{cr}$ then the test indicates the difference between both samples is due to random deviations only. Congratulations – you have got a robust process regarding the parameters you changed!

Else continue.

3. Let your *WP43* compute a new critical limit t_{cs} for f and 99.5%:

.995 $t^{-1}(p)$	get t_{cs}.

If $d \geq t_{cs}$ then this test indicates a *significant difference* between both samples. Congratulations – your parameter change caused a significant effect!

Else (i.e. for $t_{cr} \leq d < t_{cs}$) you simply can't decide seriously based on the information you have – your samples may contain too little data, your measurements were not precise enough, the process observed is scattering too far, or whatever else. Though don't let your excited audience lead you in temptation: stay silent or mumble something like "*investigation in progress*" at the utmost – management might want a quick answer but there is no solid basis for it yet.

[157] This test assumes your samples were both drawn from a *Gaussian* process which is frequently the case in real life (but shall be verified).

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Application 4 (operating characteristics):

Assume you draw a representative sample of 20 parts out of a production batch of 100 parts and check this sample thoroughly. What is the probability ***P*** to find at least one random defect in such a sample if the overall probability for a defect in such a batch is known to be 5% or 1%, for instance?

This is almost a textbook **example** for applying the *hypergeometric* distribution. P(***n*** ≥ 1) = 100% – p(***n*** = 0). Thus, the solution is:

DSP 3		
100 STO K	batch size	
20 STO J	sample size	
0.05 STO I	5% overall defect probability	
0 PROB Hyper: Hyper▲	returns	0.319 for p(***n*** = 0)
1 − +/−	returns	0.681 for P(***n*** ≥ 1)
0.01 STO I	1% overall defect probability	
0 Hyper▲	returns	0.800 for p(***n*** = 0)
1 − +/−	returns	0.200 for P(***n*** ≥ 1)

Even with 5% defects in the batch, the odds are about 1 out of 3 that no defect at all is detected in such a relatively large sample. Such sample tests are popular but definitely inadequate for controlling industrial processes with overall (target) defect probabilities less than 1%.[158]

Application 5 (lifetime of punching dies):

Assume being responsible for a workshop punching lots of small precision parts out of sheet metal. The dies you use, manufactured in batches according to the state of the art, will only withstand a limited number of punching cycles. For each batch of dies supplied, your technicians meticulously record the number of cycles performed until a die breaks.

Example with FIX **3**:

The following cycle numbers were recorded for a batch of 20 dies (sorted according to their lifetimes): 37 974, 51 340, 62 590, 77 837, 79 637,

[158] Traditionally, such problems were solved approximately using the *Poisson* or *binomial* distributions since these mathematical models are significantly easier to calculate. But you are equipped with your *WP43* which does the drudgery for you – be happy!

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106 147, 110 632, 112 434, 130 502, 156 003, 166 979, 167 339, 183 419, 205 518, 213 999, 252 171, 254 619, 301 190, 322 290, and 386 315. Derive a characteristic lifetime for this batch of dies from the data collected.

Solution:

Such lifetime data follow a *Weibull distribution*. For evaluating above record, we shall compute summed transformed 'frequencies' with $n = 20$ and i counting the dies:

$$y_i = ln\left\{ ln\left[\frac{1}{1-\left(\frac{i-0.3}{n+0.4}\right)} \right] \right\}$$

Solving this problem cries for a program. Programming will be covered comprehensively in *Section 3* – for the time being, just press GTO . . P/R and key in:

LBL α √x x e^x 9 ENTER↑ which will be echoed LBL 'FREQ'.

.3 − 20.4 / 1 x≷y − completes the denominator.

1/x ln ln

RTN

Press P/R again to leave *PEM*.

Now enter 1 XEQ PROG FREQ and you will get y_1. Repeat for 2 to 20. Results are:

i	1	2	3	4	5	6	7
y_i	−3.355	−2.442	−1.952	−1.609	−1.340	−1.116	−0.921

i	8	9	10	11	12	13	14
y_i	−0.747	−0.587	−0.438	−0.297	−0.160	−0.026	0.107

i	15	16	17	18	19	20
y_i	0.243	0.384	0.535	0.704	0.910	1.216

Call FIT CLΣ. Then enter these y_i and the corresponding numbers of recorded cycles c_i via Σ+ into STATS as usual. Plotting these twenty data points (c_i, y_i) over a logarithmic horizontal cycle scale should result in a straight line.

Instead of using such a scale you can let your *WP43* perform a logarithmic curve fit through these data points: choose LOGF and call ASSESS – it will return a plot as shown overleaf. The resulting *characteristic lifetime* of this batch of twenty dies is

0 x̂

Logarithmic x̂ = 191 185.

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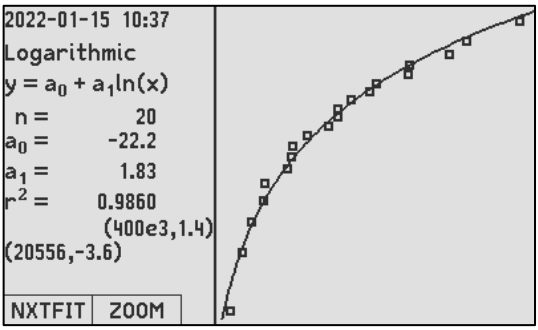

After tool composition or manufacturing processes were refined, you receive a new batch of dies. Based on a record of their lifetime data, you can then assess the benefit of these changes if there is any (see *Weibull distribution* in the *IOI*).

Note you must not use neither $\bar{x}$ nor s here.

STAT and FIT contain many more statistical functions (like *covariances*, means and standard deviations for weighted data, *geometric means* and *scattering factors*) – just look them up there and check the respective entries in the *IOI*.

More examples of statistical applications can be found in the manuals of various vintage *HP* calculators, especially of the *HP-27* and *HP-21S*.

We sincerely recommend consulting a good statistics textbook for more information about statistical methods in general, the terminology used, the mathematical models and their constraints, before actually applying them. Many methods only make sense from a certain minimum sample size on.

Reals: Summary of Functions

The majority of the functions your *WP43* features are for calculations operating on *reals*. It provides many more than the numeric functions shown on pp. 21ff, 33ff, and 88ff in various applications and examples. See all *real number* functions listed below:

- **General mathematics:**
  - *Monadic* functions:

    [1/x], [|x|], [+/-], [√x] and [x²], [∛x] and [x³], [√(1+x²)], [2^x] and [lb x], [10^x] and [lg], [e^x] and [ln], [sin], [cos], [tan], and their inverses work as shown above and you learned in school (see also pp. 136ff for more information about angular I/O),

for [sinh], [cosh], [tanh], and their inverses compare pp. 102f,

[e^x-1] and [ln(1+x)] return more accurate results for $x \approx 0$,

[ceil] returns the smallest integer $\geq x$, while [floor] returns the greatest integer $\leq x$,

[SDL] n shifts digits left by n decimal positions, equivalent to multiplying x times 10^n,

[SDR] n shifts digits right by n decimal positions, equivalent to dividing x by 10^n,

[(-1)^x] returns $cos(\pi x) + i\ sin(\pi x)$ for non-integer x (see pp. 166ff);

[#] converts a closed *real* to a *short integer* of base 2…16 (see *Integers* some chapters below); [→BIN], [→OCT], [→DEC], and [→HEX] cover the most popular bases.

- *Dyadic* functions:

  [+], [−], [×], [/], [y^x], and [ˣ√y] work as was shown above;

  use [IDIV] for integer division (and [IDIVR] if you want also the remainder returned in Y),

  (**Example:** **7.8** [ENTER↑] **3.2** [INTS] **IDIV** returns 2)

  [log$_x$y] for the logarithm of y for the base x

  (**Example:** **655.5** [ENTER↑] **5** [EXP] log$_x$y returns 4.029 6…),

  [RMD] for the remainder of y/x (see p. 157 for examples),

  [MOD] for $y \bmod x$ (see p. 158 for examples),

  [max] (or [min]) for the maximum (or minimum) of x and y; and

  [||] returns $\left(\frac{1}{x}+\frac{1}{y}\right)^{-1}$ for $x \times y \neq 0$, else **0**, being handy in electrical engineering in particular.

- *Triadic* functions:

  [×MOD] returns $(z \times y) \bmod x$ for $x > 1$, $y > 0$, $z > 0$;

  [^MOD] returns $(z^y) \bmod x$ for $x > 1$, $y > 0$, $z > 0$.

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- **Isolating parts of numbers:** Use the monadic functions …

  [EXPT] for the exponent of x and [MANT] for its mantissa,

  [|x|] for the absolute value of x,

  [FP] (or [IP]) for the fractional (integer) part of x, preserving its sign,

  and [sign] for the *signum* of x; SIGN will return **1** for $x > 0$, **–1** for $x < 0$, and **0** for $x = 0$ or non-numeric data.

- **Rounding:**

  [RDP] ***n*** rounds x to n decimal places in FIX format (e.g., **1.234 567 89×10⁻⁹⁵** RDP **99** returns 1.2346×10^{-95}),

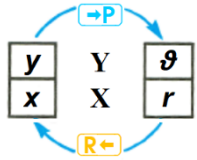
  [ROUND] rounds x using the current display format,[159]

  [ROUNDI] rounds x to next integer (½ rounds to 1),

  [RSD] ***n*** rounds x to n significant digits, and

  [SHOW] displays full precision of x in top numeric row until next keystroke

- **Conversions:**

  [→P] converts rectangular to polar *coordinates* (cf. pp. 21f), while [R←] converts vice versa.

  For *angular*, *time*, and *date conversions* see pp. 139ff and 199ff, for *unit conversions* look up pp. 299ff.

- ***Boole's* algebra:**

  [AND], [NAND], [OR], [NOR], [XOR], [XNOR], and [NOT] operate on *reals* like they did in the *HP-28S*, i.e. x and y are interpreted before executing the operation. Zero is 'false' (= **0**); any other number is 'true' (= **1**).

  Example: **13.5** [ENTER↑] **–7.2** [BIT] [AND] returns 1 .

- **Probability & statistics** (unless introduced on pp. 105 - 129 already):

  [x!] returns the factorial of x (cf. p. 103) or $\Gamma(x+1)$,

[159] Works like RND did on *HP-42S*.

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[C_{yx}] / [P_{yx}] compute combinations / permutations (cf. pp. 103f),

[Γ(x)] computes the *gamma function* of x,

[lnΓ] returns the natural logarithm of the *gamma function*, allowing also for computing great factorials.

Example: What is *5432!* ?
Remember $\Gamma(x+1) = x!$ So, entering **5433** [X.FN] **lnΓ** **10** [ln] [/] returns 17 931.480 374 010 87 as decadic logarithm of the result. Then calling [PART] **FP** [10^x] will return 3.022 55… for its mantissa.
Thus, $5\,432! \approx 3.022\,55 \times 10^{17\,931}$.

[RAN#] returns a uniformly distributed (pseudo) random *real* $x \in [0, 1)$,

[SEED] stores a seed (i.e. a start value) for RAN#,

Beyond, PROB covers the 14 distributions introduced on pp. 105ff.

[Σ] contains all the accumulated sums of your STATS data, callable by their *names*. Summon each of them by calling its *name* (no need to memorize any register numbers in this matter anymore).

[STAT] comprises the summation commands [Σ+], [Σ-], and [CLΣ], various mean values ([$\bar{x}$], [$\bar{x}_w$], [$\bar{x}_G$], [$\bar{x}_H$], [$\bar{x}_{RMS}$]), sample standard deviations ([s], [s_w]) and standard errors ([s_m], [s_{mw}]), population standard deviations ([σ], [σ_w]), various scattering factors ([ε], [ε_m], [ε_p]), furthermore [MEDIAN] and related values, **HISTOG** for histograms, and a link to FIT.

[FIT] contains [Σ+], [Σ-], and [CLΣ] again, then correlations and covariances ([r], [cov]), all commands and menus concerning curve fitting ([L.R.], the ten fit models, **ASSESS** for plotting and assessing fits, as well as forecasting for *x* and *y*) and **PLOT** for assessing the real-life performance (or quality) of measuring instruments and systems (cf. the examples above).

Turn to the *ReM* for comprehensive information about all probability and statistical functions provided on your *WP43*, in particular to the *IOI* and *ReMA I*.

DRAFT

- **Percentages:**

  [%] calculates $^{xy}/_{100}$, leaving y as is (so you can easily calculate another percentage of the same base after CLX).

  Example (from the *HP27 OH*):

  If you buy a new car, you have to figure the sales tax percentage, then add that to the purchase price to find the total cost of the car. ... For example, if the sales tax on a $6200 car is 5%, what is the amount of the tax and total cost of the car?

  6200 [ENTER↑] **5** [FIN] [%] returns 310. US$ for the sales tax;
  [+] returns 6 510. US$ for the total cost.

  If the dealer[160] gives you a 10% discount on the car, what will be your total cost?

  6200 [ENTER↑]
  10 [%] [−] returns 5 580. US$ for the discounted price;
  5 [%] [+] returns 5 859. US$ for the total cost.

  [Δ%] calculates the percentage of change from y to x, returning $100\ ^{x-y}/_{y}$, leaving y as is (for same reason as with [%]). Feel free to use [Δ%] also for calculating *markup* or *margin*:

  Example:

  You purchase ink cartridges for 21.99 US$ wholesale and retail them for 26.50 US$. What percent is your *markup*[161] and what percent is your *margin*?[162]

  21.99 [ENTER↑] **26.5** [Δ%] returns 20.5 % *markup*.
  [x≷y] **26.5** [x≷y] [Δ%] returns −17.0, i.e. 17 % *margin*.

  [%MRR] computes the mean rate of return in % per period with y = present value, x = future value after z periods,[163]

---

[160] *TNG:* Dieser Dealer ist ein ehrenwerter Mann.

[161] *Markup* is the price difference as a percentage of cost (or wholesale) price.

TNG: *Markup* = *Aufschlag* (z.B. auf die Produktionskosten), *Handelsspanne* (aber siehe nächste Fußnote).

[162] *Margin* is the price difference as a percentage of selling (or retail) price.

TNG: *Margin* = *Handelsspanne, Marge* (bezogen auf den Endpreis). Der Begriff *Handelsspanne* ist also im Deutschen leider mehrdeutig.

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〔%T〕 calculates $100\ x/y$ (called "% of total"), leaving y as is,

〔%Σ〕 returns $100\ x/\sum x$, and

〔%+MG〕 calculates a sales price by adding a *margin*[162] of x % to the cost y;[163] you may use %+MG for calculating net amounts as well – just enter a negative percentage in x.

Example:
Total billed = 221,82 €, VAT = 19%. What is the net?

221.82 〔ENTER↑〕 **19** 〔+/−〕 〔FIN〕 〔%+MG〕 returns 186,40 (€).[164]

- **Advanced mathematics** (see the *IOI* and *ReMA I* for comprehensive information about the functions following):
  - *Monadic* functions:

    〔B_n〕 and 〔B_n*〕 return the *Bernoulli numbers*,

    〔erf〕 and 〔erfc〕 the *error function* and its complement,

    〔FIB〕 the extended *Fibonacci number*,

    〔g_d〕 and 〔g_d^{-1}〕 the *Gudermann function* and its inverse, and

    〔NEXTP〕 the next *prime number* greater than x (note 〔PRIME?〕 tests if $|\mathrm{IP}(x)|$ is *prime*, see *Section 3*);

    〔sinc〕 returns $sin(x) / x$ and 〔sincπ〕 returns $sin(\pi x) / \pi x$ for $x \neq 0$ and **1** for $x = 0$,

    〔W_p〕 returns the principal branch of *Lambert's W* for given $x \geq -1/e$, 〔W_m〕 the negative branch of it, and

[163] See the *ReM* for the formula.

[164] Every engineer or scientist should be able to produce the very same result distinctly faster via **221.82** 〔ENTER↑〕 **1.19** 〔/〕.

Finding functions like **%+MG** and **%T** provided on financial calculators, one might get the impression that average financial people might be mathematically slightly challenged and need some extra support. On the other hand, there was a saying in technical quarters (before 2008 already):

"*Wenn man sieht, was Kaufleute allein mit Plus und Minus alles anstellen, sollte man sie an höhere Rechenarten erst gar nicht ranlassen.*" (translated: '*Looking at the results financial/business people produce with plus and minus alone, their access to more advanced operations should be denied.*')

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〔W^{-1}〕 returns x for given W_p (≥ −1),

〔K(m)〕 and 〔E(m)〕 return the *complete elliptic integrals of the 1st* and *2nd kind*, and

〔ζ(x)〕 *Riemann's Zeta function* at location x .

Call 〔H_n〕 for the *Hermite polynomials* for probability and

〔H_{np}〕 for the *Hermite polynomials* for physics,

〔L_n〕 for *Laguerre's polynomials* and 〔$L_{n\alpha}$〕 for his *generalized* ones,

〔P_n〕 for the *Legendre polynomials*,

〔T_n〕 for the *Chebyshev polynomials of 1st kind* and 〔U_n〕 for his *polynomials of 2nd kind*.

- *Dyadic* functions:

  〔AGM〕 returns the *arithmetic-geometric mean*,

  〔$J_y(x)$〕 (〔$Y_y(x)$〕) the *Bessel function of 1st (2nd) kind and order* y ,

  〔sn(u,m)〕, 〔cn(u,m)〕, and 〔dn(u,m)〕 the *Jacobi elliptic sine*, co*sine*, and *amplitude functions*,

  〔β(x,y)〕 *Euler's Beta function* and 〔lnβ〕 its natural logarithm,

  〔Γ_p〕 and 〔Γ_q〕 return the *regularized gamma functions*,

  〔γ_{xy}〕 the *lower* and 〔Γ_{xy}〕 the *upper incomplete gamma function,*

  〔Z(φ,m)〕 returns *Jacobi's Zeta function*, and

  〔Π(n,m)〕 the *complete elliptic integrals of the 3rd kind*,

- *Triadic* function*:*

  〔I_{xyz}〕 returns the *regularized Beta function*.

DRAFT

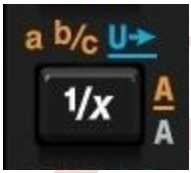
Rational Numbers (Fractions)

In addition to *reals*, you can also work with fractions on your *WP43*, with their absolute values between 1 000 000 and 0.000 1. Maximum denominator is 9 999 (DENMAX accepts greater values but will reduce them as soon as input is closed).[165]

Enter a fraction directly by keying in a 2nd [.] in numeric input. Herein, the 1st [.] is interpreted as a blank space, the 2nd as a fraction mark:

Examples:

Key in:	… and get in *SDC* (~~FRCSRN~~)	
[1][2] [.] [3] [.] [4] [EXIT]	$12\ ^3/_4$	
[.] [1] [.] [2] [EXIT]	$0\ ^1/_2$	
[3] [.] [1] [.] [EXIT]	$3\ ^1/_2$	takes the last denominator entered [166]
[1] [.] [.] [4] [EXIT]	$1\ ^0/_1$	[167] since input was **$1\ ^0/_4$**
[.] [.] [1][5] [EXIT]	$0\ ^0/_1$	input was **$0\ ^0/_{15}$**



Any closed *real* on the *stack* will be displayed as a fraction after [a b/c] was pressed, after a fraction is entered as demonstrated, or after x is combined with a fraction by an arithmetic operation. If the fraction displayed is slightly greater or slightly less than the underlying *real*, [<] or [>] will precede this fraction, respectively (see examples following). Get a more verbose notation (e.g., $x = {}^1/_2$ or $x < {}^3/_7$) with FRCSRN set, applying to the other *stack registers* by analogy.[168]

[165] This works almost like on the *HP-32SII* and its successors but with higher precision.

[166] … following an idea of *Craig Finseth*. *Startup default* denominator is **4**.

[167] This display of a pure integer tells you unambiguously your *WP43* is in *proper fraction display mode*. In *improper fraction display mode*, $^1/_1$ will be displayed instead.

(Note the *HP-32SII* reads [1] [.] [.] [4] as ¼ – though this is neither coherent with its other input interpretations nor does it save any keystrokes. The calculator cannot and shall not know better than the user what (s)he meant.)

[168] Your *WP43* continues calculating with *reals*, it will just display fractions as close to them as possible within the constraints set by you. FRCSRN means *show register names*.

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Vice versa, any closed number displayed as a fraction will show its *real* value again after [.d] or after [DISP] **ALL**, **FIX**, **SCI**, or **ENG**. And a closed fraction will be decomposed to its *long integer* numerator in **Y** and denominator in **X** by [PART] **DECOMP**; this can be simply reverted by division.

Your *WP43* features two fraction display modes: *proper* and *improper fractions*.[169] [a b/c] toggles them, starting with *proper fractions* as illustrated below. The following **example** comprises most aspects of fraction display (assuming *SDC*):

Enter:	and you will see:	
3 [1/x]	$3.333\,333\,333\,333\,333\times10^{-1}$	
[a b/c]	$< 0\ {}^{1}/_{3}$	*proper fraction*
[a b/c]	$< {}^{1}/_{3}$	*improper fraction*
	since $x = 0.333\,333\,333\,333\,333\,3 < {}^{1}/_{3}$.	
[1/x]	${}^{3}/_{1}$	since this is exact
78.40625 [−]	$-2\ {}^{413}/_{32}$	and stays exact
[ENTER↑] [×]	${}^{5\,822\,569}/_{1\,024}$	
2 [×]	${}^{5\,822\,569}/_{512}$	

Now, press [a b/c] for displaying this *improper fraction* as a *proper* one.[170]

You will get	$11\ 372\ {}^{105}/_{512}$	
11 [/]	$> 1\,033\ {}^{4\,713}/_{5\,632}$	

This fraction is less than the underlying *real* x, deviating $< {}^{0.5}/_{5\,632}$ from it.

Now, let's reduce the maximum denominator by

64 [MODE] **DENMAX**	64	
[DROP↓]	$> 1\,033\ {}^{41}/_{49}$	
CF **SYS.FL** **DENANY**	$< 1\,033\ {}^{27}/_{32}$	

since ~~DENANY~~ allows for denominators 2, 4, 8, 16, 32, and 64 here only (DENFIX is clear since startup).

The last fraction shown is greater than the underlying *real* x; the difference is

---

[169] *TNG*: *Proper fractions* decken sowohl *echte Brüche* (wie ¾) als auch *gemischte Brüche* (wie 2 ½) ab. Bei *improper fractions* wird der ganzzahlige Anteil nicht herausgezogen, so dass hier der Zähler größer als der Nenner sein kann.

[170] This conversion was newly introduced on *RPN* calculators with the *WP 34S* in 2011.

< $^{0.5}/_{32}$ (and > $^{0.5}/_{64}$ – else the display would read 1033 $^{53}/_{64}$ instead).

DECOMP will decompose a fraction preserving full precision always; thus, it will return here y = 5 822 569 for the numerator and x = 5 632 for the denominator. So you can revert DECOMP simply by dividing.

Another **example**, now with a maximum denominator 60:

DENANY allows for any denominator ≤60 (i.e. 60, 59, 58, ..., 4, 3, and 2).

~~DENANY~~ & DENFIX allows for a fix denominator 60 only.

~~DENANY~~ & ~~DENFIX~~ allows factors of 60 (i.e. 60, 30, 20, 15, 12, 10, 6, 4, 3, and 2).[171]

Before closing this chapter about rational numbers, let's not forget those isolated irrational islands in the vast ocean of *SI* where you may come across dimensions like in the following **example**:

A calculator stand is specified to measure 9" × 3 $^{1}/_{2}$" × $^{5}/_{8}$". It goes without saying that your *WP43* will support you also in such harsh environments. Only absolute greenhorns, however, will expect that a tight thin-walled box around this stand will displace

9 [ENTER] **3** [.] **1** [.] **2** [×]	31 $^{1}/_{2}$
[.] **5** [.] **8** [×]	19 $^{11}/_{16}$ *cubic inches* of water.

Instead, a magic conversion from *cubic inches* to so-called *fluid ounces* is required now, even depending on the country you are in! [172]

Though don't despair: In *Section 5* you will learn how to do this magic – it will take just a little more time and effort than calculating with rational units.

---

[171] 60 was a sacred number in ancient Babylon, hence our sexagesimal angles and hours.

[172] No such differentiation in *SI* – here a volume stays unchanged regardless of the matter filling it, be it air, steam, liquid water, mercury, ice, or solid lead, for instance.

Honestly, unless you were raised on such an isolated irrational island, we bet you assumed *fluid ounces* being a unit of mass, didn't you? Since you have heard of *ounces* once before and just thought … terribly wrong! Don't think there – you may run into deep troubles easily (thinking less, however, may pave your road to top positions in administration – see evidence between 2016 and 2020 and after 2025-01).

Inhabitants of those irrational islands obviously have lots of spare time to devote to solving such problems. Sometimes, however, their endeavors might fail (e.g., here: *https://www.latimes.com/archives/la-xpm-1999-oct-01-mn-17288-story.html*).

DRAFT

Angles and Trigonometric Functions

For dealing with *angles*, you may choose one out of six *angular display modes* (*ADM*) on your *WP43*: DEG, RAD, GRAD, MULπ, MIL, and D.MS.[173] *Angles* are entered as *reals* and interpreted according to the current *ADM* as indicated in the *status bar* by 4°, 4ʳ, 4ᵍ, 4π, or 4⁻ (cf. p. 81) when a function expecting angular input is called.

Exception: *Sexagesimal degrees*[171] must be entered in the format ddddd.mmsshh d.ms – with ddddd standing for integer *degrees*, mm for *angular minutes*, ss for *seconds*, and hh for hundredth of *seconds*.

Examples:

12.3456543 d.ms returns 12°34'56.54",
45.6789018 d.ms returns 46° 8'29.01".

There are some functions (e.g., ARCSIN) operating on *reals* and returning *angles*. Then the returns will be automatically tagged according to the current *ADM*. Assume ALL 2 set for the following **examples**:

In *ADM* ...		... 1 TRI arcsin will return ...
decimal RADians	4ʳ	1.570 796 326 794 897ʳ
MUL*tiples of* π	4π	0.5π
decimal DEGrees	4°	90.°
Degrees Minutes Seconds	4"	90° 0' 0.00" [174]
GRADs or *gon*	4ᵍ	100.ᵍ
MIL	4⁻	1 600.⁻

[173] All commands setting *ADM* (except D.MS) live in MODE. GRAD is used in surveying only, MIL in military (artillery).

TN: The traditional calculator notations DEG and GRAD are misleading in German at least: DEG*rees* on your *WP43* mean *Grad*, while calculator GRADs are generally called *Gon* in Continental Europe. MIL corresponds to *Strich* in German.

[174] There are no leading zeros for angular *minutes* and *seconds*. And this *ADM* can neither digest nor display anything smaller than 0.01". On the other hand, it will display down to

Whenever you see a number formatted alike on your *WP43* you know it is an *angle*. – Other functions presume their inputs being *angles* (e.g., SIN). There, decimal inputs are generally interpreted as *angles* of current *ADM*. 22 angular conversions are provided in [∡→]:

From … / to …	*sexa-gesimal degrees*	*decimal degrees*	*decimal radians*	*multi-ples of π*	*grads / gon*	*mil*	current *ADM* or tagging
sexag. degrees	—	D→D.MS	—	—	—	—	→D.MS
dec. degrees	D.MS→D	—	R→D	—	—	MIL→D	→DEG
dec. radians	—	D→R	—	Mπ→R	—	MIL→R	→RAD
multiples of π	—	—	R→Mπ	—	—	—	→MULπ
grads / gon	—	—	—	—	—	—	→GRAD
mil	—	D→MIL	R→MIL	—	—	—	→MIL
current *ADM*	D.MS→	DEG→	RAD→	MULπ→	GRAD→	MIL→	—

Example (with FIX **5**):

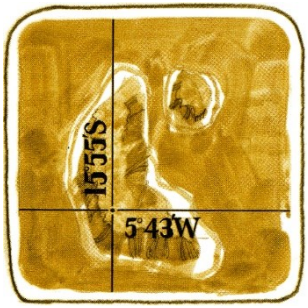
[MODE] MULπ — Choose *multiples of π* as *ADM* and ∡π will appear in the *status bar* and stay there for the time being.

150 [1/x]	0.006 67	So π/150 …
[∡→] →RAD	0.020 94ʳ	are some 0.020 94 *radians*
→DEG	1.200 00°	or exactly 1.2°
→D.MS	1°12’ 0.00”	or 72 *angular minutes*
→MIL	21.333 33⁻	or 21.33… *mil*
→MULπ	0.006 67π	equivalent to π/150 still.

Note →RAD ‘knew’ it had to convert from *multiples of π* since this function expects angular input and takes the current *ADM* setting into account. Angular output of operations is tagged and will stay so. Thus, →DEG above converted from *radians*, →D.MS from *decimal* and →MULπ from *sexagesimal degrees* since the respective inputs were tagged. Also [.d] converts *sexa-gesimal degrees* into *decimal degrees* but without tagging.

that fraction always and can’t be shortened (cf. examples above)..

You have learned about trigonometric functions in school. Thus, we demonstrate their operation on *angles* with one **example** only:



Lovesick sailor *Oscar Odysseus* dwells on the island of *Tristan da Cunha* (37°03'S, 12°l8'W), and his sweetheart, *Penelope*, lives on the nearest island. Unfortunately for the course of true love, however, *Tristan da Cunha* is the most isolated inhabited spot in the world. If *Penelope* lives on the island of *St. Helena* (15°55'S, 5°43'W), use the following formula to calculate the great circle distance that *Odysseus* must sail in order to court her.[175]

Solution:

The formula for the great circle distance *d* in *nautical miles* is:

$$d = 60 \times arccos[sin(B_s)\ sin(B_d) + cos(B_s)\ cos(B_d)\ cos(L_d - L_s)]$$

with B_s and L_s being the latitude and longitude of the start (*Tristan da Cunha*) and B_d and L_d being the latitude and longitude of the destination (*St. Helena*).[176] Hence, with the numbers inserted, this formula reads:

$$d = 60 \times arccos[sin(37°03'S)\ sin(15°55'S) + cos(37°03'S)\ cos(15°55'S) \times cos(5°43'W - 12°18'W)]$$

Set the appropriate number of decimals for display and calculate from inside out, remembering the trigonometric functions assume their input being in the current *ADM* as indicated in the *status bar*.

d.ms	*Sexagesimal degrees* will be used throughout this calculation, so set *ADM* accordingly.
DISP FIX 2	We will not need more decimals displayed.
5.43 d.ms	5°43' 0.00"
12.18 d.ms	12°18' 0.00"
(−)	-6°35' 0.00"
TRI cos	0.99

[175] This problem is found first in the *HP-25 OH*. It was reprinted thereafter in each and every *HP* scientific pocket calculator manual until the *HP-41C/41CV OHPG*.

[176] This formula means that 1 *nautical mile* (nmi) corresponds to 1 *angular minute* on a great circle. This doesn't hold exactly but precisely enough for practical sailing.

TN: *Latitude* means *geographische Breite* in German. Hence it is represented by the letter ***B*** in the formula above.

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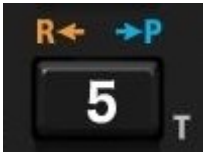
15.55 STO J **cos**	0.96	
×	0.96	
37.03 STO K **cos**	0.80	
×	0.76	
RCL K **sin**	0.60	
RCL J **sin**	0.27	
×	0.16	
+	0.93	
arccos	22°15' 36.15"	
.d	22.26	convert to a *real*.
60 × returns	1 335.60	nmi or
U→ **x:** **m ← nmi**	2 473 535.88	, i.e. some 2474 km that *Odysseus* must sail to visit *Penelope*, presuming prosperous winds.

Mixed Calculations: Coordinate Transforms in *2D*, Flight Directions, Courses over Ground, etc.



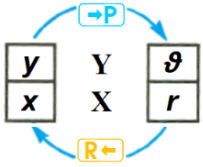
Two functions are provided for converting polar or rectangular coordinates in two dimensions. Input and output data are both in *stack registers* X and Y here. →P converts *2D Cartesian* coordinates ***x*** and ***y*** to polar magnitude or radius ***r*** in X and angle ***ϑ*** in Y.

Example (assuming *SDC*):
Convert (*x*, *y*) = (6, 4.5) to polar. Two decimals shall do.

Solution (with FIX **2** set):

4.5 ENTER↑ **6** →P returns

ϑ =	36.87°
r =	7.50



i.e. a *vector* of magnitude 7.5 pointing up at an angle of some 37° to the positive *x*-axis.

R← converts *2D* polar magnitude or radius ***r*** in X and angle ***ϑ*** in Y to *Cartesian* coordinates ***x*** and ***y***. Both functions honor the *ADM* settings

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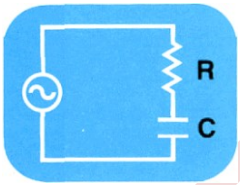
and tags as described in previous chapter.

Example (continued):
Convert the returned angle of the conversion executed above to *radians*, and then convert the resulting coordinates (*r*, *ϑ*) to rectangular.

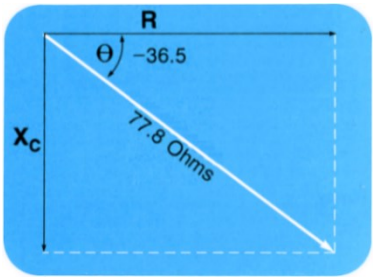
Solution:

x≷y	7.50 36.87°
→RAD	7.50 0.64ʳ
x≷y R← returns	y = 4.50 x = 6.00 as expected.



Example (from *HP-29C OHPG*):
Engineer *Trigo Slothrop* has determined that in the RC circuit shown…, the total impedance is 77.8 Ω and voltage lags current by 36.5°. What are the values of resistance ***R*** and capacitive reactance X_C in the circuit?

Solution (with FIX **1**):
See the picture: of the total impedance: ***R*** is its component parallel to the *x*-axis, and X_C is its perpendicular component parallel to the *y*-axis:



−36.5 ENTER↑ **77.8**

R← returns

y =	−46.3
x =	62.5

i.e. a resistance of 62.5 Ω and a reactance of 46.3 Ω.

Note you can use →P and R← also to convert *3D* cylinder coordinates to *Cartesian* and vice versa, since ***z*** is kept unchanged.

Having learned about →P, R←, and Σ+, we can profit from combining these functions now.

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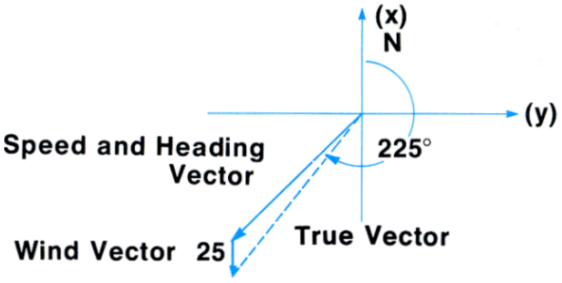
Example (from the *HP-25 OH*):

The instruments in fearless bush pilot *Apeneck Sweeney's* converted *P-41* indicate an air speed of 125 *knots* and a heading of 225°. However the aircraft is also being buffeted by a steady 25-*knot* wind that is blowing from north to south. What is the actual course and speed of the aircraft?



Solution (with FIX **2**):

Combine the *vector* indicated on the aircraft instruments with the wind *vector* to yield the actual course and speed. Convert the *vectors* to rectangular, then combine the *x*- and *y*-coordinates in the statistical storage. Finally, recall the summed *x*- and *y*-coordinates and convert them to polar coordinates giving the actual *vector* of the aircraft. (North becomes the *x*-coordinate in order that the problem corresponds with navigational convention.)

CLR CLΣ	clears the statistical storage.	
225 ENTER↑ 125	indicated heading and air speed.	
R←	returns	y = −88.39 x = −88.39
STAT Σ+	puts x and y into the statistical storage.	
180 ENTER↑ 25	north wind.	
R←	returns	y = 0.00 x = −25.00
Σ+	adds x and y to the statistical storage.	
SUM	pushes the summation *registers* Σx and Σy on the stack.	
→P	returns	θ = −142.06° r = 143.77
x≷y 360 +	returns	217.94°

(we have to change the angle to become positive for being in line with navigational convention).

Mr. *Sweeney* is actually flying at 143.77 *knots* on a course of 217.94° over ground.[177]

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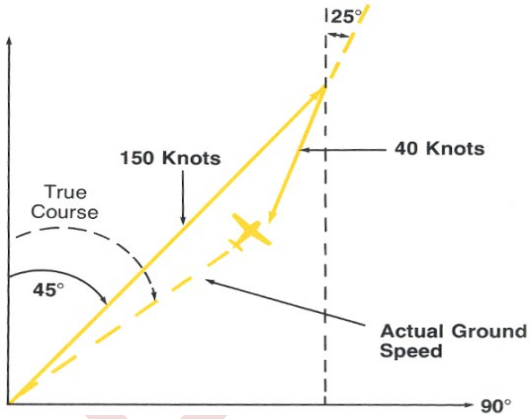
A similar problem appeared first in the *HP-55 OH* and was copied for some years. We quote it from the *HP-33 OH*:

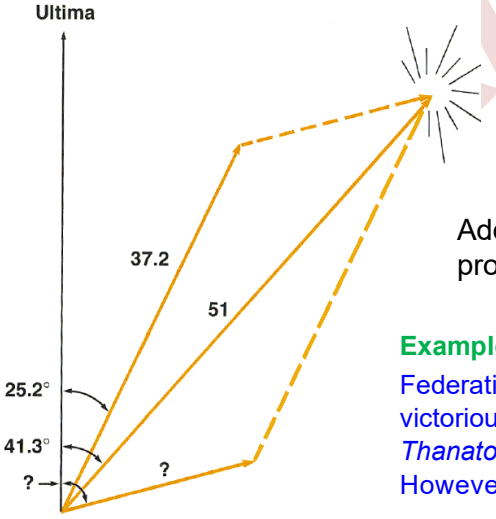


Example:

On his way to search for an albino caribou, grizzled bush pilot *Apeneck Sweeney's* converted *Swordfish* aircraft has a true air speed of 150 *knots* and an estimated heading of 45°. The *Swordfish* is also being buffeted by a headwind of 40 *knots* from a bearing of 25°. What is the actual ground speed and course of the *Swordfish*? [178]

Start of solution:

A bearing of 25° equals a heading of 25° + 180° = 205°,so the corresponding headwind *vector* may be added (cf. the *HP-97 OHPG*). – We leave it to you to solve this problem (results for crosschecking: 51.94° and 113.24 *knots*).



Additionally, here is an advanced problem from a universe far, far away:

Example from the *HP-32 OH*:[179]

Federation starship *Felicity* has emerged victorious from a furious battle with the starship *Thanatos* from the renegade planet *Maldek*. However, its automatic pilot is kaput, and its

[177] Another way of solving such *2D vector* problems is shown on p. 174.

[178] This kind of plane is still in service after 50 years. Our grandparents could not imagine.

[179] This problem is from 1978. The 1st episode of *Star Wars* was launched in 1977.

main thrust engine is locked on at 37.2 *meganewtons* (MN) directed along an angle of 25.2° from the star *Ultima*.[180] Consulting the ship's star map, the navigator reports a hyperspace entrance *vector* of 51 MN at an angle of 41.3° from *Ultima*. To what thrust and angle should the auxiliary engine be set, for *Felicity* to achieve alignment with the hyperspace entrance *vector*?

Solution (with FIX **1**):

The required thrust *vector* of the auxiliary engine is equal to the hyperspace entrance *vector* minus the thrust *vector* of the main engine. The *vectors* are converted to rectangular coordinates using [R←], and their difference is calculated using [Σ+] … This difference is recalled to … **X** and **Y** … using [SUM]. Then, these rectangular coordinates of the auxiliary engine thrust *vector* are converted to polar coordinates using [→P].

[CLR] CLΣ	clears all statistical data.
41.3 [ENTER↑] **51**	hyperspace entrance *vector*
[R←]	returns y = 33.7 x = 38.3
[STAT] Σ+	puts the ***x*** and ***y*** components of the hyperspace entrance *vector* into the statistical storage.
25.2 [ENTER↑] **37.2**	main engine thrust *vector*
[R←]	returns y = 15.8 x = 33.7
[+/-] [x≷y] [+/-] [x≷y]	inverts both its *Cartesian* components.[181]
Σ+	adds the ***x*** and ***y*** components of the negative main engine thrust *vector* to the statistical storage.
SUM	recalls the summation *registers*: Σy = 17.8 Σx = 4.7

[180] *TN*: a) *Thanatos* (Θάνατος) is the ancient Greek name of death, pronounced like 'Túnnatoss' in English but like 'Tánatos' in Spanish, Italian, French, German, Swedish, and Finnish, for instance.

b) Why was that planet called *Maldek* instead of, e.g., *Bendek*? Subliminal reader manipulation …

c) Ultima is Latin for 'the last' (fem.).

[181] [CC] [+/-] [CC] will do the same one keystroke faster (see *Complex Numbers* below).

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→P returns

ϑ =	75.4°
r =	18.4

meaning the auxiliary engine shall be set at 18.4 MN and an angle of 75.4° from *Ultima*.[182]

See the operating manuals of other vintage *HP* calculators (especially the *HP-27*) for further applications in the area of angular mathematics (e.g., triangle solutions, navigation, and surveying).

Angles: Summary of Functions

The number of functions operating on and with *angles* is quite limited. They are important nevertheless. All are listed below:

- General mathematics:
  - *Monadic* functions:

    [sin], [cos], and [tan] operate on *angles* and return *reals*,

    [arcsin], [arccos], and [arctan] operate on *reals* and return *angles*,

    [+/−] returns $x \times (-1)$ for closed input (a.k.a. 'unary minus').

    [SHOW] displays the full precision of x in top numeric row until next keystroke.

  - *Dyadic* functions:

    [+], [−], [×], and [/] work as specified in the matrices on pp. 76f,

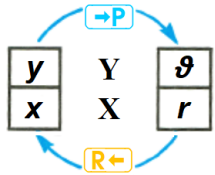
    [max] (or [min]) returns the maximum (or minimum) of x and y.

[182] Those looking for an extra challenge can compute next how flat the starship crew will become within *seconds* after the auxiliary engine is ignited (you can't claim knowing physics was really advantageous when solving such problems or watching *Star Wars* – on the other hand, we had a couple of good laughs). Being at it, there is generally far too much noise in space movies – *Stanley Kubrick* knew better already.

Above universal problem was reprinted in the *HP-34C OH* one year later. It vanished in hyperspace thereafter, without any trace.

- Rounding commands (RDP, ROUND, ROUNDI, and RSD) as well as SHOW operate on angles as they do on *reals* (cf. pp. 131f).

- Conversions:

  →P converts rectangular to polar *coordinates* (cf. pp. 21f), while R← converts vice versa. Cf. examples on pp. 142ff.

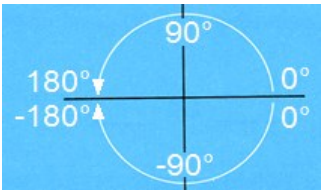
  Angular conversions are covered comprehensively on p. 140.

- Advanced mathematics (see the *IOI* and *ReMA I* for comprehensive information about the functions following):

  - *Monadic* functions:

    〔**sinc**〕 returns $sin(x) / x$ and 〔**sincπ**〕 returns $sin(\pi x) / \pi x$ for $x \neq 0$ and **1** for $x = 0$.

ATTENTION: Angular input is accepted from −∞ to +∞. For trigonometric functions, it will be reduced modulo 360° to a range of −180° to +180° or equivalent in *grads*, *mil*, *multiples of* π, or *radians* before letting any such function operate on it.



Angular output is limited to −180° to 180° (or its equivalents **−π** to **π** *radians*, −1 to 1 *multiples of* π, etc.) unless in angular conversions.

WARNING: Input $\gtrsim 10^{34}$ for trigonometric functions is nonsense since no partial turns can be specified anymore there.

See the *ReM* for more.

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Integers: Input and Displaying

Any number you enter without using [.], [E], or [CC] (see below for the latter) is taken as an *integer* by your *WP43* (cf. pp. 73f). It allows for *integer* computing in 15 bases from binary to hexadecimal.

Any number displayed without any decimal mark on your *WP43* is an *integer* (e.g., a counted value, see examples below). And it will stay integer as long as it is exclusively combined with other *integers* and only integer functions operate on it; else it will be converted to another *DT* (cf. the matrices on pp. 76f and see *ReMS 3*). Generally, [.d] converts any closed *integer* ***x*** to a *real*; and even an open *integer* will be converted to an *angle* by any angular conversion (cf. p. 80).

Your *WP43* features two kinds of *integers*: *integers* of finite length (called *short*) and of almost infinite length (called *long*).

Long integers are useful for numeric tasks, for instance. If you enter a number of arbitrary length just without using [.], [E], or [CC], your *WP43* will take it as a *long integer* of base 10. For **example**,

1 111 111 111 [EXP] [x^2] returns 1 234 567 900 987 654 321

Note the number is adjusted to the right again when closed, though no decimal point (or comma) is displayed. Such a 19-digit result is shown with ease. Greater *long integers* ($\geq 10^{21}$) will be displayed using a smaller font. Very big ones ($\geq 10^{43}$) will be shown with an exponent instead of their least significant digits; nevertheless, all their digits are kept internally, thus *long integers* can be of very high precision (up to 1000 digits[183]).

Example:[184]

Integers ***n*** from 1 up to 100 could all[185] be expressed as sum of three cubes $n = i^3 + j^3 + k^3$ – except 42 (sic!). In 2019-09, two mathematicians of Bristol and Boston published that the triple 12 602 123 297 335 631,

[183] Check the *ReM, App. B*, for the exact limits.

[184] See https://phys.org/news/2019-09-sum-cubes-solvedusing-real-life.html

[185] Unless $\mathrm{mod}(n; 9) = 4 \text{ or } 5$. See two chapters below for an explanation of the function mod.

80 435 758 145 817 515, and −80 538 738 812 075 974 should solve this problem. Verify!

12 602 123 297 335 631 EXP x^3 returns

2 001 387 454 481 788 542 313 426 390 100 466 780×10^12

SHOW shows all digits of this number:

2 001 387 454 481 788 542 313 426 390 100 466 780 457 779 044 591

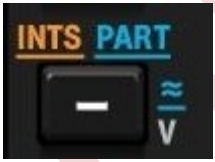
80 435 758 145 817 515 x^3 + SHOW returns

522 413 599 036 979 150 280 966 144 853 653 247 149 764 3 62 110 466

−80 538 738 812 075 974 x^3 + returns 42 !

This is all you need to know about entering and displaying *long integers* – go to p. 157 for further information about calculating with them.

Short integers feature a finite *word* size (user settable up to $64 bits \approx 18 \times 10^{18}$) and are especially useful for assembler programming and close-to-hardware system design including debugging.[186] A *short integer* of a certain base (2, 3, …, 16) is entered specifying its digits trailed by # ***base***.



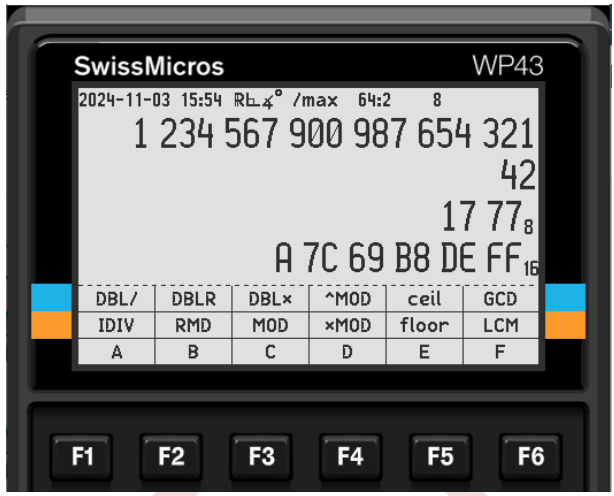
You may abbreviate # 10 by # D, and # 16 by # H. Open INTS (see its top *view* overleaf) for direct access to the digits **A** … **F** required for numeric input in bases >10. From the 2nd integer input on, you can save keystrokes: If you enter a new number omitting #, ., and E, your *WP43* takes it as a *short integer* of the base you specified last before – as long as you did not enter any other *DT* in between.[187]

[186] Your *WP43* encompasses all integer and bit manipulation operations of the dedicated *Computer Scientist's HP-16C*, plus all bases and the entire extended function set of *WP 34S*, actually making a second calculator obsolete. It can perform some clever calculations with hexadecimal and binary numbers that most calculator apps on a mobile phone can't do, and neither *Excel* nor whatever else one uses on the *PC* in the office.

[187] This shortcut will close when you enter ., E, CC, or # in input, even if you delete this keystroke thereafter.

Illegal digits keyed in (e.g., **2** in base 2 or **B** in base 10) can be detected no earlier than

Word size and *integer sign mode* (*ISM*) set are indicated in the *status bar* using a format `ww:x` (e.g., **64:2** here). Therein, `ww` denotes the *word* size in *bits*, and `x` is `1` or `2` for *1's or 2's complement*, `u` for *unsigned*, or `s` for *sign-and-mantissa mode* (cf. p. 81); these *ISM's* control the handling of negative *integers* (see examples below).



CARRY and OVERFLOW – if set – will be shown as `c` or `o` or `oc`, respectively, trailing *ISM* display in the *status bar*. Both are *system flags*[188] (cf. p. 59) – if you want to set, clear, or check them individually, use the commands provided in FLAG.

Example:

Enter [SF] SYS.FL (LEAD.0) (or [SF] [L])

[INTS] [▲] WSIZE [1] [2] This allows seeing all bits at a glance.

147 [EXIT] Enter 147 as a (decimal) *long integer*.

[▲] →BIN Convert 147 to a binary *short integer*

1COMPL 1's complement, and you will see [189]

0000 1001 0011$_2$ and – after [+/−] – 1111 0110 1100$_2$.

Obviously [+/−] in 1COMPL flips every bit, equivalent to NOT here.

---

said input is completed. And you may key in more than the current *word* size can take; also this will be checked when input is closed. So an error message can be thrown only after closure.

[188] They behave like they did on *HP-16C* and *WP 34S*.

[189] Note the gap automatically inserted every four *bits* here for easier reading.

Return to the original number via [+/−], press **2COMPL**, and you will get

$0000\ 1001\ 0011_2$ and – after [+/−] – $1111\ 0110\ 1101_2$.

Note the negative number equals the inverse + 1 in 2COMPL.

Return via [+/−] again, press **SIGNMT** and you will see

$0000\ 1001\ 0011_2$ and – after [+/−] – $1000\ 1001\ 0011_2$.

Negating a number will just flip the top bit in SIGNMT (hence its *name*).

Return via [+/−] once more, press **UNSIGN** and you will get

$0000\ 1001\ 0011_2$ and – after [+/−] – $1111\ 0110\ 1101_2$.

The 2nd number looks like in 2COMPL but an *overflow* is indicated here – see ° in the *status bar* trailing the *ISM*.[190] Thus, pressing [+/−] will not suffice for returning to the original number here anymore; you must clear OVERFLOW explicitly by [CF] **SYS.FL** (**OVERFL**) (or [CF] [B]).

So positive numbers stay unchanged in all these four modes. Taking a negative *short integer*, however, in one *ISM* and switching to another one will lead to different interpretations and, thus, different displays.

Example (continued):
The fixed bit pattern representing

-147_{10} in **12:2** will be displayed as…
-146_{10} in **12:1** , as…
$-1\ 901_{10}$ in **12:s** , and as…
$3\ 949_{10}$ in **12:u** . You can verify this easily.

[190] Actually, changing signs should have no meaning in unsigned mode per definition. Thus, [+/−] should be illegal here or result in no operation at least. "In unsigned mode, the most significant bit adds magnitude, not sign, so the largest value represented by a 12-*bit* word is 4095 instead of 2047" (quoted from the *HP-16C Computer Scientist Owner's Handbook* of April 1982, p. 30).

[+/−] in unsigned mode was allowed, however, by the designers of the *HP-16C* and implemented as shown above. So we follow that implementation for sake of backward compatibility (as we did with *WP 34S*), though frowning.

Luckily, this inherited error carries a benefit: You can easily (by a single key press) determine that a very big positive unsigned number in display may correspond to a small negative number for the *word* size chosen. This may be quite useful.

Keeping the *ISM* and changing bases will produce different views on the constant bit pattern as well.

Example (continued):
Compare the outputs for different bases in 16:2 :

−16123 EXIT	−16 123	to base 10 corresponds to
# 2	$1100\ 0001\ 0000\ 0101_{2}$	to base 2,
# 3	$-211\ 010\ 011_{3}$	to base 3,
# 4	$30\ 01\ 00\ 11_{4}$	to base 4,
# 5	$-1\ 003\ 443_{5}$	to base 5,
# 6	$-202\ 351_{6}$	to base 6,
# 7	$-65\ 002_{7}$	to base 7,
# 8	$14\ 04\ 05_{8}$	to base 8,
# 9	$-24\ 104_{9}$	to base 9,
# 1 1	$-11\ 128_{11}$	to base 11,
# 1 2	$-9\ 3B7_{12}$	to base 12,
# 1 3	$-7\ 453_{13}$	to base 13,
# 1 4	$-5\ C39_{14}$	to base 14,
# 1 5	$-4\ B9D_{15}$	to base 15, and
# H	$C1\ 05_{16}$	to base 16.

You may have noticed that the displays for bases 2, 4, 8, and 16 look similar, presenting all 16 *bits* to you, while a signed mantissa is displayed for the other bases instead. There are also different separator intervals (four bits for binary, two digits for bases 4, 8, and 16, and three for all other bases); they are fixed unless GAP **0** is set by you.

These different display formats consider that bases 2, 4, 8, and 16 are most convenient for bit and byte manipulations and further close-to-hardware applications. The bases in between will gain most interest in dealing with different number representations and calculations therein, where base 10 is the common reference standard.[191]

Let's turn to bigger *words* now:

Example (continued):

Enter [CF] [L]

[INTS] [▲] WSIZE [6][4]

UNSIGN

[▼] 9 3 A 1 4 C 6 [#] [H] (cf. the *menu* INTS shown on p. 151).

Then your *WP43* will display

$$9\ 3A\ 14\ C6_{16}$$

In binary representation this number will need 28 digits, looking like

$$1001\ 0011\ 1010\ 0001\ 0100\ 1100\ 0110_2$$

Obviously, your *WP43* can't display such a big binary number in one row this way. Look what it does instead – type [#] [2] for converting x to binary and you will get:

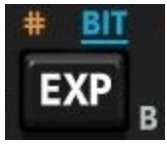
$$1001\ 0011\ 1010\ 0001\ 0100\ 1100\ 0110_2$$

This number is shown using the small font provided. If also leading zeros were turned on via [SF] [L], all 64 *bits* will be displayed in one row using a minimal font:

$$\ldots\ldots\ \ldots\ldots\ \ldots\ldots\ \ldots\ldots\ \ldots\ldots\ \ldots\ldots\ \ldots\ldots\ \ldots\ldots\ \ldots\ldots\ 1..1\ ..11\ 1.1.\ ...1\ .1..\ 11..\ .11._2$$ [193]

Integers: Bitwise Operations on Short Integers

As mentioned, your *WP43* carries all the bitwise operations you may know from the vintage *HP-16C Computer Scientist*, plus some more you may have learned with *WP 34S*. You find them all in BIT.



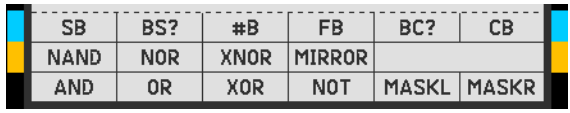
[191] During numeric input, gaps are inserted suiting the base chosen (cf. previous page). If there is no such base yet, GAP **3** applies since your *WP43* cannot know your intentions in advance.

[192] Up to 16 hexadecimal digits are possible for unsigned *short integers*.

[193] If you want to convert hexadecimal numbers to *split octal*, you can do so inserting a **0** after every 2 hex digits. Vice versa you shall insert a **0** after every 3 octal digits.

Example: AB**0**CD**0**EF**0**FF$_{16}$ = 253**0**315**0**357**0**377$_8$.

Generally, bits in a *word* are counted from right to left, starting with number 0 for the least significant bit (this is important for specifying correct bit numbers in the operations BC?, BS?, CB, FB, and SB).



The examples below all deal with 8-*bit words* showing leading zeros for easy reading:

SF L BIT ▼ WSIZE 8

Let's look at the bitwise *dyadic* functions, all found in the 1st view of BIT:

Common input	Y	0110 1011₂
	X	1011 1001₂
Operation	Symbol	Output
AND	∧	0010 1001₂
NAND	$\overline{\wedge}$	1101 0110₂
OR	∨	1111 1011₂
NOR	$\overline{\vee}$	0000 0100₂
XOR	$\underline{\vee}$	1101 0010₂
XNOR		0010 1101₂

Find seven shift and rotate functions in the table below (all taken from the 2nd view of BIT) with schematic pictures here like they were printed on the backplane of the *HP-16C* (the boxed C represents the *carry* bit, indicated in the *status bar* if set).

Common input 1011 0011₂

Operation	Schematic picture if applicable	E. g.	Output
Clear Bit		CB **4**	1010 0011₂
Flip Bit		FB **5**	1001 0011₂
Set Bit		SB **6**	1111 0011₂

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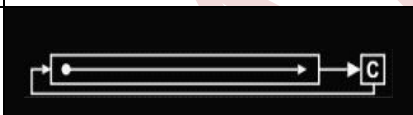
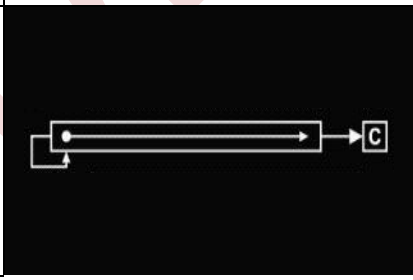
		Common input	$1011\ 0011_2$
Operation	Schematic picture if applicable	E. g.	Output
Negate[194]		NOT (¬)	$0100\ 1100_2$
Mirror		MIRROR	$1100\ 1101_2$
Rotate Left		RL **1** RL **2**	$0110\ 0111_2$ C $1100\ 1110_2$
Rotate Left through Carry		RLC **1** RLC **2**	$0110\ 0110_2$ C $1100\ 1101_2$
Rotate Right		RR **1** RR **2** RR **3**	$1101\ 1001_2$ C $1110\ 1100_2$ C $0111\ 0110_2$
Rotate Right through Carry		RRC **1** RRC **2** RRC **3**	$0101\ 1001_2$ C $1010\ 1100_2$ C $1101\ 0110_2$
Shift Left		SL **1** SL **2**	$0110\ 0110_2$ C $1100\ 1100_2$
Shift Right		SR **1** SR **2**	$0101\ 1001_2$ C $0010\ 1100_2$ C
Arithmetic Shift Right		ASR **3**	in 1/2COMPL: $1111\ 0110_2$ in UNSIGN: [195] $0001\ 0110_2$ in SIGNMT: $1000\ 0110_2$

[194] *TN*: Remember: $(🍏\wedge\neg🍐)\vee(\neg🍏\wedge🍐)=(🍏\vee🍐)\wedge(\neg🍏\vee\neg🍐)$.
What the English do with apples and oranges, Germans do with apples and pears.

[195] The picture correctly describes this operation for *ISM* 1 and 2 only. Executing

See the *IOI* for these and more commands operating on bit level on *short integers* (LJ, RJ, MASKL and MASKR, #B, and the tests BS? and BC?).

Finally, note that no such operation will set OVERFL. CARRY is only settable by shift or rotate functions as demonstrated above. And ASR is the only bitwise operation being sensitive to *ISM* – ASR is the link to integer arithmetic operations.

Integers: Arithmetic Operations

Of the four basic arithmetic operations (+, −, ×, and /), the first three work with *integers* as they do with *reals*; the only difference lies in precision: up to 64 digits precision for *short integers* in binary representation on your *WP43* or even up to 1000 digits precision for *long integers*. Take [+/-] as a multiplication times −1, and [y^x] as repeated multiplication. Depending on input parameters and mode settings, OVERFL or CARRY may be set in such an operation (see pp. 160ff).

Divisions, however, must be handled differently in integer domain since the quotient can't feature a fractional part here. Generally, the formula

$$\frac{a}{b} = (a \operatorname{idiv} b) + \frac{1}{b} \times \operatorname{rmd}(a;b)$$

applies where the horizontal bar denotes *real* division, idiv represents integer division, and rmd stands for the remainder of the latter. So

$$\operatorname{rmd}(a;b) := a - b \times (a \operatorname{idiv} b)$$

Examples:

$\frac{25}{7} = 3 + \frac{1}{7} \times 4$ (and for a *real* case: $\frac{25}{7.5} = 3 + \frac{1}{7.5} \times 2.5$)

$\frac{-25}{7} = -3 + \frac{1}{7} \times (-4) \quad \Rightarrow \quad \operatorname{rmd}(-25;7) = -4$

$\frac{25}{-7} = -3 + \frac{1}{-7} \times 4 \quad \Rightarrow \quad \operatorname{rmd}(25;-7) = 4$

$\frac{-25}{-7} = 3 + \frac{1}{-7} \times (-4) \quad \Rightarrow \quad \operatorname{rmd}(-25;-7) = -4$

ASR three times on the *HP-16C*, however, equals a signed division by 2^3 in all its modes, hence the different results for the latter two *ISMs* above.

There is another function doing almost the same: it is called mod (read *modulo*). With the same input as above, it returns:

$$\mathrm{mod}(25;7) = 4\ ,$$
$$\mathrm{mod}(-25;7) = 3\ ,$$
$$\mathrm{mod}(25;-7) = -3\ ,$$
$$\mathrm{mod}(-25;-7) = -4\ .$$

So mod returns the same as rmd only if both parameters have equal signs. The general formula for mod reads:

$$\mathrm{mod}(a;b) := a - b \times \mathrm{floor}\left({}^{a}/_{b}\right)$$ with, e.g., $\mathrm{floor}\left(\frac{25}{7}\right) = 3$ and $\mathrm{floor}\left(-\frac{25}{7}\right) = -4$.

Also this formula applies to *reals* as well. Use it straightforwardly for calculating, e.g.,

$$\mathrm{mod}(25.3;-7.5) = 25.3 - (-7.5)\cdot(-4) = -4.7$$

These four functions are called IDIV, RMD, MOD, and FLOOR[196] on your *WP43* for obvious reasons. They are found in INTS (cf. p. 151), together with more integer operations like CEIL, ×MOD, and ^MOD (see p. 164 for an instance). MOD is also printed on the keyboard as 🟦-shifted function of /.

Furthermore, many exponential and logarithmic operations, x^2 and $\sqrt{x}$, x^3 and $\sqrt[3]{x}$, $x!$, COMB, PERM, as well as SIN, COS, and TAN operate on *integers*, too. Some of these functions will stay in integer domain while others may return *real* or even *complex numbers*. See the summary on pp. 163f for further information.

[196] Both FLOOR and CEIL operate also on *reals* and return *long integers*.

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Integers: Overflow and Carry with Short Integers

In arithmetic operations on *short integers* on your *WP43*, OVERFLOW and/or CARRY may be touched under certain conditions. – Note for each *word* size and *ISM* there is a maximum and a minimum integer displayable: let's call them I_{max} and I_{min}.

Example:

For 4-*bit words* (i.e. WSIZE **4**), we get

- I_{max} = 15 and I_{min} = 0 for 4:U, while
- I_{max} = 7 and I_{min} = −8 for 4:2,
- I_{max} = 7 and I_{min} = −7 for 4:1 and 4:S.

Let's start from 1 incrementing by 1 and see what will happen in these various modes. And whenever OVERFLOW and/or CARRY will be lit in the *status bar* in this course, we will clear them (using

CF SYS.FL (OVERFL) (or CF B) and/or

CF SYS.FL (CARRY) (or CF C))

before next increment:

4:U		4:2		4:1		4:S	
0001_2	1	0001_2	1	0001_2	1	0001_2	1
0010_2	2	0010_2	2	0010_2	2	0010_2	2
…	…	…	…	…	…	…	…
0111_2	7	0111_2	7	0111_2	7	0111_2	7
1000_2	8	1000_2 O	−8	1000_2 O	−7	1000_2 O C	−0
1001_2	9	1001_2	−7	1001_2	−6	0001_2	1
…	…	…	…	…	…	0010_2	2
1110_2	14	1110_2	−2	1110_2	−1		
1111_2	15	1111_2	−1	1111_2	−0		
0000_2 O C	0	0000_2 C	0	0001_2 C	1		
0001_2	1	0001_2	1	0010_2	2		

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For comparison, we start another turn from 1 following the same rules but decrementing by 1 instead:

4:U		4:2		4:1		4:S	
0001_2	1	0001_2	1	0001_2	1	0001_2	1
0000_2	0	0000_2	0	0000_2	0	0000_2	0
1111_2 o c	15	1111_2 c	−1	1110_2 c	−1	1001_2 c	−1
1110_2	14	1110_2	−2	1101_2	−2	1010_2	−2
…	…	…	…	…	…	…	…
1001_2	9	1001_2	−7	1000_2	−7	1111_2	−7
1000_2	8	1000_2	−8	0111_2 o	7	1000_2 o	−0
0111_2	7	0111_2 o	7	0110_2	6	1001_2	−1
0110_2	6	0110_2	6	…	…	1010_2	−2
…	…	…	…	…	…		
0010_2	2	0010_2	2	0001_2	1		
0001_2	1	0001_2	1	0000_2	0		

The most significant bit is #2 in 4:S and #3 in all other modes here.

With these results, I_{max}, and I_{min}, the general rules for setting and clearing CARRY and OVERFL are as printed here:

Operation	Effect on CARRY	Effect on OVERFL
Shift and rotate	As demonstrated on pp. 155f.	None.
Boole's, MIRROR	None (cf. pp. 155f).	None.
|x|, ABS	None.	Clears OVERFL (but sets it for $x = I_{min}$ in 2COMPL).
+, RCL+, STO+, INC, etc.	Sets CARRY if there is a carry out of the most significant bit, else clears CARRY.	Sets OVERFL if the result exceeds $[I_{min}; I_{max}]$ else clears OVERFL.

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Operation	Effect on CARRY	Effect on OVERFL
−, RCL−, STO−, DEC, etc.	Sets CARRY in a subtraction $m - s$ • in 1COMPL or 2COMPL if the binary subtraction causes a *borrow* [197] into the most significant bit, • in UNSIGN if $m < s$, • in SIGNMT if $m < s$ & $m \cdot s > 0$ Else clears CARRY.	Sets OVERFL if the result exceeds $[I_{min}; I_{max}]$, else clears OVERFL.
×, RCL×, STO×, +/−, $(-1)^x$, x^2, x^3, LCM, $x!$, etc.	None.	Thus, in UNSIGN, +/− always sets OVERFL and $(-1)^x$ does so for odd x.
2^x	Clears CARRY. Sets CARRY only if $x = -1$, or in UNSIGN if $x = wsize$ or in the other modes if $x = wsize - 1$	
y^x, 10^x	Sets CARRY for $x < 0$ (as well as for 0^0), else clears CARRY.	Sets OVERFL if the result exceeds $[I_{min}; I_{max}]$, else clears OVERFL.
e^x	Sets CARRY for $x \neq 0$, else clears CARRY.	
DBL×	None.	Clears OVERFL.
/ , RCL / , STO / , DBL / , LN, LOG_{10}, LOG_2, $LOG_x y$,	Sets CARRY if the remainder is ≠ 0, else clears CARRY.	Clears OVERFL (but sets it in 2COMPL for the division $I_{min} / (-1)$).

[197] See the examples on previous page.

TN: The so-called *borrow* in subtraction seems to be a specialty of the *USA*. Compare the subtle methodic differences in manual subtracting explained in *http://de.wikipedia.org/wiki/Subtraktion#Schriftliche_Subtraktion* (the corresponding article in the English *Wikipedia* is distictly less instructive).

TNG: Sowohl *carry* wie *borrow* entsprechen einem *Übertrag*.

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Integers Applied: A Pseudo Random Number Generator

A straight *PRNG* **example** program for the *HP-16C* was posted (search for '*63-bit LFSR*'). Here it is translated for your *WP43* (starting in *SDC*):

```
567890 1234567890 123 # D  STO K   store seed
P/R  GTO . .                       turn to program entry mode and go to free space
LBL α A ENTER↑                     begin of program
RCL K
BIT ▲ RR 62                        rotate bit 62 into bit 0
RCL K
RR 61                              rotate bit 61 into bit 0
▼ XOR                              bit 0 = bit 61 XOR bit 62
▲ SR 01                            shift bit 0 into CARRY
RCL K
RLC 01                             multiply k times 2 and add CARRY
▼ MASKR 63                         set up a 63-bit mask right adjusted
AND                                ensure a 63-bit output
STO K                              store next seed
RTN                                end of program
P/R                                return to run mode
```

program A

```
XEQ PROG A   returns   2 134 430 432 281 004 439₁₀
XEQ PROG A   returns   4 268 860 864 562 008 878₁₀
XEQ PROG A   returns   8 537 721 729 124 017 757₁₀
XEQ PROG A   returns   7 852 071 421 393 259 706₁₀
XEQ PROG A   returns   6 480 770 805 931 743 604₁₀
XEQ PROG A   returns   3 738 169 575 008 711 401₁₀
XEQ PROG A   returns   7 476 339 150 017 422 803₁₀
```

This *PRNG* has a period of $2^{63} - 1 \approx 9 \times 10^{18}$, so it is 'pretty random'. The sequence of its returns, however, lacks randomness – hence it is called *pseudo* (see *ReMA J*). After all, this is for demonstration purposes only.

Being at it, an astounding good while simple *PRNG* is given by the sequence $\boldsymbol{r} := \mathrm{FP}(\mathbf{997} \times \boldsymbol{r})$.[198]

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Integers: Summary of Functions

Many of the numeric functions operating on *reals* also work for *integers*. Functions operating on *short integers* will return *DT* 10 except for the cases given on pp. 76ff. Functions operating on *long integers*, on the other hand, may return results of *DT* 1, 2, 3, or 4:

- General mathematics:
  - *Monadic* functions:

    [√x], [∛x], [lb x], and [lg] return *long integers* if possible (else *real* or *complex numbers*) for *long integer* input[199] or just the integer part of the solution for *short integer* input;

    [e^x] and [ln] operating on *long integers* return *real* or *complex numbers* always; for *short integers*, they act in analogy to [√x];

    [#] converts a closed *integer* to another base; [→BIN], [→OCT], [→DEC], and [→HEX] cover the most popular bases.

    [|x|], [x^2], [x^3], [2^x], and [10^x] return *integers* as you expect, and

    [+/-] works for *short integers* as demonstrated on pp. 151f. For *long integers*, it works as for *reals*.

    [SHOW] displays full precision of x in top numeric rows until next keystroke. For very *long integers* >10^{296}, see the *IOI*.

  - *Dyadic functions:*

    [+], [−], and [×] return *integers* as expected (cf. pp. 76f),

    [/] returns an *integer* for *short integer* input while it may return a *real* for *long integer* input as specified in f. 90 on p. 77,

    [IDIV] returns just the integer part of the division,

    [RMD] returns the remainder of y/x (cf. pp. 157f for examples),

---

[198] Thanks to *Namir Shammas* for pointing me to it.

[199] For instance, **64** [√x] returns *integer* 8 while **65** [√x] will return 8.062… By analogy, **8** [∛x] returns 2, **1024** [lb x] returns 10, **1000** [lg] returns 3, **1296** [ENTER↑] **4** [$\sqrt[x]{y}$] returns 6, and **625** [ENTER↑] **5** [$\log_x y$] returns 4 (all being *integers*). For **−64**, [√x] may return 0.+*i*×8. (see *Complex Numbers* below).

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[IDIVR] combines IDIV and RMD,

[MOD] returns $y \bmod x$ (cf. p. 158 for examples),

[y^x], [$\sqrt[x]{y}$], and [$\log_x y$] work as specified on p. 78f;[199]

[max] (or [min]) return the maximum (or minimum) of x and y,

[GCD] returns the *Greatest Common Divisor* of x and y and

[LCM] their *Least Common Multiple*.

- *Triadic* functions:

  [×MOD] returns $(z \times y) \bmod x$ for $x > 1$, $y > 0$, $z > 0$, and

  [^MOD] returns $(z^y) \bmod x$ for $x > 1$, $y > 0$, $z > 0$
  (**Example:** **73** [ENTER↑] **55** [ENTER↑] **31** [INTS] [^MOD] returns **26**).

- *Boole's* algebra:

  [AND], [NAND], [OR], [NOR], [XOR], [XNOR], and [NOT] operate bitwise on *short integers* as shown on p. 155. They operate on *long integers* like they do in the *HP-28S*, cf. p. 131.

- Bitwise operations are for *short integers* exclusively:

  [CB], [FB], [SB], [ASR], [SL], [SR], [RL], [RLC], [RR], [RRC], and [MIRROR] work as demonstrated on pp. 154ff.

  [LJ] ([RJ]) justifies the bit pattern to the left (right) within its *word* size,

  [MASKL] and [MASKR] create mask *words*,

  [BC?] and [BS?] test if the specified bit is clear or set,

  [#B] counts the number of bits set in x.

- Probability (cf. pp. 104f):

  [x!] returns the factorial of $x \leq 450$,

  [C_{yx}] computes the number of *combinations* of x and y,

  [P_{yx}] the number of *permutations*; and

  [RANI#] returns a (pseudo) random *integer number* $\in [x, y]$.

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- Advanced mathematics (see *ReMA I* for comprehensive information about the functions following):

  〔B_n〕 and 〔B_n*〕 return the *Bernoulli numbers* and

  〔FIB〕 the *Fibonacci number*;

  〔FACTOR〕 outputs the *prime factors* of x in a *real matrix* (see the chapters about *matrices* below and the *ReM*):

  Example:

  123 456 789 012 FACTOR returns

```
[ 2.   3.   10288065751.]
[ 2.   1.              1.]
```

  since 123 456 789 012 = $2^2 \times 3 \times 10\,288\,065\,751$

  〔PRIME?〕 tests if $|x|$ is a *prime number* and

  〔NEXTP〕 returns the next *prime number* $> |x|$.

Many more functions accept *integer* input but will return different, mostly *real* output. See the *IOI* and *ReMS 3* for more about them.

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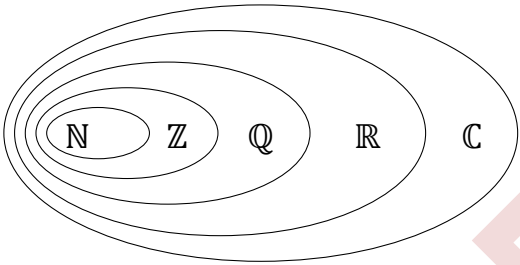
Complex Numbers: Introduction

So far, we dealt with *reals* only (fractions and *integers* are mere subsets of *reals*). Your *WP43* can do more for you. Mathematicians can deal with more complex numbers than *reals*; these are called *complex numbers* (sic!). If you don't know of them, feel free to skip the next three chapters; you can profit from your *WP43* perfectly without them.



If you know of *complex numbers*, however, note your *WP43* supports most functions operating on *reals* also for *complex numbers*.

***Complex* results in calculations**: As long as you put in just *reals*, you will get only *real* results in *SDC*. To allow for *complex* results, set CPXRES (or *flag* I for *imaginary*). Try 1 +/- √x and see the different returns.

Use CC to **enter a *complex number* directly** (cf. p. 74). With *SDC*, CC separates and concatenates the *real* and *imaginary* part in numeric input.

Examples (assuming *SDC* but FIX **5**):

3 + *i* × 4 is keyed in 3 CC 4 ENTER↑ while the display shows in **X** row:

3	3_
CC	3. +*i*×_
4	3. +*i*×4_
ENTER↑	3.000 00 +*i*×4.000 00

You enter the *real* part first – CC closes it – the *imaginary* part second as you write the number.[200] Input of negative *complex numbers* works by analogy with *real* input (cf. p. 25). Following our case above,

[200] We follow *HP-42S* tradition here – reverse *RPN* in a way. – Note that pushing four *complex numbers* on the stack is easily performed on your *WP43*. It is a challenge on *HP-42S* and *DM42* though.

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$3 - i \times 4$	is keyed in	3 CC 4 +/- ENTER↑,
$-3 + i \times 4$	is keyed in	3 +/- CC 4 ENTER↑,
$-31 - i \times 42$	is keyed in	3 1 +/- CC 4 2 +/- ENTER↑.
	Alternatively, use	3 1 CC 4 2 ENTER↑ +/- here.

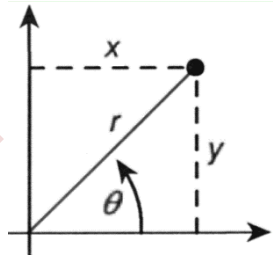
In scientific notation, e.g., SCI **5**, this last number will be displayed like

$$-3.100\ 00\times10^{1}\ -i\times4.200\ 00\times10^{1}$$

Different display formatting allows for SCI **6**:

$$-3{,}100\ 000\cdot10^{1}\ -i\cdot4{,}200\ 000\cdot10^{1}$$

With a *complex number* entered (or at least one *complex* input parameter in arithmetic operations or function calls), your *WP43* will set CPXRES automatically (indicated by ℂ in the *status bar*).



Instead of rectangular notation, a *complex number* may be written in **polar notation** as well. Polar notation is set by SF SYS.FL (POLAR) (or SF X), causing ⊙ lit in the *status bar*. Then, for a new *complex number*, its *magnitude* (or *radius*) ***r*** shall be entered first and its *phase* (or *argument*) ***ϑ*** second. Note ***ϑ*** may be entered in any *ADM* (cf. p. 139); though often *radians* or *multiples of* π will make most sense here.

Example:

Set polar notation and MODE RAD. Then a *complex number* (**5**; **1.2**^{r}) is keyed in 5 CC 1 . 2 ENTER↑ while the display (set, e.g., to FIX **2**) shows in X row successively:

5	5_
CC	5. ∠ _
1 . 2	5. ∠ 1.2_
ENTER↑	5.00 ∠ 1.20^{r}

Special cases: If a negative magnitude is entered, it is made positive and ***ϑ*** is increased by ***π*** and then normalized (i.e. ***ϑ*** will stay within

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$(-\pi, \pi]$ – cf. p. 148; greater phase input is legal, but output will be normalized always). If **0** is entered for ***r*** then ϑ will be set to **0** as well.

Composing and decomposing a *complex number*: Instead of entering a *complex number* directly, you can generate it from two closed *reals* provided in **X** and **Y**. If ⌞ is lit, CC will take y as *real* part and x as *imaginary* part composing the *complex number*. If ⊙ is lit, CC will take y as magnitude and x as phase of the new *complex number* (compare numeric input above).

Example:

MODE DEG These three entries return to *startup default* settings.

CF X

CF I

4 ENTER↑ −3 EXIT

```
                                    4
                                   -3
```

Note EXIT closed input without disturbing the stack.

CC composes a *complex number* out of x and y now, lights ℂ and returns[201]

```
                              4.-ⅈ×3.
```

SF X turns ⌞ to ⊙ and displays

```
                  5.∠-36.869 897 645 844 02°
```

Vice versa, CC may also cut a closed *complex number* x into two *reals* in **X** and **Y** following the same rules.

Example (continued):

CC returns

```
r =                                 5.
ϑ =                 -36.869 897 645 844 02°
```

ℂ⊙ remain lit in the *status bar*.

RAD CC returns

```
                  5.∠-6.435 011 087 933×10^-1 r
```

CF X turns ⊙ to ⌞ and shows

```
                              4.-ⅈ×3.
```

[201] Whenever a *complex number* is returned, your *WP43* will set CPXRES and light ℂ in the *status bar*.

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CC	returns	Re = 4. Im = −3.

ℂ⊾∡ʳ remain lit in the *status bar.*

Generally, *complex* outputs follow *real* formatting (cf. pp. 85ff). The number of displayable decimals, however, may be limited by screen space.[202] If you want to see more digits of both parts of a *complex* x, either press CC, watch, and press CC again, or call SHOW; the latter will show full precision of both parts of x until next keystroke.

In calculations, press ENTER↑ for separating *complex number* input as you do in *real* domain.

Example (assuming *SDC*):
(1 + 2 *i*) × (3 + 4 *i*) is entered and solved like this:

1 CC **2** ENTER↑		1. +*i*×2.
3 CC **4**		3. +*i*×4
×	returning	−5. +*i*×10.

With a *complex* x closed and ~~POLAR~~, …

- +/− will change the signs of both the *real* and the *imaginary* part (as shown above),
- CPX conj will change the sign of the *imaginary* part only, and
- CPX Re⇄Im will swap *real* and *imaginary* parts.
- CC cuts it so you can continue with either part of it. This also allows returning from a *complex number* with zero imaginary part to a *real*.

---

[202] Choosing *rectangular* notation and multiplication dots allows for displaying both components using large font in SCI **4** within [10^{-999}, 10^{1000}) together. It will work in SCI **5** for both components within [10^{-99}, 10^{100}). Staying with the *SDC* (i.e. MULT×) instead will cost you one displayed decimal in *complex* domain.

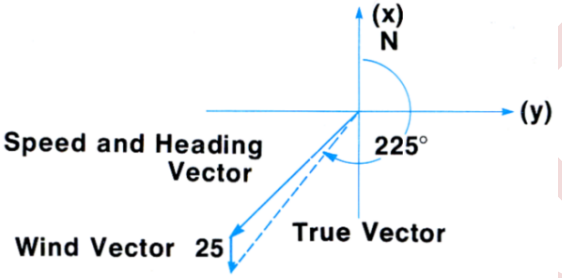
In *polar* notation, angles will be normalized to (**–π**, **π**], so there will be no space needed for a power of ten for them. Hence this will allow for SCI **6** within [10^{-999}, 10^{1000}) regardless of the multiplication symbol chosen, and for SCI **7** within [10^{-99}, 10^{100}).

See HIDE and RANGE in the *IOI*.

Also many transcendental functions will operate on *complex numbers* (sin, cos, tan, ln, e^x, y^x, $\sqrt{x}$, etc.). Please check pp. 174ff.

Complex Numbers Used for *2D* Vector Algebra

You can use *complex* domain for *2D vector* algebra as demonstrated below. The functions |x|, +, –, CROSS, DOT, and UNITV wait for you – see the contents of CPX and the *IOI*.



We can, for **example**, compute Mr. *Sweeney's* ground course (as explained on p. 144) according to the following alternative method:

DISP FIX 2

MODE DEG

SF SYS.FL POLAR (or SF X)

125 CC 225	125. ∠ 225_	indicated air speed & heading.
ENTER↑	125.00 ∠-135.00°	normalized angle (cf. p. 167)
25 CC 180	25. ∠ 180_	for the north wind
+	143.77 ∠-142.06°	the resulting *vector*.
CC	r = 143.77 ϑ = -142.06°	
360 +	143.77 217.94°	(according to navigational convention, the angle must be positive. – Compare with pp. 144f).

Two complex **examples** more (from the *HP-42S OM*):

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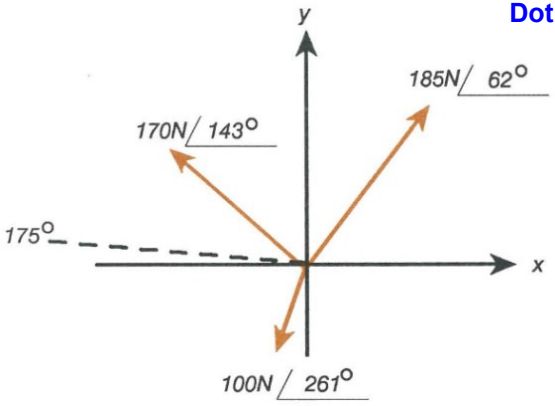


Dot Product of *Complex Numbers*:

The figure … represents three *2D* force *vectors*. Use *complex numbers* and add the three *vectors*. Then use the DOT (*dot product*) function to find the component of the resulting *vector* along the 175° line.

Solution:

DISP FIX 2

MODE DEG

SF X ensure proper *ADM* and coordinate system.

170 CC 143 ENTER↑	170.00 ∠ 143.00°
185 CC 62	185. ∠ 62_
+	270.12 ∠ 100.43°
100 CC 261	100. ∠ 261_
+	178.94 ∠ 111.15°

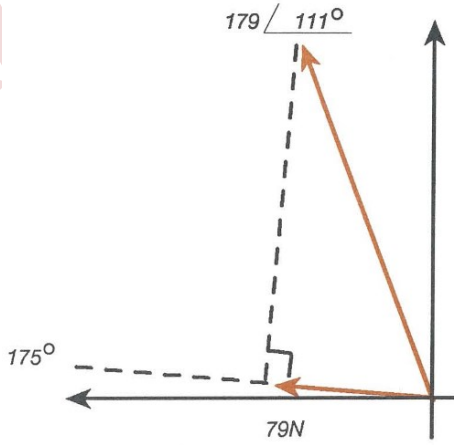
Now, take the *unit vector* at 175°:

1 CC 175 1. ∠ 175_

and multiply

CPX dot 78.86

Thus, the resulting *vector* sum has a component of approximately 79 N in the direction of 175°.



Computing Moments:

To compute the moment of two *vectors*, use the CROSS (*cross product*) function. The cross product of two *vectors* is a third orthogonal *vector*. However, when two *complex numbers* are crossed, the *WP43* simply returns a *real number* that is equal to the signed magnitude of the resulting moment *vector*.

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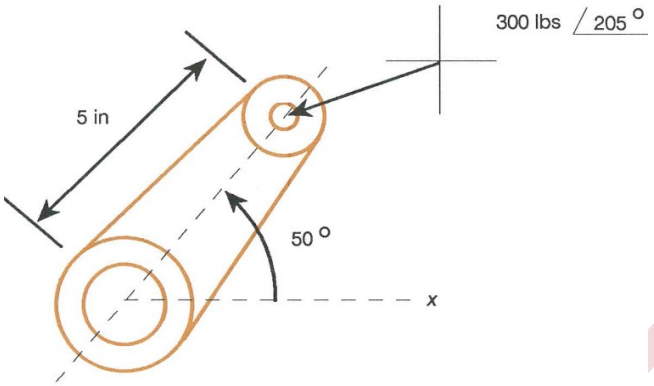


Find the moment generated by the force acting through the lever in the illustration …, where $\vec{M} = \vec{r} \times \vec{F}$.

Note lever and force are both acting in the drawing plane.

Solution (continued with the settings of previous problem):

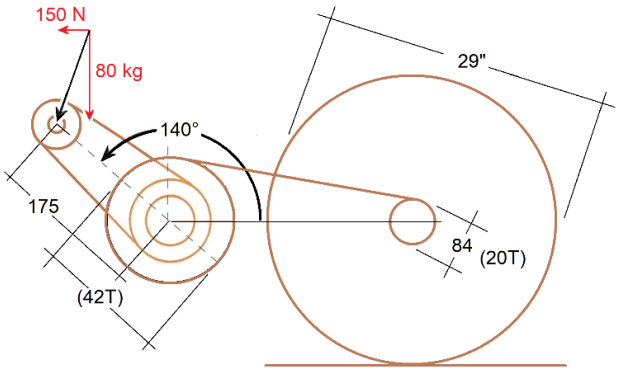
Key in the radius *vector* and the force *vector*:

5 CC **50** ENTER↑	5.00 ∠ 50.00°
300 CC **205**	300. ∠ 205_
CPX **cross** returns $y \times x$	633.93

The moment *vector* has a magnitude of 634 *pounds times inches* and, since the result is positive, the *vector* points up, perpendicular to the plane of this page.[203]

Another **example** using a bicycle:

Assume an 80 kg bicycle rider starting his 29" bike with his weight on the right pedal pointing 40° up. By drawing at the handlebars, he generates an additional horizontal force of 150 N. The pedal crank is 175 mm long. The front sprocket has 42 teeth, the rear sprocket of 20 teeth measures 84 mm diameter. What moment is exerted by him initially? What force is acting along the chain then? How big is



[203] If the problem you're working requires a true (three-dimensional) *vector* as a result, use a 1×3 or 3×1 *matrix* to represent each *vector* in three dimensions. See the chapters after next.

the force at the contact area between back wheel and ground? (Extra points: What about sliding?) Assume full pressure in the tires.

Solution (continued with the settings of previous problem):

See the problem sketched above (not to scale at all!). Finding the forces and moments is complicated by the different units used, but such is real life. So let's begin with bringing everything to coherent units:

80 [+/-] [CONST] [$g_\oplus$] [×] returns **−784.53** N.

Set rectangular coordinates by [FLAG] **CF** **SYS.FL** [POLAR] and enter

150 [+/-] [CC] **−784.53 −*i*×150.00**

Set polar coordinates by **SF** **SYS.FL** [POLAR] and enter the *vector* of the pedal crank **.175** [CC] **140**. Since $\vec{M} = \vec{r} \times \vec{F}$, we shall swap x and y before multiplying:

[x≷y] [CPX] **cross** returns **108.36** Nm for the moment (cf. previous page).

Now, what about the forces? The effective radius of the front sprocket is

84 [ENTER↑] **42** [×] **20** [/] **2** [/] i.e. **88.20** mm or

[E] **3** [/] **0.09** m.

Hence the force acting in the chain is [/] **1 228.56** N.

This force acts also on the rear gear. The force at the circumference of the rear tire results in

.084 [ENTER↑] **29** [→U] [x:] [m←] **inch** **0.74** m

[/] [×] **140.10** N.

Actually, it will be slightly greater since the effective radius of the tire will become about 2 cm smaller under load.[204]

[204] "Bei keiner anderen Erfindung ist das Nützliche mit dem Angenehmen so innig verbunden wie beim Fahrrad." (*Adam Opel*; he started as producer of sewing machines, turned to making bicycles, then automobiles; translation: '*No other invention links use/benefit and convenience/pleasure as closely as a bicycle.*')

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Complex Numbers: Summary of Functions

Many of the functions operating on *reals* also work for *complex numbers*:

- General mathematics:
  - *Monadic* functions:

    [√x] and [x²], [∛x] and [x³], [√(1+x²)], [2ˣ] and [lb x], [10ˣ] and [lg], [1/x], [eˣ] and [ln], as well as [sin], [cos], [tan], [sinh], [cosh], [tanh], and their inverses work as usual (note that [arcsin] etc. returning *complex* results will display them in *radians* but not tagged),

    [eˣ-1] and [ln(1+x)] return more accurate results with $x \approx 0$,

    [+/-] returns $x \times (-1)$ (a.k.a. 'unary minus') for closed input and ~~POLAR~~, while it turns x by 180° for POLAR, and

    [(-1)ˣ] returns $cos(\pi x) + i\, sin(\pi x)$ for non-integer x.

  - *Dyadic* functions:

    [+], [−], [×], [/], [yˣ] and [ˣ√y] work as usual,[205]

    [$\log_x$y] computes the logarithm of y for base x,

    [dot] and [cross] allow for using *complex numbers* for *2D vector* computations, and

    [||] returns $\left(1/x + 1/y\right)^{-1}$ for $x \times y \neq 0$ and 0 else.

- Isolating and manipulating parts of *complex numbers*:

  Use [CC] for composing and cutting,

  [Re] for returning the *real* part of x and [Im] for its *imaginary* part,

  [Re⇄Im] for swapping its *real* and *imaginary* part,

  [|x|] for the *magnitude* of x and [∡] for its *phase* (a.k.a. *argument*),

---

[205] ATTENTION: For odd integer x and $y < 0$, $y^{1/x} \neq \sqrt[x]{y}$. You should know the latter returns the correct solution.

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〔FP〕 (〔IP〕) for the fractional (or integer) parts of x (preserving their signs);

〔UNITV〕 returns the *unit vector* of x, and

〔conj〕 returns its *complex conjugate*.

- Rounding:

  〔RDP〕 n rounds x to n decimal places in FIX format (**Example: 0.023456789 + i × 98.7654321** RDP **5** will return 0.023 46+i×98.765 43 , cf. p. 131),

  〔ROUND〕 rounds x using the current display format,[206]

  〔RSD〕 n rounds x to n significant digits, and

  〔SHOW〕 displays full precision of x until next keystroke.

- Advanced mathematics (see the *IOI* and *ReMA I* for comprehensive information about the functions following):

  - *Monadic* functions:

    〔FIB〕 returns the extended *Fibonacci number*,

    〔g_d〕 and 〔g_d^{-1}〕 the *Gudermann function* and its inverse,

    〔sinc〕 returns $sin(x) / x$ and 〔sincπ〕 returns $sin(\pi x) / \pi x$ for $x \neq 0$ and **1** for $x = 0$,

    〔W_p〕 returns the principal branch of *Lambert's W*,

    〔W^{-1}〕 returns x for given W_p ,

    x! (= $\Gamma(x + 1)$) and 〔Γ(x)〕 calculate the *complex gamma function*, and 〔lnΓ〕 returns its natural logarithm.

  - *Dyadic* functions:

    〔AGM〕 returns the *arithmetic-geometric mean*,

    〔C_{yx}〕 and 〔P_{yx}〕 calculate with *complex gamma*,

    〔β(x,y)〕 returns *Euler's Beta function*, and 〔lnβ〕 its natural logarithm.

[206] Works like RND did on *HP-42S*.

Vectors and Matrices: Introduction and Input

So far, we dealt with just one or two or (seldom) three numbers at once. Your *WP43* can do more for you – e.g., manipulate a set of numbers in a column or a row or even in an array of 4, 6, 8, 9, 10, 12, or more numbers simultaneously. Such number columns or rows are called *vectors* and the arrays are called *matrices* by mathematicians. If you don't know of *vectors* and *matrices* yet, feel free to set them aside; your *WP43* will serve you perfectly without them.

If you know of them, however, note the function set of your *WP43* covers *vector* operations and also allows for adding, subtracting, multiplying, inverting, and transposing *matrices*, as well as for editing and manipulating parts of such *matrices*. It also provides functions for computing *determinants*, *eigenvalues* and *eigenvectors*, and for solving *systems of linear equations*. Most of these operations are collected in MATX.

Generally, we talk of an ***n×m*** *matrix* if it features ***n*** rows and ***m*** columns. A *vector* may be regarded as a special *matrix* featuring one column or one row only.

Example:

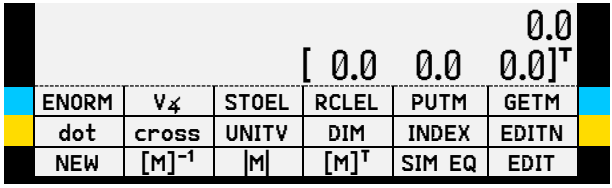
Enter a *vector* $\begin{bmatrix} 4 \\ -5 \\ 6.7 \end{bmatrix}$ and a *matrix* $\begin{bmatrix} -1 & 12 & 7 \\ 25 & 0 & 3 \end{bmatrix}$ subsequently. The *stack* shall be clear at beginning.

Solution:

DISP FIX 0 1

3 ENTER↑ 1 MATX NEW to initialize the *3D* column *vector* (i.e. a 3×1 *matrix*). See the new *matrix* in X:

The top *view* of MATX is displayed in the *menu section*:



For saving screen space, your *WP43* displays each column *vector transposed*

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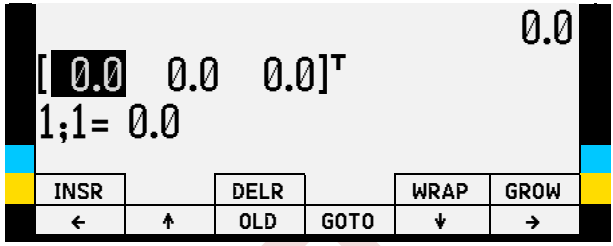
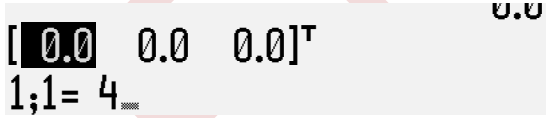
(thus the superscript **T** trailing it), i.e. in one row instead of one column on the screen. The *vector* is initialized with all its components being zero. To enter the *vector* components, press EDIT and the *Matrix Editor* will appear in the *menu section*:

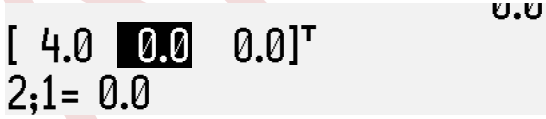


The 1st *element* of the *vector* is displayed inverted now, indicating the position of the edit cursor. This particular *element* is shown below in the format set (i.e. FIX 1 here). So we need two display rows for **X**.

Now press 4

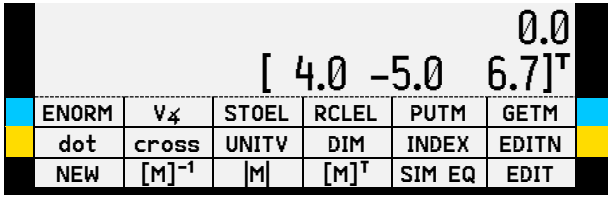


Move the cursor to the next *element* (2nd row, 1st column): →



Continue editing: **5** +/- → **6.7**

EXIT



This EXIT closed the last *matrix element*, left the *Matrix Editor* returning to the top *view* of MATX, closed input for the *matrix*, and shifted ***x*** to the right.

Now, let's initialize the 2×3 *matrix* via
2 ENTER↑ **3** NEW and begin editing once again by EDIT

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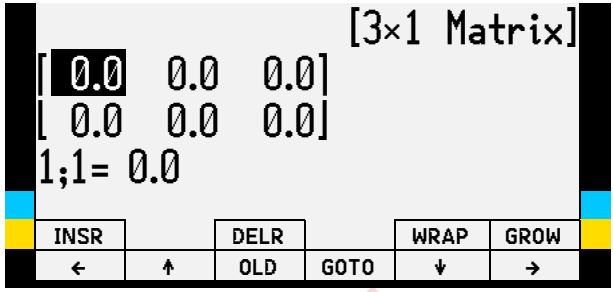
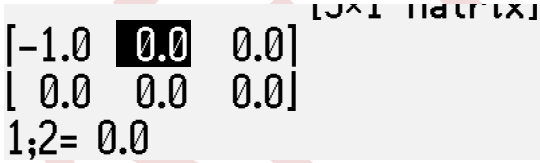


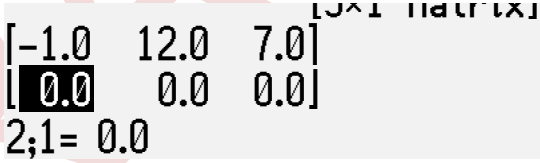
Three numeric rows are required for editing ***x*** now. The 3×1 *matrix* in **Y** is the *vector* we entered just before; note every *matrix* is displayed in this short form (a.k.a. *matrix descriptor*, using a multiplication cross even for ~~MULT×~~) in any *stack register* but **X**.

Again, all *elements* of the new *matrix* comprise zero at the beginning. Its 1st *element* is displayed inverted as the 1st *element* of the *vector* was above. *Matrix* editing will continue in the same way:

1 **+/−** **→**



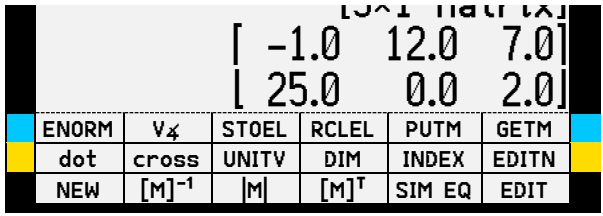
12 **→** **7** **→**



Entering the last **→** moved the cursor from the last (3rd) *element* of row 1 to the 1st *element* of row 2. So you can simply continue editing row-wise:

25 **→** **→** **2**

EXIT



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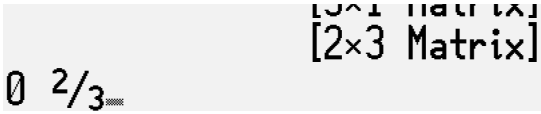
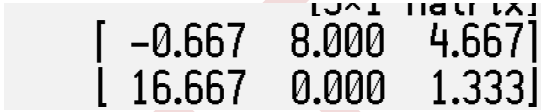
Now, also this *matrix* is closed and ready for calculating. Assume you want to multiply it by $^2/_3$ and see three decimals in the result:

DSP 3
. 2 . 3



Press × and you will get all *matrix elements* multiplied by $^2/_3$ at once:



You may store such *matrices* in any *RoV* . So let's store our result in **R00** – just press STO 0 0 for this.

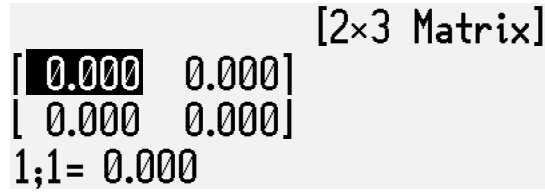
You can also create and fill a matrix directly in a *variable* (i.e. you don't have to create the matrix on the *stack* and store it afterwards).

Example:

Create a square *matrix* $[MA] = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$ and fill it directly.

2 ENTER↑ DIM α M A ENTER↑ creates **MA** as a 2×2 *matrix*.

EDITN VAR MA



4 → 3 +/- → 2 +/- → 1

Now, press EXIT and you are done with **MA** as well – and the screen looks just as before again:

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```
[3×1 Matrix]
[ -0.667   8.000   4.667]
[ 16.667   0.000   1.333]
```

Vectors & Matrices: Displaying and Editing Larger Objects

Whenever **X** contains a *matrix*, your *WP43* will try to show it completely (i.e. display all its *elements* in the format you chose for *reals*). Objects in higher *stack registers* will be indicated in a single row each (abbreviated if necessary) or shifted out of the display window.

If space doesn't suffice for showing the entire current *matrix* in the format chosen, your *WP43* will switch to the small font automatically.

Example (continued):

DSP 5

```
[3×1 Matrix]
[ -0.666 67   8.000 00   4.666 67]
[ 16.666 67   0.000 00   1.333 33]
```

If font switching should not suffice for gaining enough screen space, your *WP43* will automatically turn to abbreviated SCI 3 format for the *elements* of the respective *matrix*. This allows for showing arbitrary *real matrices* up to 5×4 entirely.[207] If a *real matrix* exceeds five rows, its 5th row is displayed filled with ellipses (…); if it exceeds four columns, its 4th column is shown filled with ellipses.

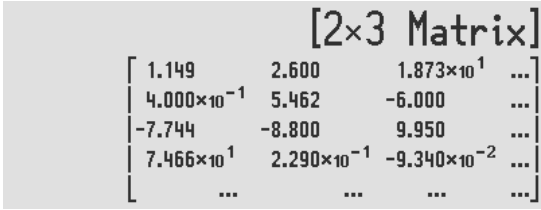
Example:
Assume a 6×5 *matrix*

$$x = \begin{bmatrix} 1.1493 & 2.6 & 18.725 & 3 & 9.2 \\ 0.4 & 5.462 & -6 & 95.1 & 51.6 \\ -7.744 & -8.8 & 9.95 & 54.5 & 0.17 \\ 74.66 & 0.229 & -0.0934 & 2 & -3.829 \\ 33.9 & -79.4 & 3.436 & 9.08 & 4.256 \\ 0.0488 & 7 & 5.98 & -0.68 & -22.492 \end{bmatrix}$$

was entered on the present *stack* and is in **X** now. Then the screen will look

[207] ALL may be an even more favorable format sometimes. It will be not set automatically, however.

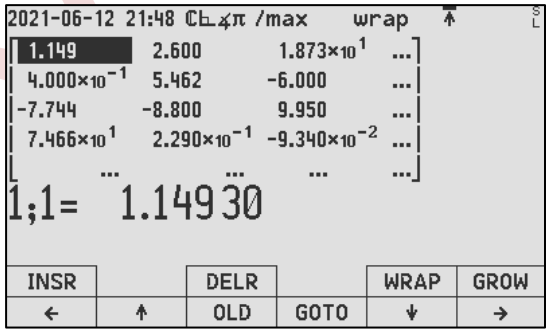
like this:



Editing such a large *matrix* will push also y from the screen as long as the *Matrix Editor* stays open. You can browse the entire *matrix* regardless of its size always.

For *matrices* larger than 5 rows and/or 4 columns, the display may vary depending on the cursor position: ellipses may appear on top and bottom, left and right side. A view of 3×3 *real matrix elements* including the one selected by the cursor can be seen always at least – this selected *element* is also displayed below of the *matrix* in the format you have chosen for *reals*. Since the indices of this *element* are shown there as well you always know where you are.

Example (continued):
Press MATX EDIT and you will see:



The 1st *matrix element* is selected; and the lowest numeric row shows it in FIX **5** as we had chosen.

Now, go to the bottom row of this *matrix* by pressing ↓ five times and you will get:

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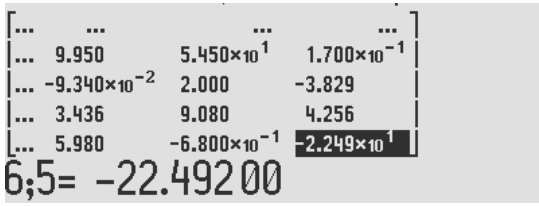
Go to the very last (bottom right) *element* of this *matrix* by pressing → four times:



Wherever you are within a *matrix*, you can replace or modify the currently selected *element* in three different ways:

1. Recall a number from any *RoV* via RCL.
2. Simply key in a new number replacing the old one.

Example (continued):
Replace the last *matrix element* by **17.435**.

Key in 1 7 . 4 3 5

3. You can calculate a new value either operating on the present value of the current element or entirely from scratch, using the full function set of your *WP43*. If you need any *menus* to reach a function, they will temporarily replace the *Matrix Editor menu*; exiting those *menus* will return to the *Matrix Editor menu*.

   Example (continued):
   If you now decide you want to keep the old *element* (**−22.492**), however, call OLD. This old content will stay available until you leave this *matrix element* by pressing ←, ↑, ↓, →, or EXIT after entering the new number.

You can also store (copy) the current *matrix element* into an arbitrary destination using STO as long as the *matrix* is open for editing.

Repeatedly pushing the cursor in one direction (e.g., by →) will jump through the *matrix* rows to the last one and then return from the bottom

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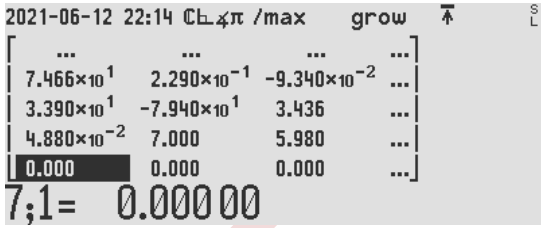
right *element* to the top left in default *wrap* mode. If *grow* is set instead, then → from the bottom right *element* will add a new row to the *matrix*.

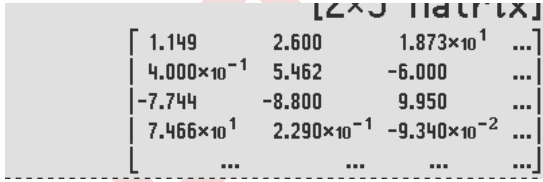
Example (continued):

GROW

→

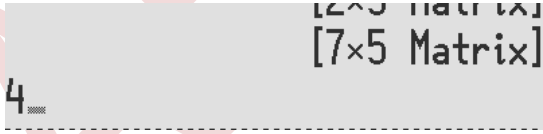


Here, we are done with this *matrix* for now. So press EXIT and you will see again:



The 1st *matrix element* is not highlighted anymore since you left the *Matrix Editor*.

Thus, just entering 4 will display, due to *automatic stack lift*:



So *matrix* editing is easy and straightforward. The *IOI* contains additional information, also about the commands DELR, INSR, and R⇄R showing up in the *Matrix Editor menu*.

Vectors & Matrices: Complex Stuff

Your *WP43* supports also *complex vectors* and *matrices*, i.e. *matrices* comprising *complex elements*. They are created and initialized like *real* objects via NEW or DIM as explained above (luckily, *matrix* dimensions stay integer). Or you can recall a *real matrix* and edit it; if you enter one or more *complex numbers* for its *elements* it becomes a *complex matrix* – you can store it at the same or another place after editing.

Example (continuation of p. 179):

Create and store a *complex matrix* $\begin{bmatrix} 5+8i & \pi i \\ -2 & 4-3i \end{bmatrix}$.

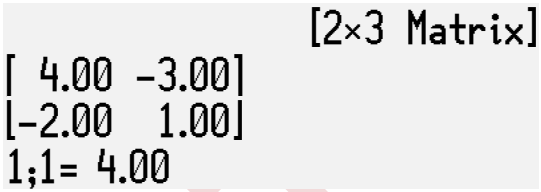
Solution:

Remember we have created a 2×2 *matrix* just a few pages ago. So it is most easy to recall it for using it as a template:

RCL VAR MA

DSP 2 since this will suffice for the process following.

EDIT

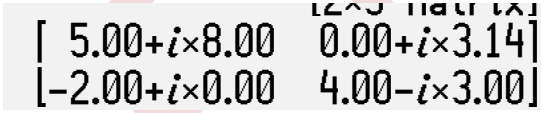


We can now enter the new *elements* here as we have done before:

5 CC 8 → 0 CC π → → 4 CC 3 +/−

EXIT

STO 0 1



Since we edited on the stack and stored the resulting new *complex matrix* in a new location, the old *real matrix* **MA** is not affected at all.

Compare pp. 166f for the input and formatting of *complex numbers*. Everything else works as it does for *real* matrices. You see *complex matrices* are no complex topic at all for you with your *WP43*.

Vectors & Matrices: Calculating

As we have seen on p. 179, your *WP43* can multiply a *matrix* by a plain number (a.k.a. a *scalar*); doing this, each *element* of said *matrix* is multiplied by said number. Additions, subtractions, and divisions work alike for a *matrix* y combined with a *scalar* x. Vice versa, with a *scalar* y and a *matrix* x, additions, subtractions and multiplications will work the same way (divisions are special, see below). Also *monadic* functions will operate on each *matrix element* in your *WP43*, if applicable (except 1/x).

Examples:

With an arbitrary *matrix* in **X**, pressing...

- √x will extract the square root of each *matrix element* individually.

If CPXRES is set, a *real matrix* x comprising at least one negative *element* will become *complex* this way.

- [x^2] will square each *matrix element* individually (instead, use [ENTER↑] [×] for squaring the *matrix*).
- [|x|] will calculate the absolute value of each *matrix element* (instead, use [MATX] ENORM for calculating the *Euclidean norm* of the *matrix* or take |M| for getting its *determinant*).
- [+/−] will change the sign of each *matrix element*.

You can also let the *dyadic* functions [+], [−], [×], or [/] operate on two *matrices* or *vectors* (i.e. *DT* 8 and 9), if the rules of *linear algebra* are obeyed:

	y	x	Operation	Resulting x
[+] / [−]	[m × n Matrix]	[m × n Matrix]	$[y]+/-[x]$	[m × n Matrix]
[×]	"	[n × p Matrix]	$[y] \cdot [x]$	[m × p Matrix]
[/]	"	[n × n Matrix]	$[y] \cdot [x]^{-1}$	[m × n Matrix]

The 1st row of this table reads as follows: For adding or subtracting two arbitrary *matrices*, both must be of identical size and shape, and the result will be of the same size and shape. Subsequent rows read by analogy.[208] If either *matrix* is *complex*, the result will often be *complex* as well.

Example (continuation of p. 180):
Multiply the *matrices* in **R00** and **MA**. Output format shall be FIX **3**.

Solution (we omit the *menu section* in the following pictures):
[DSP] [3]
[RCL] VAR MA (or [RCL] [α] [M] [A] [ENTER↑] – cf. p. 61)

[208] Remember *matrix* multiplication behaves different than multiplication of plain numbers. Generally, for two arbitrary *matrices* [A] and [B] of matching sizes and shapes, [A]·[B] ≠ [B]·[A]. Only square *matrices* can be squared.

And *matrix* division is defined as multiplication of the numerator times the inverse of the denominator. Therefore, x must be an invertible (i.e. non-singular) *matrix* here – else you can't divide by that *matrix*. Only square *matrices* may be invertible.

```
[2×2 ℂ Matrix]
[ 4.000 -3.000]
[-2.000  1.000]
```

The '2×2 ℂ Matrix' in Y is the *complex matrix* we entered in previous chapter – the *stack* handles *matrices* as it handles other objects. Now let's recall ***r00***:

RCL 0 0

```
[2×2 Matrix]
[ -0.667  8.000  4.667]
[ 16.667  0.000  1.333]
```

The '2×2 Matrix' in Y now is the one we have recalled from **MA** into X before recalling ***r00***. We multiply ***y*** times ***x*** as usual by × resulting in

```
[-79.000  48.000  19.000]
[ 27.000 -24.000 -11.000]
```

You see that arithmetic operations on *matrices* are almost as easy as on *scalars* using your *WP43*.

And your *WP43* features further *matrix* operations: |M| for computing its *determinant*, $[M]^{-1}$ for inverting, $[M]^{T}$ for transposing, M.LU and M.QR for computing the LU and QR decompositions, and two *norms* (*Euclid's* ENORM and the *row norm* RNORM) – please look them up in the *IOI*.

Example:

We want to invert a 2×2 *matrix* $[M] = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

Solution:

Just enter the *matrix* as usual

2 ENTER↑ MATX DIM α M ENTER↑ creates ***M*** as a 2×2 *matrix*.

RCL VAR M

EDIT etc. until EXIT

```
[2×3 Matrix]
[ 1.000  2.000]
[ 3.000  4.000]
```

[M]⁻¹

```
[-2.000  1.000]
[ 1.500 -0.500]
```

Thus, the inverted *matrix* reads $[M]^{-1} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0{,}5 \end{bmatrix}$.

For two *vectors* of identical size, there are two special multiplications provided: DOT and CROSS. DOT will return the dot product, a *scalar* – exactly what the table above says for ***m*** = ***p*** = **1**. CROSS works exclusively for two *2D* or *3D vectors* and will return their cross product.

Example from the *HP-27 OH*:

The force $\vec{F}$ on a particle with charge ***q*** which is moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$ is given by $\vec{F} = q\,\vec{v} \times \vec{B}$. Suppose a proton ($q = -e = 1.6 \cdot 10^{-19}\,coulomb$) is moving with velocity $\vec{v} = (0.4 \quad 2.8 \quad -1.2) \cdot 10^7\ {}^{m}/_{s}$. A uniform magnetic field surrounding the proton is of a strength $\vec{B} = (1.3 \quad -0.3 \quad 0.7)\,tesla$. Calculate the force on the proton. This can be written as

$$\vec{F} = q\,\vec{v} \times \vec{B} = 1.6 \cdot 10^{-19} \cdot 10^7 \cdot (0.4 \quad 2.8 \quad -1.2) \times (1.3 \quad -0.3 \quad 0.7)$$

Solution (FIX **1** will do):

In cross products, *vectors* must be entered in proper sequence as written from left to right:

3 ENTER↑ **1** MATX DIM α ▼ V ENTER↑ creates ***v*** as a 3×1 *matrix*.

RCL VAR **v**

EDIT etc. until EXIT

```
[3×1 Matrix]
[ 0.4   2.8 -1.2]ᵀ
```

STO α B ENTER↑ creates also ***B*** as a 3×1 *matrix*.

RCL VAR **B**

EDIT etc. until EXIT

```
[3×1 Matrix]
[ 1.3 -0.3  0.7]ᵀ
```

cross returns

```
[2×2 Matrix]
[ 1.6 -1.8 -3.8]ᵀ
```

E **7** ×

CONST E F1 × returns

$$\begin{bmatrix} 2.6\times10^{-12} & -2.9\times10^{-12} & -6.0\times10^{-12} \end{bmatrix}^T$$

DRAFT

... *newtons*, of course. The total 'size' or absolute value of this force is

ENORM 7.2×10^{-12}

Compare the weight of a proton:

CONST M P F1 recall the proton mass m_p.

CONST G F6 × recall the gravity acceleration $g_\oplus$ and multiply.

1.6×10^{-26}

So this is a force ratio of

/ 4.4×10^{14}

Thus, physicists deliberately neglect gravitational effects in such microscopic calculations as long as no higher precision is required.

If you just want to perform elementary *vector* operations in *2D*, however, there are two simple alternatives (known from earlier calculators):

1. Enter the *Cartesian* components of each *vector* in **X** and **Y** (if necessary, convert polar into *Cartesian* components by R← before) and use Σ+ for additions (cf. examples on pp. 142ff). Recall the result via SUM; it may look like this, for instance:

   Σy = 1 464.21
   Σx = 123.58

2. Calculate with *complex numbers*. In *complex* domain, +, −, and also *2D vector* multiplications are viable since the commands DOT and CROSS are contained in CPX as well (cf. pp. 166ff).

Vectors & Matrices: Solving Systems of Linear Equations

Your *WP43* can also solve simultaneous linear equations (of the kind $[A]\cdot\vec{X}=\vec{B}$) for you. To deal with such problems, proceed as follows:

1. Specify the number of unknowns[209] (e.g., 3) by entering

   MATX SIM EQ 0 3

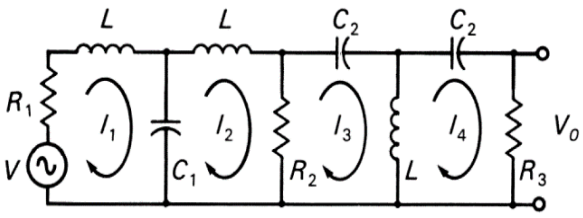
Your *WP43* automatically creates and dimensions three *matrices* (if necessary): Mat.A, Mat.B, and Mat.X. You will see a new *menu* showing up:

2. Press **Mat A**. The *Matrix Editor* will open and you can enter the 16 *elements* of the 4×4 *coefficient matrix* **Mat.A** (you learned on pp. 176ff how to do this). When done, leave the *Matrix Editor* by EXIT to return to the *menu* shown in step 1.
3. Press **Mat B** and enter the *elements* of the 4×1 *constant matrix* **Mat.B** the same way (this is a *vector* actually).
4. Press **Mat X** to let your *WP43* compute the 4×1 *solution matrix* **Mat.X** (a *vector* again). You are done!

To work another problem with the same number of unknowns, return to step 2 or 3. For a problem with a different number of unknowns, press EXIT for leaving the *menu* shown above, and start over with step 1. It goes without saying this works for *real* and *complex matrices*.

Example taken from the *HP-15C Advanced Functions Handbook*:

Find the output voltage V_0 for the filter network shown. Input voltage shall be $V = 10$ V at frequency $\omega = 15 \times 10^3$ rad/s, and $R_1 = 100\ \Omega$, $R_2 = 1\ \mathrm{M}\Omega$, $R_3 = 100\ \mathrm{k}\Omega$, $C_1 = 250$ nF, $C_2 = 25\ \mu$F, and $L = 10$ mH.



Solution:

Describe the network using loop currents (via *Kirchhoff* equations):

$$\begin{bmatrix} (R_1+i\omega L-i/\omega C_1) & (i/\omega C_1) & 0 & 0 \\ (i/\omega C_1) & (R_2+i\omega L-i/\omega C_1) & (-R_2) & 0 \\ 0 & (-R_2) & (R_2-i/\omega C_2+i\omega L) & (-i\omega L) \\ 0 & 0 & (-i\omega L) & (R_3+i\omega L-i/\omega C_2) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} V \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

[209] This procedure works as it did on the *HP-42S*. The number of unknowns is only limited by the free memory available in your *WP43* at execution time.

This system shall be solved for the four unknown loop currents I_1 through I_4. Then, $V_0 = R_3 \, I_4$. ENG **4** will suffice for the output.

With the values given above, this system of linear equations will look like:

$$\begin{bmatrix} 100 - i\,116.67 & i\,266.67 & 0 & 0 \\ i\,266.67 & 10^6 - i\,116.67 & -10^6 & 0 \\ 0 & -10^6 & 10^6 + i\,147.33 & -i\,150 \\ 0 & 0 & -i\,150 & 10^5 + i\,147.33 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Start the solver via [MATX] SIM EQ [0][4].

Press Mat A and enter the big *complex matrix* using the *Matrix Editor* as explained above.

Press Mat B and key in the voltage *vector*.

Finally, call Mat X and you will get the *vector* of the loop currents:

[199.50×10⁻⁶ +i×4.096 4×10⁻³ ...]ᵀ

Its last (4th) *element* is the current we need. Separate it with:

INDEX VAR Mat X

4 [STO] [I] set the index pointers to 4th row …

1 [STO] [J] … and 1st column and recall this *element*:

RCLEL 53.446×10⁻⁶ −i×2.259 9×10⁻⁶

100 [E] 3 [×] multiply this current with R_3 and set POLAR

[SF] [X] 5.349 4 ∠ −2.421 2°

So the output voltage is $V_0 = 5.35\ \mathrm{V}$ with a little phase shift.

Vectors & Matrices: Eigenvalues and Eigenvectors

An *eigenvalue* is a *real* or *complex number* λ solving the matrix equation $[A] \cdot \vec{X} = \lambda \cdot \vec{X}$. [210] Then, the *vector* $\vec{X}$ is called an *eigenvector* of $[A]$. Usually, there will be more than one λ and a multitude of *vectors* $\vec{X}$ solving this problem. Thus, the simplest set of linearly independent *vectors* $\vec{X}$ is

[210] *TN*: The German term *eigen* may translate here into English *inherent*.

chosen to build the base of the *eigenspace* belonging to a particular *eigenvalue* found. And the simplest set of *eigenvectors* building a base of a space of the same dimension as $\vec{X}$ are called the *eigenvectors* of $[A]$. Your *WP43* can solve such problems for you as well:

Example 1:

What are the *eigenvalues* of the *matrix* $[M] = \begin{bmatrix} 2 & 1 \\ 6 & 1 \end{bmatrix}$?

Solution:

We have got a 2×2 *matrix* named **M** already. We don't need its old contents anymore so we simply recall and edit it:

RCL VAR M

DISP FIX 0 1

MATX EDIT etc.

```
[ 2.0   1.0]
[ 6.0   1.0]
```

STO VAR M

The *eigenvalues* are the solutions of the *characteristic polynomial* of this problem $(2-\lambda)(1-\lambda)-6=0$

▲ EIGVAL returns

```
M = [2×2 Matrix]
[ 4.0   0.0]
[ 0.0  -1.0]
```

being the *matrix* with the *eigenvalues* as its diagonal *elements*. This resulting diagonal *matrix* is pushed on the *stack*.

Example 1 (continued):

Now, what are the *eigenvalues* of $[N] = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$?

Solution:

▲ EDIT etc.

```
[ 3.0   4.0]
[-4.0   3.0]
```

STO α N ENTER↑

▲ EIGVAL returns

```
N = [2×2 Matrix]
[ 3.0+i×4.0   0.0+i×0.0]
[ 0.0+i×0.0   3.0-i×4.0]
```

DRAFT

Note that we got *complex eigenvalues* here although **N** comprises only *reals.*

Example 2:

What are the *eigenvalues* of $[Q] = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$?

Solution:

3 ENTER↑ MATX DIM α Q ENTER↑ creates **Q** as a 3×3 *matrix*.

RCL VAR Q

EDIT etc.

```
[2×2 ℂ Matrix]
[ 0.0   0.0 -2.0]
[ 1.0   2.0  1.0]
[ 1.0   0.0  3.0]
```

▲ EIGVAL returns

```
Q = [3×3 Matrix]
[ 1.0   0.0  0.0]
[ 0.0   2.0  0.0]
[ 0.0   0.0  2.0]
```

Note one *eigenvalue* comes twice here.

Let's get the *eigenvectors* of **Q** now – they will be put out as row *vectors* of a *matrix*:

x≷y returns **Q** into **X**

```
[3×3 Matrix]
[ 0.0   0.0 -2.0]
[ 1.0   2.0  1.0]
[ 1.0   0.0  3.0]
```

EIGVEC pushes this *matrix* on the *stack*

```
Q = [3×3 Matrix]
[ 1.0   0.0 -2.0]
[ 0.0   1.0  1.0]
[-1.0   0.0  1.0]
```

STO α V ENTER↑

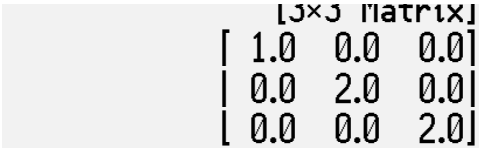
× returns

```
[3×3 Matrix]
[ 2.0   0.0 -2.0]
[ 0.0   2.0  1.0]
[-2.0   0.0  1.0]
```

DRAFT

RCL L recalls **V**.

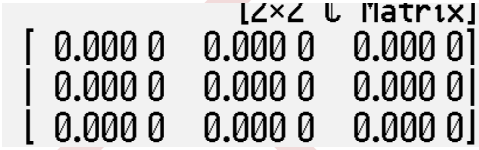
1/x x≷y × returns



This looks very much like what was returned for the *eigenvalues* of **Q** above. Let's check:

DSP 4

− returns



So the result of $[V]^{-1} \cdot [Q] \cdot [V]$ with **V** being the *matrix* of the *eigenvectors* of **Q** is exactly the diagonal *matrix* of the *eigenvalues* of **Q**. This holds generally for any square *matrix*.

Your *WP43* can compute *eigenvalues* and *eigenvectors* for *matrices* featuring rational *elements* as well:

Example 3:

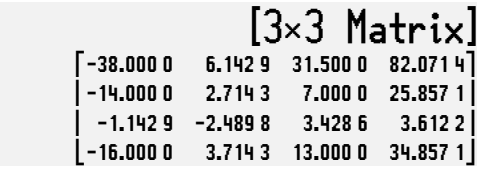
What are the *eigenvalues* of $\begin{bmatrix} -38 & 43/7 & 63/2 & 1149/14 \\ -14 & 19/7 & 7 & 181/7 \\ -8/7 & -122/49 & 24/7 & 177/49 \\ -16 & 26/7 & 13 & 244/7 \end{bmatrix}$?

Solution:

4 ENTER↑ MATX **NEW** creates a 4×4 *matrix*.

EDIT **38** +/− → **.43.7** → **.63.2** → **.1149.14** → etc.

Each *matrix element* can be entered as an integer or fraction but is converted to a *real* following the current display format as soon as said *element* is closed:



DRAFT

DSP 0 1 shall suffice here

$$\begin{bmatrix} -38.0 & 6.1 & 31.5 & 82.1 \\ -14.0 & 2.7 & 7.0 & 25.9 \\ -1.1 & -2.5 & 3.4 & 3.6 \\ -16.0 & 3.7 & 13.0 & 34.9 \end{bmatrix}$$

MATX ▲ EIGVAL returns

Note the 2nd *eigenvalue* is zero.

$$\begin{bmatrix} -5.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -2.4\times10^{-32} & 0.0 & 0.0 \\ 0.0 & 0.0 & 5.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 3.0 \end{bmatrix}$$

RCL L
DSP 2
EIGVEC displays

$$\begin{bmatrix} 4.00 & 1.25 & 5.00 & 1.33 \\ 3.00 & -0.50 & -4.00 & 0.67 \\ 1.00 & -1.00 & 5.00 & -1.00 \\ 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix}$$

Generally, your *WP43* solves *characteristic polynomials* numerically.

Vectors & Matrices: Dealing with Statistical Data

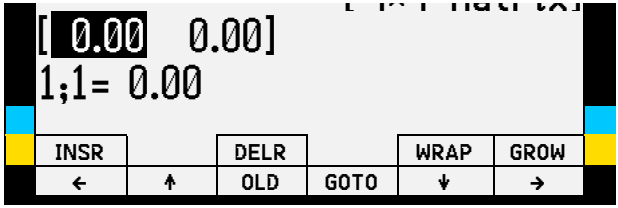
You can enter *2D* data using an input *matrix* as well as keying them in point after point. How is this done?

Let's return to the *MSA* introduced on p. 119. Resume it with its step 4 – remember there were ≥ 30 samples measured twice in a special way using the measuring system under investigation, resulting in a corresponding number of pairs of measured values recorded:

4. Create a 1×2 named *matrix* and open it for editing:

1 ENTER↑ 2 MATX DIM α M S A ENTER↑ creates **MSA** accordingly.

EDITN VAR MSA



DRAFT

GROW allows the *matrix* to grow with data entered.

Now key in all recorded pairs of measured values: The 1st value shall be y, the 2nd x – so the keystroke sequence will be ***mv1*** → ***mv2***. Then, another → lets the *matrix* grow by one row for the next data point.

With all points entered in the *matrix*, eventually key in

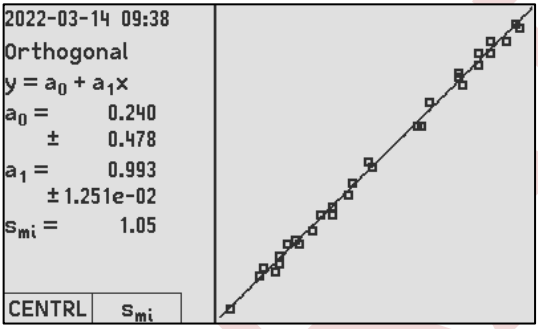
EXIT

FIT CLΣ

RCL VAR MSA

Σ+ Calling Σ+ with such a *matrix* in **X** will accumulate all your data at once and display the very last point in **X** and **Y** (and save a copy of the input *matrix* in **L**).[211]

5. Call ▲ PLOT for plotting these points in a new window.



6. Press CENTRL to fit a straight line through all these data points. Check if this line deviates significantly from $y = x$; coarsely (with some 95% confidence), this will apply if
$a_0 \pm 2\,s(a_0)$ excludes **0** or
$a_1 \pm 2\,s(a_1)$ excludes **1**.

If it does then your measuring system may be running up still (wait for equilibrium) and/or show an unstable zero (check and fix). Then restart the procedure with a new set of measurements (step 2 on p. 119).

7. Else (i.e. if $a_0 \pm 2\,s(a_0)$ includes **0** and $a_1 \pm 2\,s(a_1)$ includes **1**) press [smi] to push the precision of your measuring instrument investigated on the *stack*. This measured precision applies under the boundary conditions prevalent during steps 2 and 3 of this test, and is definitely more realistic than most catalog values stated by manufacturers.

Then enter

30 × 1/x

and multiply with the tolerance you have to control (**0.01** in this case). A result **≥ 1** will tell you this measuring instrument, device,

[211] The steps 5 through 7 are the same as shown in *Reals: Some Industrial Problems Solved*; they are repeated here just for your convenience.

This input method may be applied as well to other data sets – see *App. 4*.

machine, or system may be used for controlling series production with said tolerance zone (i.e. it is a *capable* instrument for this job) under said conditions; else you shall strive for a more precise device, better measuring conditions, or a wider tolerance. See *ReMA I* for more.

Vectors & Matrices: Summary of Functions

There are functions operating on an entire *matrix* (in $\mathbf{X}$, for instance) and others operating on its *elements* (x_{ij}) individually. Let's list the 1st set first:

- General mathematics:
  - Mostly *monadic* functions operating on an entire *matrix*:

    〔ENORM〕 returns the *Euclidean norm* of x (i.e. a *real number*),

    〔RNORM〕 computes the *row norm* of x (i.e. a *real number*),

    〔RSUM〕 computes the row sums of x (i.e. a *vector*),

    〔|M|〕 returns the *determinant* of x (i.e. a *real* or *complex number*),

    〔[M]^T〕 returns the transpose *matrix* of x,

    〔[M]$^{-1}$〕 and 1/x return the inverse *matrix* of x, if applicable.

    〔EIGVAL〕 returns the *eigenvalues* of x, and
    〔EIGVEC〕 its *eigenvectors* (cf. pp. 190ff), while

    〔UNITV〕 returns the *unit vector* of x (see the *ReM*);

    〔INDEX〕 indexes the *matrix* specified.

  - Functions operating on the *indexed matrix* or its pointers:

    〔STOIJ〕 sets the index pointers i and j according to x and y,

    〔RCLIJ〕 recalls i and j into $\mathbf{X}$ and $\mathbf{Y}$,

    〔I+〕, 〔I-〕, 〔J+〕, and 〔J-〕 increment or decrement i and j,

    〔STOEL〕 stores x in the current *element* a_{ij} of the *indexed matrix*,

    〔RCLEL〕 recalls the current *element* a_{ij} into $\mathbf{X}$.

    〔GETM〕 copies an IP(y) × IP(x) sub-*matrix* out of the *indexed matrix* into $\mathbf{X}$, starting with its *element* a_{ij} and

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〔PUTM〕 inserts a (smaller) *matrix* x into the *indexed matrix*, starting with target *element* a_{ij} (see the *IOI* for GETM and PUTM).

〔R⇄R〕 swaps row IP(x) and row IP(y) of the *indexed matrix*,

〔FIND〕 searches for the value x in this *matrix*,

〔CMAX〕 (〔CMIN〕) finds the greatest (smallest) *element* in column j.

- *Monadic* functions operating on each *element* x_{ij} of x individually:

  +/−, |x|, ∠, $\sqrt{x}$ and 〔x^2〕, 〔$\sqrt[3]{x}$〕 and 〔x^3〕, 〔$\sqrt{(1+x^2)}$〕, 〔2^x〕 and 〔lb x〕, e^x and 〔ln〕, 10^x and 〔lg〕, 〔sin〕, 〔cos〕, 〔tan〕, 〔sinh〕, 〔cosh〕, and 〔tanh〕 as well as their inverses work as explained above,

  〔e^x-1〕 and 〔ln(1+x)〕 return more accurate results with $x_{ij} \approx 0$;

  〔sinc〕 (〔sincπ〕) returns a *matrix* containing $\frac{sin(x_{ij})}{x_{ij}}$ $\left(\frac{sin(\pi\, x_{ij})}{\pi\, x_{ij}}\right)$ for $x_{ij} \neq 0$ and **1** for $x_{ij} = 0$,

  〔(-1)^x〕 returns $cos(\pi\, x_{ij}) + i\, sin(\pi\, x_{ij})$ for non-integer x_{ij}.

  〔RDP〕 n rounds x_{ij} to n decimal places in FIX format,

  〔ROUND〕 rounds x_{ij} using the current display format, and

  〔RSD〕 n rounds x_{ij} to n significant digits;

  〔erf〕 and 〔erfc〕 return the *error function* and its complement for each x_{ij},

  〔FIB〕 the extended *Fibonacci number* for each x_{ij},

  x! calculates the *factorial* of x_{ij} or $\Gamma(x_{ij}+1)$, respectively,
  〔Γ(x)〕 the *gamma function* for each x_{ij}, while
  〔lnΓ〕 computes its logarithms, and

  〔ζ(x)〕 returns *Riemann's Zeta function* for each x_{ij}.

  For *real matrices*,

  〔ceil〕 returns a *matrix* with the smallest integers ≥ x_{ij} and
  〔floor〕 with the greatest integers ≤ x_{ij} ,

  〔FP〕 returns a *matrix* with the fractional parts of x_{ij} and
  〔IP〕 with their integer parts, preserving their signs,

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〔EXPT〕 returns a *matrix* with the exponents of each x_{ij} and
〔MANT〕 with their mantissas, while
〔sign〕 replaces each x_{ij} by its *signum* returning **1** for $x_{ij}>0$,
–1 for $x_{ij}<0$, and **0** for $x_{ij}=0$ or non-numeric data.

For *complex matrices*, 〔conj〕 returns a *matrix* comprising the *complex conjugates* of x_{ij}.

- *Dyadic* functions operating on x and y:

  〔+〕, 〔−〕, 〔×〕, and 〔/〕 work as explained on pp. 184ff,

  〔dot〕 represents the dot product operating on two arbitrary *vectors* or *matrices* of matching size (cf. pp. 187f),

  〔cross〕 computes the cross product operating on two *real 2D* or *3D vectors* of identical size (cf. pp. 187f), and

  〔V∠〕 returns the angle between two *real 2D* or *3D vectors* of identical size.

- *Dyadic* functions operating on each element y_{ij} of y individually:

  〔y^x〕 raises y_{ij} to the power of x, while 〔x√y〕 extracts the xth roots of y_{ij}.

  〔log_x y〕 calculates the logarithms of y_{ij} for base number x.

- Isolating and manipulating bulk parts of a *complex matrix* x:
  Use …

  〔CX→RE〕 for cutting x in its two parts and storing them in $\mathbf{X}$ and $\mathbf{Y}$,

  〔RE→CX〕 for composing a complex x from its two parts x and y,

  〔Re〕 for returning its *real* part and 〔Im〕 for getting its *imaginary* part,

  〔|x|〕 for isolating its *magnitude* and 〔∠〕 for returning its *phase*, and

  〔Re⇄Im〕 for swapping its *real* and *imaginary* part.

The *Matrix Editor* was demonstrated comprehensively on pp. 176ff. Turn to the *IOI* for additional information about all *matrix* operations provided, in particular about those in MATX not mentioned in this summary.

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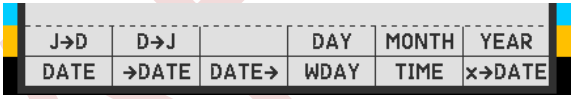
Dates

For date calculations, choose one out of three *date display modes* (*DDM*) on your *WP43*: Y.MD, D.MY, and M.DY (these mode-setting commands are contained in the 2nd *view* of CLK). *ISO* YMD is *startup default*.

Date input is decimal according to the *DDM* chosen. It is terminated by .d (as shown on pp. 73f).

Example:
The 26th of November in 2024 is entered
2024.1126 .d in Y.MD,
26.112024 .d in D.MY, and
11.262024 .d in M.DY.



Any *real* may be converted into a *date* via CLK **x→DATE** (digits exceeding those needed for a *date* in the *DDM* chosen will be interpreted as daytime – see next chapter), and any triple of *reals* or *integers* on the *stack* via **→DATE** (cf. p. 43).

Vice versa, **DATE→** splits a *date* in three *integers* and pushes them on the stack as demonstrated on p. 43. Both **DATE→** and **→DATE** observe the current *DDM*. If you want to extract particular information from a *date* independent of current *DDM*, we recommend using one of the operations CLK **DAY**, **MONTH**, or **YEAR**.

Like in the *status bar*, a closed *date* input or a *date* output returned by a function (e.g., by **DATE**) is displayed like in the following **example**:

Thursday 2023-03-09 18:29:39 in Y.MD,
Thursday 09.03.2023 18:29:39 in D.MY,
Thursday 03/09/2023 18:29:39 in M.DY (with TDM24 set)..

So you know the present *DDM* immediately from looking at the date format in the *status bar*.

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CLK **WDAY** takes a *date* from the *stack* – or a decimal input of, e.g., **2013.0504** in Y.MD (equivalent to inputs of **4.052013** in D.MY or **5.042013** in M.DY) – and returns an *integer* indicating the position of this day in the corresponding week, temporarily headed by the name of this weekday:

```
Saturday                                6.
```
[212]

Expect similar returns after CLK **DATE**.

Use + to add an *integer* (or *real*) *number*, representing an integer number of days, to a *date* – and − to subtract such a number from a *date*. The result will be a *date* again. And a *date* may be subtracted from another *date*, resulting in a difference of days. Compare the matrices on pp. 76f.

Example (assuming *SDC*):
The interval between 1931-11-09 and 2022-03-26 amounts to 90 years and almost five months. How many days were these? And how many hours?

[212] *TN*: Numeric output of WDAY corresponds directly to Chinese weekdays 1 to 6. For Portuguese weekdays (*segunda-feira* etc.), add 1 to days 1 to 5.

Calculation of weekdays for the past depends on the calendars used at that time – there may be different true results for different countries depending on the date the particular country introduced the *Gregorian calendar*. Officially, it became effective at 1582-10-15 for the catholic world. Large regions took their time and switched later. Store the proper switch date for your geographic area of interest using the command J/G (cf. *Localizing Calculator Output* and check *Wikipedia* for the precise date applicable).

Dates before the year 8 AD may be indicated differently than they were experienced at that time due to inconsistent application of the leap year rule. We count on your understanding and hope this shortcoming will not affect too many calculations.

Note that 8 AD should better read AD VIII instead. Nobody, however, counted years this way at that time – around the Mediterranean Sea, it was the year DCCLXI A.V.C. in best case (actually, this notation was broadly implemented some XL – or even CD – years later). Also note the *Julian calendar* was launched and became valid not earlier than DCCVIII A.V.C. – before, months were organized differently. *Julius Caesar* was daggered in DCCIX A.V.C. so he did not live long with 'his' calendar. (The Roman way of writing numbers was hampering calculations severely – one reason why Europe fell back in mathematics until Arabic numerals were taken over, including the zero invented in ancient India or Babylon.)

Calendars may be a sensitive topic. Note there are also other calendars wide spread on this planet today, e.g., in the Muslim or Buddhist world.

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Solution:

2022.0326 [.d]	2022-03-26 0:00:00	
1931.1109 [.d]	1931-11-09 0:00:00	
[−]	33 010	days.
24 [×]	792 240	hours.

Using [CLK] **J→D** and **D→J**, you can deal with *Julian day numbers* useful in astronomy (please see the *IOI*). And there is SETDAT, serving obvious purposes on your *WP43* (note it may take up to one minute until you see the date updated in the *status bar*). But that's it – these are all the legal operations on *dates*.

GAP, ALL, ENG, FIX, or SCI have no effect on *dates*.

Extended Dates

You may perform more precise calculations with *dates* if and when you add time-of-day information:

Example (assuming *SDC*):

Enter **2023.1219142634567** [.d] and your *WP43* will interpret this input as

2023-12-19 14:26:34.

Note the *seconds* are truncated. Now, if you want to know the date and time, e.g., 300 000 seconds later (for whatever reason) then key in

300000 [U→] **Time: s → day** returning 3.472 22... days.

Press [+] and you will get

2023-12-23 01:46:34.

Subtracting two such *dates* results in a difference in decimal *days*. Multiply its fractional part times 24 and press [h.ms] to get this part converted into *hours*, *minutes*, and *seconds* formatted `hh:mm:ss.fff` (see next chapter).

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Times

There is also a dedicated *data type* for time calculations on your *WP43*. *Sexagesimal times* are entered most easily using the format `hhh.mmssfff` terminated by [h.ms] – with `hhh` standing for *hours*, `mm` for *minutes*, `ss` for *seconds*, and `fff` for decimal fractions of *seconds* (these fractions as well as the *hours* may take more or less than three digits).

Example (assuming *SDC*):
Enter 45 *hours*, 39 *minutes*, and 7.8642 *seconds*:

45 [.] 39078642 [h.ms]

This is displayed with *startup default* TDISP **0**: 45:39:07.864 2

Choosing [CLK] [▲] TDISP [3] will return 45:39:07

… instead, while TDISP [2] will return 45:39

The latter two formats also allow for compact returns when calling TIME.

There is no display rounding for times. – The **:** (or a trailing **s**) is the unambiguous indicator for a *time* displayed on your *WP43*. There may be leading zeros in the *minutes* and *seconds* sections and a settable number of digits after the 2nd **:**. You can choose 12- or 24-hour display for *time of day* output.

Example (continued):
Call TIME in the evening and you might get 21:47

[CF] SYS.FL TDM24 will return[213] 9:47p.m.

Add and subtract *sexagesimal time intervals* simply using [+] and [−]. Multiply or divide such intervals by any *integer*, rational, or *real* – the result will stay a *time*. If you add any *integer*, rational or *real* to a *time*, it will be taken as decimal hours and automatically converted to a *time* before adding. This applies to subtractions by analogy. If you divide by a *time*,

[213] When *time of day* is returned by a function, it will be displayed according to your choice – it is stored, however, as standard 24-hour *time* for further calculations.

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it will be converted to decimal hours before. Compare the matrices on pp. 76f.

Example (assuming *SDC*):
To meet your date at 5:25 p.m. at Stanford, you need 15' from your office to get your car out of the parking garage, 1.5 *hours* for the ride, and 12' for walking from the parking lot to the lecture hall. Being careful, you count in another quarter of an hour for a possible traffic jam on the expressway. When do you have to leave your office? If Stanford is 57 *miles* away, what time do you need for a *mile* on average?

Solution:

CLK ▲ TDISP 2		
.15 h.ms 1.5 +	returns	1:45
.12 h.ms +		1:57
.1.4 +		2:12
5.25 h.ms x⇄y −		3:13.

You shall leave at 3:13 p.m. the latest.

This result looks like a 12h-time here even with *SDC* – your *WP43* can't know better based on the input given.

And TDISP 4 1.5 ENTER↑ 57 / h.ms returns 0:01:34.7 per *mile*.

If a result of your time calculations will become less than the display limit you set via TDISP (so no non-zero digit of the resulting *time* can be shown within the limit set) the result will be displayed in a fixed format like SCI 4 or ENG 4, depending on ALLENG. With ALLENG set, the following three *times* will be displayed like this, for **example**:

Time:	5 min 8.432 1 s	6.140 32 s	0.093 421 5 s
TDISP **1** or **2**	0:05	6.140 3s	93.421×10⁻³s
TDISP **3**	0:05:08	0:00:06	
TDISP **4**	0:05:08.4	0:00:06.1	
TDISP **5**	0:05:08.43	0:00:06.14	0:00:00.09
TDISP **6**	0:05:08.432	0:00:06.140	0:00:00.093
TDISP **0**	0:05:08.432 1	0:00:06.140 32	0:00:00.093 421 5

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To convert such a closed *sexagesimal time* to decimal hours, e.g., for further calculations, use [.d]. Press [h.ms] to reconvert closed decimal hours to a *sexagesimal time*. For converting a bigger number of *seconds* (e.g., 1234) into HMS format do as follows:

1234 [ENTER↑] **3600** [/] [h.ms] returning 0:20:34.

There is only one more dedicated *time* command – SETTIM – on the calculator (enter 24h time and note it may take up to one *minute* until you see the time updated in the *status bar*).[214] The commands GAP, ALL, ENG, FIX, or SCI have no effect on *times*.

See *Section 5* for the stopwatch a.k.a. TIMER.

Proposal for Configuration Setting

Based on the discussions above, the following settings (deviating from *SDC*) may be beneficial for your scientific or engineering work and provide an uncluttered screen as well:

1. Choose your regional preferences (cf. pp. 84f): just call [DISP] [▲] CHINA, EUROPE, INDIA, JAPAN, UK, or USA.
2. Enter **99** RANGE, then [▼] SCI **5** or ENG **5** (whatever you prefer).
3. [SF] SYS.FL [SPCRES] (see the *ReM* for *system flags*).
4. If you want to allow for complex results, set CPXRES.
5. Does MULT× suit your preferences? Else clear it.
6. If you want to work with fractions, **1000** [MODE] DENMAX should suffice. Even 64 or 120 may be sensible alternatives. 4800 catches almost all.

Having made your choices, call STOCFG to save them in a *user variable* (cf. pp. 63f). Alternatively, store them in a named *state file* (see p. 249).

[214] The real time clock in your *WP43* may deviate from true time by up to one *minute* per month or two *seconds* per day (i.e. ± 25 ppm approximately, caused by parts tolerances; you live with these inaccuracies wearing a quartz watch as well – mechanical watches are less accurate). This doesn't affect neither real-world time calculations nor the TIMER application described in *Section 5*.

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Alpha Input Mode: Introduction and Virtual Keyboard

This mode is designed for text entry, e.g., for keying in messages, prompts, comments, and answers (for applications in programs, see *Section 3*). It is entered via [α] typically. Within *AIM*, …

1. primary function of most keys will be appending the letter printed in grey bottom right of the respective key to x – see the *virtual keyboard* overleaf;
2. the *menu* Myα will pop up immediately, containing your favorite special characters;[215]
3. [ ] will lead to homophonic Greek letters.[216]

Upper and lower case are set by [▲] and [▼], respectively, also for the letters in Myα and CATALOG'CHARS'αINTL (cf. p. 82 and see pp. 207f).

Wherever a default primary function is not primary anymore in *AIM* but continues being meaningful, it is reached via [ ]. For instance, [ ] is required for appending a digit to x. And [ ] is also a shortcut to some special symbols, like [ ] [+/-] calling **±** or [ ] [XEQ] appending **qu**.

---

[215] For people writing German, for instance, Myα might look like pictured overleaf. Feel free to put other letters in – see *Section 6* for how to populate Myα.

[216] This works wherever applicable, with "homophonic" following classical Greek pronunciation. We assigned **Gamma** also to [C] following the alphabet, and **Chi** to [H] since this letter comes next in pronunciation.

Three Greek letters require special handling since they lack single-letter equivalents in English: **Psi** is accessed via [ ] [W] (since **ψ** just looks like **w** in a way), **Theta** via [ ] [U] (following **T**), and **Eta** via [ ] [J]; these three letters are printed in blue on the physical keyboard as reminders. **Omicron** is not needed since looking exactly like the Latin letter **O** in either case. There is an '*alpha helper*' on the calculator backside.

While we are at it, kudos to *Thales*, *Pythagoras*, *Heraclitus*, *Leucippus*, *Democritus*, *Aristotle*, *Archimedes*, *Euclid* (i.e. ὁ Θαλῆς ὁ Μιλήσιος, ὁ Πυθαγόρας ὁ Σάμιος, ὁ Ἡράκλειτος ὁ Ἐφέσιος, ὁ Λεύκιππος, ὁ Δημόκριτος, ὁ Ἀριστοτέλης, ὁ Ἀρχιμήδης ὁ Συρακούσιος και ὁ Εὐκλείδης), and their colleagues for laying the foundations of logics, mathematics, and physics (i.e. τῆς λογικὴς και μαθηματικής τέχνης και τῆς φυσικής ἐπιστήμης) as we know them today – starting 2600 years ago. Note that the first two disciplines were called *practical arts* and just the latter *theoretical science*.

And kudos to all those unnamed Babylonian mathematicians who laid the foundations for the Greek, actually recording what we now call "*Pythagoras' theorem*" as early as 1200 years (!) before him, for instance. We all stand on the shoulders of giants.

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Two additional *prefixes* operate in *AIM*: [f] + [R↓] makes the next directly keyboard-accessible input a subscript if provided, while [f] + [E] makes it a superscript. See the golden arrows printed to the right of these keys, above the letters I and M.

Note you can move the edit cursor at an arbitrary position in string x and correct or improve x as long as it is open for editing. 3 alpha *menus* become accessible for more (e.g. accented) letters, punctuation marks, mathematical and further symbols (you find the abbreviated labels for αINTL (A), α● (•), and αMATH (≈) printed on the keyboard). Look up their contents in *ReMS 2*. Outside of *AIM*, you can call [α.FN] FBR to browse the entire character set provided.

Alpha Input Mode: Entering Simple Text and More

Your *WP43* features a large font mainly for numeric output and a small font for *text strings*. See here all small font characters directly evocable through the *virtual keyboard* shown above:

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

a b c d e f g h i j k l m n o p q r s t u v w x y z

Α Β Γ Δ Ε Ζ Η Θ Ι Κ Λ Μ Ν Ξ Ο Π Ρ Σ Τ Υ Φ Χ Ψ Ω

α β γ δ ε ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ φ χ ψ ω

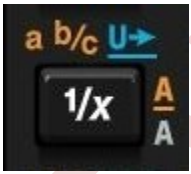
0 1 2 3 4 5 6 7 8 9 + - × or · / . , ? ! ≷ ± ( ) √ # 🖶

and the subscripts A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

a b c d e f g h i j k l m n o p q r s t u v w x y z α δ μ 0 1 2 3 4 5 6 7 8 9 + -

as well as the corresponding superscripts A ... Z , a ... z , and 0 ... - .

The 26 plain Latin letters can be also found in CATALOG'CHARS'αINTL (together with 73 more, supporting international communication). Short-cut to αINTL is [A] in *AIM.* This *catalog* will stay open until closed explicitly by [EXIT].



The 24 plain Greek letters (plus 11 accented ones) are also found in CATALOG'CHARS'A...Ω.

See *ReMS 2* for both these catalogs.

So you may, for **example**, easily store and display an actual modern Greek message like

Οι μελλοθάνατοι σε χάιρετουν.[217]

Actually, we could have written the major part of this *Owner's Manual* just using said small font. It covers at least 47 languages from Afrikaans to Zhōngwén (see the *ReM*), providing the means

[217] This corresponds to Latin MORITVRI TE SALVTANT. I created this case in 2015 looking at a banner shown in Athens during the Greek economic crisis..

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that your display messages or prompts can be read and understood most easily by more than 50% of all mankind.

Example:

You can even spell Dèng Xiǎo-píng de famous and evidently successful slogan in Pīnyīn straight ahead correctly: [218]

Bùguǎn bái māo, hēi māo, dàizhù lǎoshǔ jiù shì hǎo māo!

Taking advantage of this character set, it is also absolutely easy spelling, e.g., French, Spanish, or German prompts correctly « en français », "en Español", or „auf Deutsch", as well as message strings in many more languages using extended Latin alphabets.

Your *WP43* supports you in climbing the very first step of politeness and respect by allowing you to adapt the software you write to the language your customers speak - instead of hacking in everything in English, using merely the very meager plain Latin letter set.

Two more *menus* (αMATH, called via [f] + [−], and α•, called via [f] + [.] in AIM) contain further symbols and punctuation marks (cf. p. 206 and see ReMS 2):

! ¡ ¿ : ; " ' § $ £ ¥ € % & * < ≤ = ≔ ≝ ≙ ≈ ≠ ≧ > @ [ ] { }

\ ^ _ | ~ ° « » ÷ ← ↑ → ↓ ↕ ⌀ ∞ ⊾ ⊥ ∡ ‖ ¬ ∧ ∨

∫ ⊙ ◷ ℝ ℂ $^{-1}$ ^{T} * E ∞ ⊙ ⊕ etc.

Pressing [USER] in AIM toggles USERα, allowing you to assign your favorite symbols to particular locations on the keyboard or in menus (see pp. 315ff for how to do this). Such assignments will then become accessible when USERα is set (indicated by U and A or α being both lit in the status bar).

Open alpha input can be deleted character by character using [←]. If there is no character left anymore, input will be canceled (cf. p. 25).

AIM is typically terminated either by [ENTER↑] (duplicating string x in Y) or

[218] Alas, he deviated from his own pragmatic motto later. And his successors follow that deviation so far, for whatever reasons or fears.

EXIT (just closing string x) pressed in Myα. Then you will return to the menu displayed before entering AIM.

Example (continued):

Pressing EXIT with Dèng Xiǎo-píng's slogan keyed in will display it in X as shown above.

Pressing ENTER↑ instead will display the text twice - entirely in X and abbreviated to one line in Y, showing only its first seven words trailed by an ellipsis here:

```
Bùguǎn bái māo, hēi māo, dàizhù lǎoshǔ …
Bùguǎn bái māo, hēi māo, dàizhù lǎoshǔ jiù
                              shì hǎo māo!
```

Text strings exceeding two display lines will unveil their full contents after SHOW only.

Combining Text Strings and Numeric Data

Due to the *data type* concept of your *WP43*, adding numeric data to a *text string* is as simple as pressing +.

Example:

Assume the two lowest *stack registers* look like this:

```
The train will arrive at
                    23:55
```

So here is a *text string* in Y and a *time* in X. Pressing + will merge x and y, returning

```
The train will arrive at 23:55
```

Here, x was converted to a *text string*, taking its present display format into account, and was appended to y (cf. the matrix on p. 76).

Let's enter a 2nd string now by pressing α and typing the necessary letters, starting with a blank:

```
                    The train will arrive at 23:55
sharp at Victoria station.
```

Press [EXIT] to close x and leave *AIM*:

```
                    The train will arrive at 23:55
                          sharp at Victoria station.
```

Now we have got *strings* in **X** and **Y**, so pressing [+] will append string x to string y, returning:

```
The train will arrive at 23:55 sharp at
                          Victoria station.
```

Any *string* may contain up to 196 characters in total (about five lines of text on display). Once numeric data (like the *time* above) became part of such a string, they are fixed and will not vary even if format will be changed.[219] Easy, isn't it?

Working with Text Strings

Your *WP43* provides some more commands for dealing with strings. You find them all in α.FN:

x→α ***destination*** converts a character code x to the corresponding symbol and appends it to the destination; the character code is saved in **L**. For destination **X**, **x→α** returns the symbol only.

If **X** contains a *string* already, the entire *string* is appended to the destination (in this case **x→α** executes like STO+).

[219] Some *text strings* displayed may be hard to tell apart from numbers. *Text strings* are, however, always shown in small font and there are no automatic gaps in them.

If **X** contains a *matrix* instead, **x→α** uses each *element* in the *matrix* as a character code. It starts with the 1st *element* (1; 1) and continues following usual reading habits until reaching the end of the *matrix*.

α→x ***source*** converts the leftmost symbol in the source string to the corresponding code, pushes this on the *stack*, and removes said symbol from said string. If the source is empty, **α→x** returns **0**.

αRL ***source*** rotates the source string by ***x*** symbols to the left.

αRR ***source*** rotates the source string by ***x*** symbols to the right.

αSL ***source*** shifts the source string by ***x*** symbols to the left, deleting the first ***x*** symbols from the string.

αSR ***source*** shifts the source string by ***x*** symbols to the right, deleting the last ***x*** symbols from the string.

αLENG? ***source*** pushes the length of the source string on the *stack*.

αPOS? ***source*** searches the source string for the target symbol or string given in **X**. If a match is found, αPOS? returns in **X** the position where the target was found starting in the source (counting the leftmost symbol as position 0); else it returns **−1**. The target is saved in **L**.

FBR shows all characters defined in both fonts provided.

You can also compare *text strings* using commands found in TEST, e.g., to create a sorted list. String ***A*** is called “smaller” than string ***B*** if ***A*** precedes ***B*** in sorting.

Nevertheless, don’t forget that your *WP43* is mainly designed as a programmable calculator. Please turn overleaf to see what could be performed with such devices.

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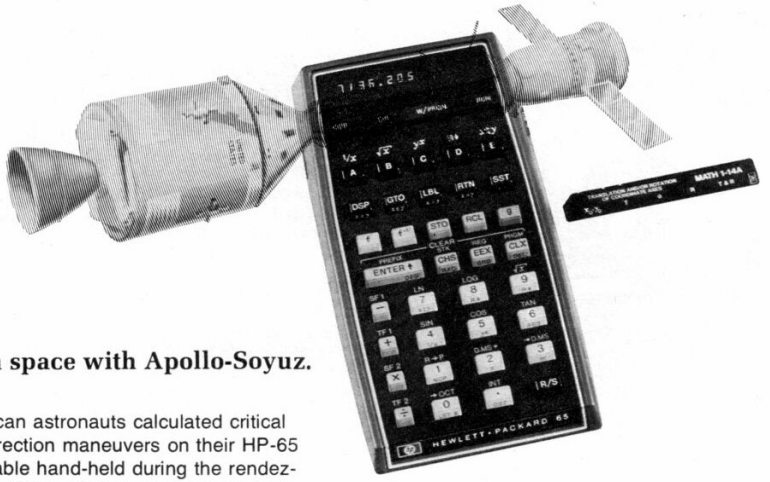


HP-65 in space with Apollo-Soyuz.

The American astronauts calculated critical course-correction maneuvers on their HP-65 programmable hand-held during the rendezvous of the U.S. and Russian spacecraft.

Twenty-four minutes before the rendezvous in space, when the Apollo and Soyuz were 12 miles apart, the American astronauts corrected their course to place their spacecraft into the same orbit as the Russian craft. Twelve minutes later, they made a second positioning maneuver just prior to braking, and coasted in to linkup.

In both cases, the Apollo astronauts made the course-correction calculations on their HP-65. Had the on-board computer failed, the spacecraft not being in communication with ground stations at the time, the HP-65 would have been the only way to make all the critical calculations. Using complex programs of nearly 1000 steps written by NASA scientists and pre-recorded on magnetic program cards, the astronauts made the calculations automatically, quickly, and with ten-digit accuracy.

The HP-65 also served as a backup for Apollo's on-board computer for two earlier maneuvers. Its answers provided a confidence-boosting double-check on the coelliptic (85 mile) maneuver, and the terminal phase initiation (22 mile) maneuver, which placed Apollo on an intercept trajectory with the Russian craft.

Periodically throughout their joint mission, the Apollo astronauts also used the HP-65 to calculate how to point a high-gain antenna precisely at an orbiting satellite to assure the best possible ground communications.

The first fully programmable hand-held calculator, the HP-65 automatically steps through lengthy or repetitive calculations. This advanced instrument relieves the user of the need to remember and execute the correct sequence of keystrokes, using programs recorded 100 steps at a time on tiny magnetic cards. Each program consists of any combination of the calculator's 51 key-stroke functions with branching, logical comparison, and conditional skip instructions.

The HP-65 is priced at $795*. See it, and the rest of the HP family of professional hand-helds at quality department stores or campus bookstores. Call 800-538-7922 (in California, 800-662-9862) for the name of the retailer nearest you.

HEWLETT hp PACKARD

Sales and service from 172 offices in 65 countries

An *HP* advertisement of 1975 (above) and one of the Eighties (overleaf). Compare the capabilities of your *WP43*. Imagine the opportunities.

NASA
Columbia
United States
Quando sono
i risultati che contano
HEWLETT
PACKARD
Gli astronauti della "Columbia" hanno utilizzato due HP-41C per effettuare alcuni calcoli, giudicati "critici" dalla NASA, al momento del rientro nell'atmosfera: posizione del centro di gravità della navetta spaziale e continua visualizzazione dei parametri e dei dati inviati dalle stazioni a terra in costante contatto con la "Columbia".
HP-41C
ON
USER
PRGM
ALPHA
ENTER
CHS
EEX
R/S

SECTION 3: PROGRAMMING AND AUTOMATIC PROBLEM SOLVING

Your *WP43* is a powerful *keystroke-programmable* calculator. If already this statement makes you smile with delight, this section is for you. Else we will bring a smile on your face by mentioning the following facts:

Your *WP43* allows you to store a sequence of keystrokes like you would use them to solve a problem manually; this will save you time on repetitive calculations (cf. pp. 23ff). Once you have written the keystroke procedure (or *routine*) for solving a particular problem and recorded it in your *WP43*, you need no longer devote attention to the individual keystrokes that make up the procedure. Instead, you can let your *WP43* solve each similar problem for you. And because you can easily check the routine stored, you have more confidence in your final answer since you don't have to worry each time about whether or not you have pressed an incorrect key. Your *WP43* performs the drudgery, leaving your mind free for more creative work.

And even better: you may use *program memory* (*PM*) for storing many routines containing thousands of keystrokes – cf. p. 59. For telling your *WP43* where a particular routine begins and ends, each one is confined by two steps: It starts with LBL (for LaBeL) and typically ends with RTN (for ReTurN) – cf. p. 24. These two steps separate it from other routines you may add for other tasks. And LBL puts a *label* (a.k.a. tag) on your routine so you can find and call it when you want it to be executed.

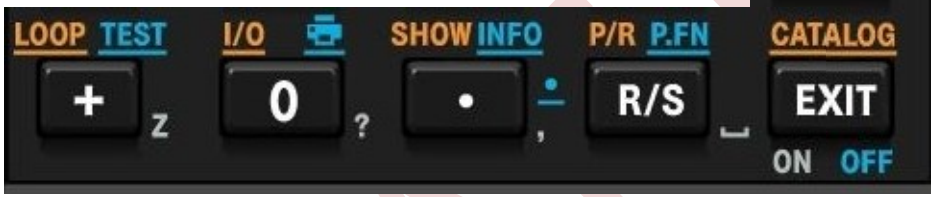
You may structure *PM* even more: Collect a set of routines and separate them by END from other routines or sets. What is found between two END steps is called a *program* – they are the basic building blocks within *PM*. Think of the beginning and end of the entire used *PM* section containing implicit END steps;[220] there will be at least one program in *PM* at any time.

Within routines, you may store any sequence of keystrokes representing commands, operations, labels, or constants. Most executable operations are programmable. The commands in your routine may also access each

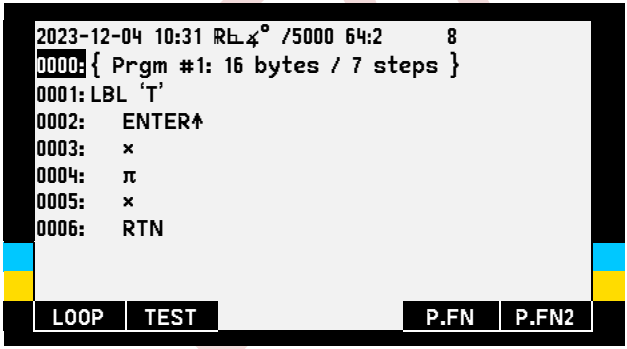
[220] You can't see that first END but the last one is visible – pictured, e.g., on p. 219.

and every *register*, *variable*, or *flag* provided – there are almost no limits. You are the sole and undisputed master of *WP43* memory.[221]

Each such routine itself may contain *subroutines* – also these start with LBL and typically end with RTN. Actually, *sub*routines may look exactly like routines: the only difference is that a *sub*routine is called from another routine, while a main (or top) routine is called from the keyboard. Thus there is no need for differentiating these two kinds of routines further on.



Enough of theory – press [RTN] and switch to *program entry mode* (*PEM*) by [P/R]. The display of your *WP43* will change to something like this:



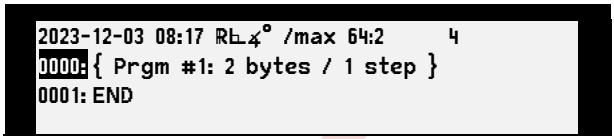
This screen shows the routine you entered on p. 24 (the *status bar* on top will differ, of course, according to your time and settings). In the *LCD* section used for numeric output so far, the first 6 steps in *program memory* are shown. Label steps are 'outdented' for visual structuring. The position of the *program pointer* (the *current step*)

[221] This freedom has a price: Take care that your routines don't interfere with each other in their quest for storage space. Recording the *global labels* and *registers*, *variables* and *flags* a particular routine uses, and documenting their purposes and contents for later reference is good practice.

An alternative – using *local registers* and *flags* instead of *global* ones – will be covered further below.

is highlighted by inversion; the routine the *program pointer* is in is called the *current routine*; the corresponding program is the *current program*. MyPFN is shown in the *menu section*.

Turning to *PEM* the very first time after unpacking your *WP43* (or after CLPALL or RESET), the display may look like this, instead.



Writing and Recording a New Routine

Whenever you want to create a new routine, ensure you are in *PEM*. Then enter GTO . . – this will bring you to the very end of the used section of *program memory*, so you can start keying in your new routine right there without interfering with anything coded previously.

Start the routine with LBL giving it a descriptive *label*. Then press the keys as you would do in manual problem solving (cf. pp. 23f). Each new step will be inserted right after the *current step*. You find the following keys and labels related to programming and programs on your keyboard as shown on previous page; clockwise, these are:

- LBL for LaBeLing a routine or just the program section following,
- RTN for ReTurNing to the caller of the *current routine*,
- GTO for unconditionally Going TO a specified label (i.e. moving the *program pointer* onto the respective LBL step),
- XEQ for eXECUting or calling a specific (sub-) routine,
- ▲ and ▼ (or ≡△ and ≡▽ if there is an open *multi-view menu*) for browsing program steps,
- EXIT for exiting *PEM* if just MyPFN is open (meant to be your personal *menu* of programming commands), returning to *RUM*,
- R/S for Running or Stopping the *current routine*, and
- P/R for toggling *PEM* and *RUM*.

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These are continued to the left by the *menus* for Programming FuNctions (P.FN, containing END for finishing and closing the *current program*), system INFOrmation, I/O, LOOPs, and TESTs.

Example (from the *HP-15C OH*):

Mother's Kitchen, a canning company, wants to package a ready-to-eat spaghetti mix containing three different cylindrical cans: one of spaghetti sauce, one of grated cheese, and one of meatballs. *Mother's* needs to calculate the base areas, total surface areas, and volumes of the three different cans. It would also like to know, per package, the total base area, surface area, and volume.

Solution:

The program to calculate this information uses these formulas and data:

$$base\ area = \pi\, r^2$$
$$volume = base\ area \times height = \pi\, r^2\, h$$
$$surface\ area = 2 \times base\ area + side\ area = 2\,\pi\, r^2 + 2\,\pi\, r\, h$$

r	*h*	Base Area	Volume	Surface Area
2.5	8.0	?	?	?
4.0	10.5	?	?	?
4.5	4.0	?	?	?
	TOTALS	?	?	?

Method:

1. Enter an ***r*** value into the calculator and save it for other calculations. Calculate the base area ($\pi\, r^2$), store it for later use, and add the base area to a *variable* which will hold the sum of all base areas.
2. Enter ***h*** and calculate the volume ($\pi\, r^2\, h$). Add it to a *variable* to hold the sum of all volumes.
3. Recall ***r***. Divide the volume by ***r*** and multiply by **2** to yield the side area.
4. Recall the base area, multiply by **2**, and add to the side area to yield the surface area. Sum the surface areas in a *variable*.

Don't enter the actual data while writing the program – just provide for their entry. These values will vary and so they will be entered before and / or during each program run.

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Turn to *PEM* using P/R and go to free *program memory* by GTO . . . Key in the following program to solve the above problem:

LBL K ENTER↑	LBL 'K'	For kitchen
STO α R ENTER↑	STO 'r'	Store radius
EXP x^2	x^2	
π	π	
×	×	Compute base
STO α ▲ BASE ENTER↑	STO 'BASE'	
STO + α ▲ S B ENTER↑	STO+ 'ΣB'	Sum of bases
VIEW VAR BASE	VIEW 'BASE'	Show base
P.FN PAUSE 1 0	PAUSE 10	for 1 second
INPUT α H ENTER↑	INPUT 'h'	Ask for height input
RCL VAR BASE	RCL 'BASE'	
×	×	Compute volume
STO α ▲ VOLUME ENTER↑	STO 'VOLUME'	
STO + α ▲ S V ENTER↑	STO+ 'ΣV'	Sum of volumes
VIEW VAR VOLUME	VIEW 'VOLUME'	Show volume
PAUSE 1 0	PAUSE 10	for 1 second
RCL VAR r	RCL 'r'	
/	/	
2	2	
×	×	Compute side area
RCL VAR BASE	RCL 'BASE'	
2	2	
×	×	
+	+	Compute surface
STO + α ▲ S S ENTER↑	STO+ 'ΣS'	Sum of surfaces
RTN	RTN	End of routine
P/R		Leave *PEM*

Now, let's run the program (…):

GTO K makes **K** the *current program*.

CLR CLCVAR clears all *variables* of the *current program*. By doing so, it creates the *variables* not defined yet, avoiding later errors caused by undefined *variables*.

DISP FIX 0 1

2.5	2.5	1st can: radius[222]
XEQ K	BASE = 19.6	show base for 1 s
	h? 0	ask for height input
8 R/S	VOLUME = 157.1	show volume for 1 s
	164.9	surface
4	4	2nd can: radius
XEQ K	BASE = 50.3	show base for 1 s
	h? 8	ask for height input
10.5 R/S	VOLUME = 527.8	show volume for 1 s
	364.4	surface
4.5	4.5	3rd can: radius
XEQ K	BASE = 63.6	show base for 1 s
	h? 10.5	ask for height input
4 R/S	VOLUME = 254.5	show volume for 1 s
	240.3	surface
RCL VAR ΣB	133.5	sum of bases
RCL VAR ΣV	939.3	sum of volumes
RCL VAR ΣS	769.7	sum of surfaces

If you want to run this routine again for another set of cans, remember to clear the *variables* **ΣB**, **ΣV**, and **ΣS** before. CLCVAR will do this for you most easily.

[222] No units are mentioned in this problem. Assume they talked about *inches*, so the output is in *square* and *cubic inches*. May be necessary to convert into *fluid ounces*?

The preceding program illustrates the basic techniques of programming. It also shows how data can be manipulated in *PEM* and *RUM* by entering, storing, and recalling data (input and output) using [ENTER↑], [STO], [RCL], store arithmetic, and programmed I/O.

See next chapters and the *IOI* for comprehensive information about all the commands used in this case.[223]

Labels

As mentioned above, each routine or subroutine shall begin with a LBL step. Structuring *program memory* and jumping within is eased by those *labels*. You may tag *labels* not only to the 1st but to any step in your routine. Your *WP43* allows for specifying a wide variety of alphanumeric *labels* as elaborated overleaf.

Whenever a step like, e.g., GTO ***labl*** is encountered (with ***labl*** representing an arbitrary *label*) with either a program running or a single program step executed in *RUM*, your *WP43* will **search this *label*** using one of the two following methods:

1. If ***labl*** is just numeric (**00 ... 99**) or **A … E**, it will be searched forward from the current *PP* position. When an END step is reached without finding ***labl*** so far, the quest will continue right after previous END (so the search will stay in the *current program*). This is the procedure for ***local labels*** – they may be hence reused in other programs.
2. If, however, ***labl*** is alphanumeric (up to 7 characters of arbitrary case, starting with a letter and automatically enclosed in ‘ like ‘M’ or ‘Ab1’), searching will start at the end of *PM*, go upwards, and cover the entire *PM* independent of the current *PP* position. This is the procedure for ***global labels***.[224]

[223] *Mother's Kitchen* might tend to use a spreadsheet application on its laptop or tablet to compute such outputs nowadays. Your *WP43* is smaller, boots faster, and is less prone to be hit by spaghetti sauce.

[224] These procedures for *local* and *global labels* are as established since the *HP-41C*. Though the range of *local labels* varies.

So, *global labels* can be accessed from anywhere in *PM* while *local labels* can only be reached from within their own program.

Addressing *labels*, on the other hand, follows the rules given below:

1	User input	XEQ, GTO, LBL, LBL?, SOLVE, ∫, f'(x), f''(x), Π_n, or Σ_n		
	Echo	OP _ (with *TAM* set) e.g., GTO __		
2	User input	α [225] sets *AIM*.	→ [226] opens indirect addressing.	*local label* 00 … 99, A … E, or 1-letter *global label* (see next page)
	Echo	OP '_	OP → _	OP *nn* e.g., LBL 07 or LBL C
3	User input	*Alphanumeric global label* (up to 7 chars[227])	*Stack or lettered register* X, Y, ..., K	*Register number* 00 … 99, .00 … .98[228]
	Echo	OP '*label*' e.g., SLV 'F1μ'	OP → *x* e.g., ∫fd →T	OP → *nn* e.g., XEQ →44

SLV 'F1μ' solves the function given in the routine **F1μ** (see pp. 268ff).

∫fd →T integrates the function given by PGMINT over the *variable* whose label is on *stack register* **T** (see pp. 275ff).

XEQ →44 calls and executes the routine whose label is found in **R44**.

---

[225] You can skip pressing here – see overleaf. See also an alternative there.

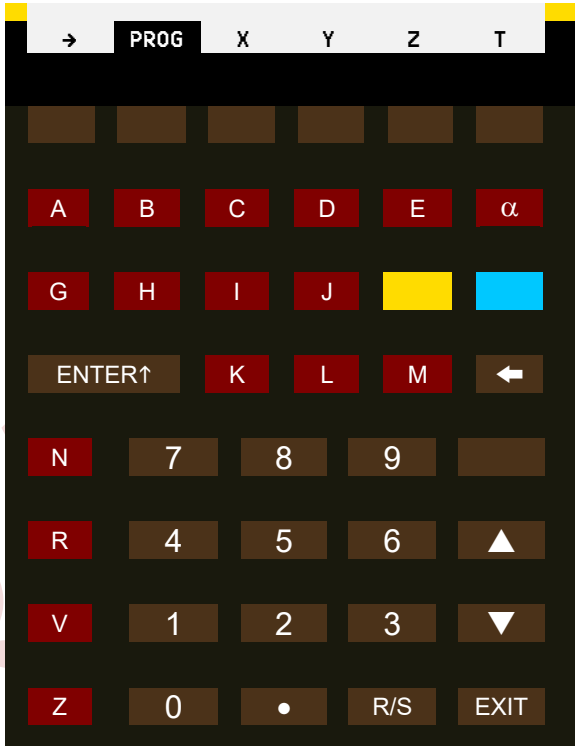
[226] Indirect addressing works with all these operations except LBL .

[227] Said *label* must contain at least one letter. *Labels* are case sensitive. The 7th character will terminate entry and close *AIM* – shorter *labels* need a closing ENTER↑.

[228] ... if the respective *local register* is allocated. Note XEQ . is not allowed neither in *RUM* nor *PEM*. Some lettered *registers* may be dedicated to special applications. Cf. *Addressing and Manipulating Objects in RAM* in *Section 1*.

Furthermore, GTO provides four special cases: see GTO↟, GTO↡, GTO**.**, and GTO**..** in next chapter.

And remember *TAM* is set also during *label* addressing, so the *virtual keyboard* of your *WP43* will look like this:



You can access the *local labels* **A** - **E** directly as well as the *global labels* **G** - **N**, **R**, **T**, **V**, **X**, **Y**, and **Z**. This virtual keyboard allows for calling 14 programs and five different subroutines per program by just two keystrokes each.

Note also the changed assignment of [F2] compared to p. 61. Thus, instead of typing a longer *global label* in *AIM*, press [PROG] and choose the label from this *submenu* (comprising all *global labels* defined at execution time) – this may well be easier and faster.

Editing a Routine

Whenever you want to edit (modify, correct, expand, or shorten) an existing routine, start in *RUM* entering [GTO] ***labl***. This will move the *PP* onto the corresponding step **LBL** ***labl*** (as explained on p. 220). Then turn to *PEM* using [P/R] and start browsing from this LBL step on.

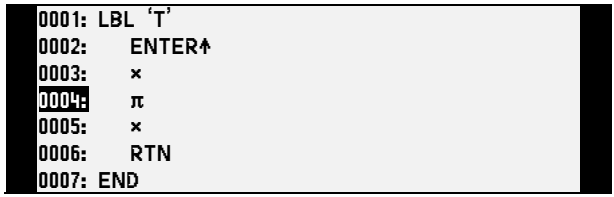
Example:

Let's browse the program steps in our first routine. Press

GTO T

P/R

▼ ▼ ▼



Unless you are next to the very begin or end of a program, the *program pointer* (*PP*) will always be placed in the middle of the display with three steps displayed above and three steps below of it.

Navigating in *PM*, you may execute various actions. If you want to ...

- continue browsing forward, press ▼ (or ≡▽ if a *multi-view menu* is displayed); when reaching the END, browsing will start with the 1st step of the same program again.
- browse backwards, press ▲ (or ≡△ if a *multi-view menu* is displayed); when reaching program top, browsing will stop.
- delete a program step, go to said step (make it the *current step* by moving the *PP* onto it), then press ←. It will vanish (this can't be undone!) and the *PP* will move on the step before.
- insert something, go to the program step before, and then press the keys to be inserted after it.

Example (continued):
Replace **ENTER↑ ×** in our routine above by **x^2** doing the same.

Solution:

Press ▲ –	the *PP* is on step	0003: × now.
←	This step vanished; *PP* is on step	0002: ENTER↑ now.
←	This step vanished; *PP* is on step	0001: LBL 'T' now.
EXP x^2	This command will be stored in while π moved to step 3.	0002: x^2

- jump to a particular *global label* (without inserting a GTO in the *current routine*), press GTO . α ***label*** ENTER↑ or GTO . PROG (*label*). For a *local label*, press GTO . ***nn*** instead. 1-letter *labels* can be

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accessed, e.g., via GTO . D (cf. p. 222).

- jump to the top (i.e. the 1st step) of next program, press GTO ▼.
- jump to the top of current program, press GTO ▲. If the *PP* is there already, GTO ▲ will bring it to the top of previous program.
- start writing a new routine, press GTO . ., then LBL... (as experienced in *Mother's Kitchen* on pp. 217f).

That's almost all. When you are done, press P/R or EXIT to leave *PEM*, returning to *RUM*.

Please note your *WP43* can't know in advance if all the necessary pre-conditions for a certain program step will be fulfilled at the time of its execution. Thus, respective error messages can appear no earlier. Debugging may be required.

Running a Routine from the Keyboard (also for Debugging)

Whenever you want to execute an existing routine, ensure you are in *run mode*. Then there are three alternatives:

1. **Automatic execution of the *current routine*:** Press RTN to return the *PP* to the 1st step of the *current routine*. Then press R/S. This will run this routine, i.e. start executing the following steps automatically until an error happens, a STOP, a final RTN, or an END will be encountered[229] where it will halt and display x.

2. **Automatic execution of a selected routine:** Press XEQ and specify the *global label* of the program you want to execute (or press PROG and pick it from this *menu*).[230] This will move the *PP* to the corresponding LBL (cf. p. 220) and start executing the following steps automatically until an error happens, a STOP, a final RTN, or an END will be encountered[229] where it will halt and display x (cf. pp. 23f).

[229] ... or you interrupt it by pressing R/S or EXIT – then it will stop after the *current step* is executed completely. For resuming its execution, press R/S again.

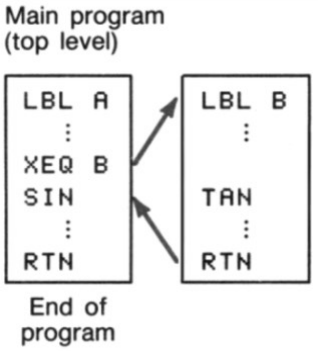
[230] This is the standard way to run routines. Furthermore, you can define shortcuts to your favorite routines by customizing your *WP43* as described in *Section 6*.

3. **Stepwise execution of a selected routine:** Press GTO and specify the *global label* of the program you want to execute (or press PROG and pick it from this *menu*). This will move the *PP* to the corresponding LBL (cf. p. 220) and wait. Each following program step will then be executed one at a time as you press ▼ (or ≡▽ if a *multi-view menu* is displayed): pressing the key will show the *current step* at left end of the top numeric row, releasing will execute it.[231]

   Where your routine asks for input (see pp. 234ff), close this input by ▼ (or ≡▽). Reaching the end of the *current routine*, ▼ (or ≡▽) will return to its 1st step. Following this procedure, you will go through the routine as in normal execution but slower – and you may call RBR or STATUS or perform additional checks after each program step.[232] This procedure is especially useful for *debugging*.

If an error occurs while a routine is running, it stops immediately at the step generating said error and throws the corresponding error message (see *ReMA C* for a compilation of all error messages provided). To view the program step causing said error, press P/R.

Subroutines: Running a Routine from another Routine

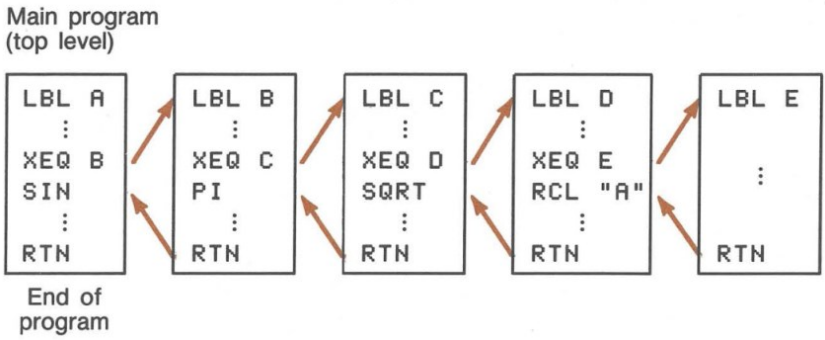


The command XEQ is programmable as well. Whenever a running routine encounters an XEQ, it will search for the associated label as described on p. 220, move the *PP* to it, and continue program execution with the step after this LBL until it encounters a RTN; this will return the *PP* to the step right after said XEQ, where execution will continue. Compare the picture where routine **A** calls routine **B**.

[231] Pressing ▲ (or ≡△ if a *multi-view menu* is displayed), on the other hand, moves the *PP* backwards in the *current routine* without executing anything.

[232] Watch that your additional checks don't alter the status of your *WP43* in a way deviating from its status in automatic execution; else you shall compensate. Also take care when browsing backwards.

You can also nest subroutines – your *WP43* can remember as many pending subroutine return addresses as *RAM* allows. Every routine may allocate its own *local registers*, up to 99 each (see p. 251).

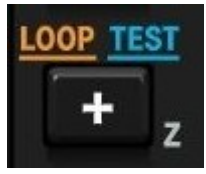


Be aware that all return addresses will be lost for the *current program* if you alter the *PP* while this program is stopped – pressing R/S, ≡▽, or ▼, however, will not cause such a loss.

If you need any of your subroutines elsewhere in another routine then you can call it again and again at no expense of memory. If you want to call a particular subroutine from another program than the one it is defined in, the label of the subroutine you call must be global.

Automatic Binary Testing and Conditional Branching

So far, we were talking about linear programs, running straight from their beginning (LBL) to end (final RTN or END). Your *WP43* can do more for you: like other keystroke-programmable calculators before, it features a set of binary tests checking various calculator states. Most of the binary tests provided are collected in TEST. There are also two tests in BIT, and 8 tests of *flags* are stored in FLAG. *Names* of binary test commands contain a '?', mostly as their last character.



Generally, binary tests will return `true` or `false` as *temporary information* at left end of the **Z** numeric row if called from the keyboard. If

called in a running routine automatically instead, they will execute the next program step if the test is true at execution time, else skip that step. So the general rule reads "do if true". Think of the step immediately following the test containing a GTO and you see how conditional branching comes into play.

Example:

```
...
0020:    x≤? Y
0021:    GTO 'Join'
0022:    x≷y
...
0032: LBL 'Join'
0033:    ln x
...
```

If this test is true (i.e. if x is not greater than y)
then go to the label **Join** (at step 32);
else swap x and y
and continue working here.

Most binary tests operate on x. Six tests check its *data type*:

- REAL? **tests** if x is a *real* object (*DT* 2 or 8)
  and executes the next program step if true, else skips it.
- CPX? tests if x is a *complex* object (*DT* 3 or 9) with an *imaginary* part $\neq$ **0** and …
- MATR? tests if x is a *matrix* (*DT* 8 or 9) ...
- STRI? tests if x is a *text string* (*DT* 7) ...
- SPEC? tests if x is special (i.e. ∞, $-\infty$, or 'Not a Number') ...
- NaN? tests if x is 'Not a Number' ...

Five tests check its numeric content:

- INT? **tests** if x is integer (note INT? will return `true` if FP? returns `false`, i.e. also for x being of *DT* 2 with FP(x) = 0)
  and executes the next program step if true, else skips it.
- EVEN? tests if x is integer (cf. INT?) and even ...
- ODD? tests if x is integer (cf. INT?) and odd ...
- FP? tests if x has a nonzero fractional part ...
- PRIME? tests if $|\text{IP}(x)|$ is a *prime number* …

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Seven tests compare x with 0, 1, or the content of a source specified (let's call it s, cf. also pp. 61 and 63):

- x < ? ***s*** **tests** if x is less than s
  and executes the next program step if true, else skips it.
- x ≈ ? ***s*** tests if the rounded values of x and s are equal …
- x ≤ ?, x = ?, x ≠ ?, x ≥ ?, and x > ? work in analogy to x < ?.

Four tests check its internal structure:

- BC? (or BS?) tests if **X** contains a *short integer*, then checks its bit specified and executes the next program step if said bit is clear (or set), else skips it.
- LEAP? tests if **X** contains a *date*, then extracts the year and tests for a leap year ...
- M.SQR? tests if **X** contains a *matrix*, then checks if it is square ...

Eight *flag* tests operate on the *flag* specified:

- FC? tests this *flag* and executes the next program step if said *flag* is clear, else skips it.
- FS? works like FC? but executes the next program step if said *flag* is set.
- FC?F (FS?F) works like FC? (FS?) but flips this *flag* after testing (clears it if it was set or sets it if it was clear).
- FC?C (FS?C) works like FC? (FS?) but clears this *flag* after testing.
- FC?S (FS?S) works like FC? (FS?) but sets this *flag* after testing.

Finally, there are four special tests:

- LBL? tests for the existence of the label specified, anywhere in *PM*.
- TOP? will return `true` if the *program pointer* is in the top level routine (cf. the sketches on pp. 225f).

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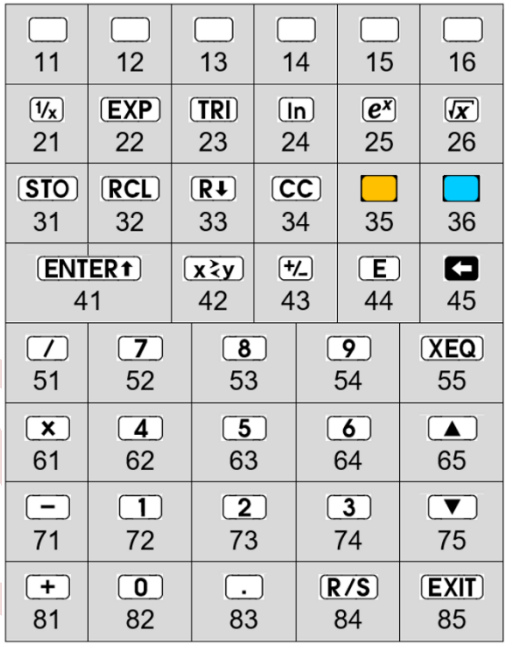
- ENTRY? checks the (internal) entry *flag*. This is set if:
  - a character is entered in *AIM* or
  - a command is accepted for entry (be it via [ENTER↑], a function key, or [R/S] with a partial command line).

  ENTRY? is useful, e.g., after PAUSE.

- KEY? tests if a key was pressed while a routine was running or paused. If no key was pressed in that interval, the next program step after KEY? will be executed (exception!); else it will be skipped and the code of said key will be stored in the address specified. Key codes reflect the rows and columns on the keyboard (see the picture and p. 237 for an application).



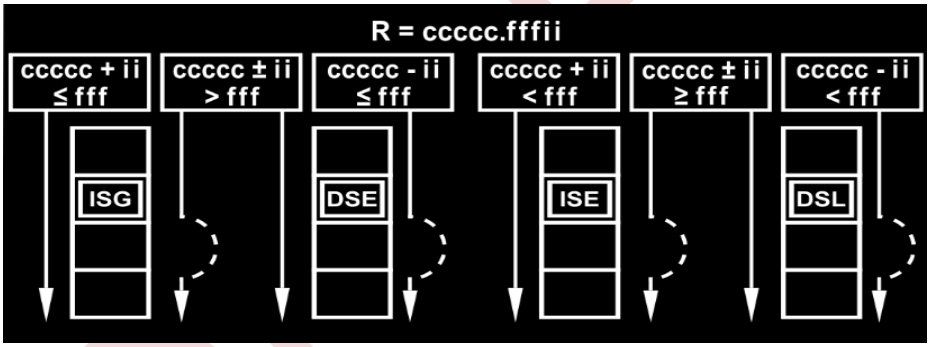
See the *IOI* for comprehensive information about all binary tests provided, also beyond the ones mentioned above.

There are further commands provided in your *WP43* featuring a trailing '?' as well but returning numbers (like WSIZE?) or codes (like KTYP?) instead of `true` or `false` – you will find these commands in INFO. Turn to the *ReM* for information about them.

As mentioned above, routines end with RTN (typically) and programs with END. Executing a program, both RTN and END work in a very similar way and show only subtle differences: An RTN immediately after a binary test returning `false` will be skipped – an END will not.

Loops and Counters

The commands DSE, DSL, DSZ, ISE, ISG, and ISZ (all contained in LOOP) are for controlling loops in routines. Each of them Decrements or Increments a counter in a *RoV* as specified, tests the result, and executes or skips the following program step. See the picture below illustrating ISG (Increment and Skip if Greater), DSE (Decrement and Skip if Equal), ISE (Increment and Skip if Equal),, and DSL (Decrement and Skip if Less).[233] The control *variable* contains `ccccc.fffii`, i.e. the start value of a counter ***ccccc***, its 3-digit final value ***fff***, and a 2-digit increment/decrement ***ii***. If ***ii*** is entered as zero, it defaults to **1**. (The commands ISZ and DSZ simply skip if Zero – anything between −1 and +1 is assessed as 0 here.)



With a GTO placed in the skipped step pointing to a label upstream in the same routine you can create loops running until the specified condition is met – see examples below. E.g., if the loop counter should go from **5** to **1**, use **5** STO K **...** DSE K; on the other hand, if it should go from **1** to **5**, use **1.005** STO K **...** ISG K.

Without such an exit condition you can deliberately create an infinite loop. Such a loop will run until you interrupt it manually by EXIT or R/S, or until battery voltage falls below the limit (note your WP43 will not turn off automatically with a program running!).

Created inadvertently, infinite loops in a running program are difficult to

[233] A similar picture is printed on the back of your *WP43*.

recognize – will stay lit in the *status bar* and your *WP43* will accept no keystrokes except [EXIT] or [R/S] – it may look like locked or caught in a hung state. Thus, implementing some outputs in regular intervals is recommended especially in programs not completely checked yet.

Let's demonstrate loops, use of *flags*, tests, and also indirect addressing – outputs will be covered in next chapter:

Example 1:

The '*Cullen numbers*' are $C_n = n\,2^n + 1$. It is proven that no C_n is prime for $n \leq 1000$ except very few cases. One is C_1. What else do you find?

Solution:

Reset the *program pointer* to the start of free *program memory* by [GTO] [.] [.].
Switch to *PEM* via [P/R] and key in:

[LBL] [α] [C] [N] [ENTER↑]	LBL 'CN'	
[RCL] [I]	RCL I	
[PART] IP	IP	
[FILL]	FILL	n
[EXP] 2^x	2^x	2^n
[x]	×	$n\,2^n$
[LOOP] INC X	INC X	$n\,2^n + 1 = C_n$
[TEST] PRIME?	PRIME?	
[GTO] [A]	GTO A	… if prime
[x≷y]	x≷y	else return n
[GTO] [B]	GTO B	and go to *local label* **B**.
[LBL] [A]	LBL A	
[x≷y]	x≷y	return n
[R/S]	STOP	… to show n. Resume with [R/S]
[LBL] [B]	LBL B	
[LOOP] ISG I	ISG I	if $n + 1 \leq f$ …
[GTO] [PROG] [Cn]	GTO 'Cn'	then repeat with $n + 1$
[P.FN] END	END	else end (with $n + 1 > f$).
[P/R]		

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Now, load the loop counter: **1.999** STO I (**999** is the maximum final count possible) and start the program by pressing XEQ PROG Cn. It will display 1 as expected. Press R/S. What will be found next? And thereafter?

Example 2 (following an idea of *Gene Wright*):

Write a little routine to store random numbers in **R25** through **R39**.

Solution:

Enter GTO . . P/R and key in:

LBL X	LBL 'X'	
PROB ▲ RAN#	RAN#	Get a new random number and…
STO → 24	STO →24	store it where *r24* is pointing to.
LOOP ISG 24	ISG 24	Increment the pointer.
GTO X	GTO 'X'	If *r24* ≤ **39** then return to label **X**
RTN	RTN	else finish this routine.
P/R		

Initialize the loop counter via **25.039** STO 24 and start the program by pressing XEQ X. It will stop with the last random number in display. Check the target *registers* using RBR.

Example 2 (continued):

Now, write a routine to sort these 15 stored numbers so the smallest number moves to the *register* with the smallest address.

Solution:

We will use the so-called '*bubble sort*' algorithm. Re-initialize the loop counter via **25.039** STO 24 (*r24* was modified by program **X** above). Enter

GTO . .		
P/R		
LBL Y	LBL 'Y'	
P.FN LocR 2 ENT↑	LocR 02	Allocate 2 local *registers*.
REM α H ▼ E …	REM 'Here the main loop begins'	
LBL A	LBL A	
RCL 24	RCL 24	Put the start pointer *r24* …
STO .00	STO .00	into local *register* **00**;

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LOOP INC X	INC X	increment this pointer and …
STO .01	STO .01	store it in local *register* **01**.
CF .00	CF .00	Clear local *flag* **00**.
LBL C	LBL C	
RCL → .00	RCL →.00	Recall the contents of the *registers* …
RCL → .01	RCL →.01	where ***r.00*** and ***r.01*** are pointing to.
TEST x<? Y	x<? Y	Is $x < y$?
GTO B	GTO B	Then go to local label **B** below …
LBL D	LBL D	else…
LOOP ISG .00	ISG .00	increment ***r.00*** and…
ISG .01	ISG .01	if ***r.00*** ≤ **39** increment ***r.01*** and…
GTO C	GTO C	if (***r.00*** > **39** or ***r.01*** ≤ **39**) then return to local label **C** above;
FLAG FS? .00	FS? .00	else check local *flag* **00**:
GTO A	GTO A	if set, return to local label **A** above,
RTN	RTN	else stop executing this routine.
LBL B	LBL B	
SF .00	SF .00	Set local *flag* **00**.
STO → .00	STO →.00	Store the smaller value …
x≷y	x≷y	where ***r.00*** and the greater …
STO → .01	STO →.01	where ***r.01*** is pointing to.
x≷y	x≷y	Restore the *stack* and …
GTO D	GTO D	return to local label **D** above.
P.FN END	END	
P/R		

Start the program by pressing XEQ Y . Then check the target *registers* using RBR. You will find the smallest value in **R25**, a greater one in **R26**, etc., up to the greatest in **R39**.

Note this program allocates two *local registers* for its exclusive use (**R.00** and **R.01**). Furthermore, it uses one *local flag* and four *local labels*.

The following sorting program is even shorter (kudos to *Jean-Marc Baillard*). Start it by **25.039** XEQ Z . Finding out how it works is left as an exercise

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for the reader:

```
LBL 'Z'
  sign
LBL A
  RCL L
  RCL L
  RCL →L
LBL B
  RCL →Y
  x>? Y
  GTO C
  x≷y
  RCL L
  +
LBL C
  R↓
  ISG Y
  GTO B
  x≷ →L
  STO →Z
  ISG L
  GTO A
END
```

Cf. *HP-42S OM*, pp. 152 – 154.

Programmed User Interaction and Dialogues

A number of commands are provided for controlling the interaction of programs with you. A program shall output a result to you at least, and it may also ask for your input. In the *IOI*, the behavior of those I/O commands is described if they are entered from the keyboard. Executed by a program, however, they will work differently.

When you start a program by [XEQ] or [R/S], the symbol will be lit in the *status bar*. While in *RUM* each command executed may change the display immediately, only AVIEW, INPUT, PAUSE, PROMPT, STOP, and VIEW will update the screen with a program running. AVIEW and VIEW will display until the next of these six commands is encountered or the

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program stops. For programmed I/O, see the following **examples**:

- Use AVIEW for displaying a text string (or message) stored in the source, without showing the *name* of the source.
- Take VIEW for displaying, e.g., intermediate results. Specify any *RoV* you want as source of information – also **X** is a valid parameter of VIEW. The *name* of the source will tag the output.

  Note frequent display updates will slow down program execution.

- Use

  ```
  VIEW xyz
  PAUSE n
  ```

  for showing output for a fixed time interval of $\frac{n}{10}\,s$.

- Ask ('prompt') for numeric input employing

  Use RCL, VIEW, or AVIEW to display the *RoV* ***xyz***, then …

  ```
  STOP
  STO xyz
  ```

  … stop and wait for user reaction, finished by [R/S].
  stores what the user entered.

  A stop sign will be displayed in the *status bar* when the program runs on a STOP. Whatever you will key in will be put into ***xyz*** when you continue program execution by [R/S].

  More elegant is using PROMPT or INPUT for this task:

  ```
  PROMPT yx
  STO xyz
  ```

  displays a message (like AVIEW **yx**) and stops, lighting .
  stores what the user entered.

  or:

  ```
  INPUT xyz
  ```

  displays and stops in just one step – also lighting .

- Prompt for text (string) input using the following steps:

  ```
  SF ALPHA
  RCL xyz
  STOP
  ```

  sets *AIM* for upcoming input.
  displays a stored message string and lights **A**.
  waits for your input. Whatever you key in now is appended to x when you continue by pressing [R/S].

  ```
  CF ALPHA
  STO yzx
  ```

  returns to the numeric mode previously set.
  stores x to wherever you like.

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Again, more elegant is using INPUT or PROMPT, e.g.:

```
SF ALPHA                sets AIM for upcoming input.
INPUT xyz
CF ALPHA                returns to the numeric mode previously set.
```

- If you need to enter values for several *variables* then the following way is most efficient (although it may look lengthy here):

```
LBL 'Var.In'            we will need this label for VARMNU later.
   REM 'Following are the variable declarations:'
   MVAR 'xy1'
   MVAR 'xy2'
   MVAR 'xy3'
   REM 'And this creates and displays a menu for all these vars:'
   VarMNU 'Var.In'
   STOP                 stops for user interaction.
   EXITall              exits the menu when program continues.
   RCL 'xy2'            recalls what you need first (it may have been
                        entered in the menu in arbitrary order).
```

The label called 'Var.In' here should be located close to the program top. It may be followed by up to 18 MVAR steps defining your *variables* required. When the program encounters the step VarMNU, it will set up a *menu* for all these *variables* and display it. In our case, this would look like

As long as such a menu is displayed and you want to…

- **write** a new value into one of these *variables*, key in the value or calculate it, then press the *softkey* corresponding to your target *variable*, and *x* will be stored in it.
- **recall** the present value of one of these *variables*, press RCL and then the corresponding *softkey*.
- **view** the present value of one of these *variables*, press VIEW and then the corresponding *softkey*.[235]

[234] P.FN REM inserts a comment in your program. It will be skipped in execution.

[235] Cf. p. 65. The *HP-42S* allowed for viewing the present value of one of the variables displayed by just pressing ▮ and the corresponding *softkey*. We can't support this on *WP43* since this offers you three *menu* rows.

- **exit** this *menu*, press [EXIT].
- **close** this menu **and continue** program execution, press [R/S].

If you continue by pressing a *softkey*, the *name* of the corresponding *variable* is stored into **K**. Your program can use this information to determine which key was pressed. Else, if you just continue by pressing [R/S], ***k*** is not altered.

- Directly react on particular keys pressed. The key codes returned by KEY? (cf. p. 229) allow for 'real time' response to user input from the keyboard. KEY? takes a *register* argument (**X** is allowed but will not lift the stack) and stores the key most recently pressed during program execution or pause in the *register* specified.[236] Although the keyboard is active during program execution it is desirable to display a message and suspend the routine by PAUSE while waiting for user input. Since PAUSE will be terminated early by any key press, simply use `PAUSE 99` in a loop to wait 10 s for input. And since KEY? acts as a test as well, a typical user input loop may well look like the following **example**:

```
LBL 'User.in'
  RCL xyz       displays the register with the message string.
  PAUSE 99      waits 9.9 s for user input unless a key is pressed.
  KEY? 00       tests for user input and puts the key code in R00;
  GTO 'User.in'   if there was no input then return to the beginning;
  LBL? →00          else if a label corresponding to the key code exists
  XEQ →00             then call it, …
  GTO 'User.in'       else return to the beginning.
```

Instead of the dumb waiting step, the routine can do some computations in between and update the display before the next call to PAUSE.

To be even more versatile, you can use KTYP? to return the type of the key pressed if its row / column code is given (see the *IOP*).

If you decide not to handle the key in your program you may feed it back to the main processing loop of your *WP43* with the command PUTK *nn* . It will cause the program to halt, and the key will be handled as if pressed after the stop. This is especially useful if you want to

[236] [R/S] and [EXIT] can't be queried since they stop program execution immediately.

allow numeric input while waiting for some special keys like the arrows. After execution of the PUTK command you are responsible for letting the routine continue its work by pressing R/S.

See the *IOI* for more about the commands mentioned in this chapter.

Solving Differential Equations

The following examples use the programmability of your *WP43* for solving differential equations, frequently appearing in various disciplines:

A simple ecological model[237] of interacting populations consists of rabbits with an infinite food supply and foxes that prey on them. The system can be approximated by a pair of nonlinear, first-order differential equations:

$$\frac{dr}{dt} = 2\,r - \alpha\,r\,f \quad \text{and} \quad \frac{df}{dt} = -f + \alpha\,r\,f$$

where r is the number of rabbits, f is the number of foxes, and α is a positive constant to show how frequently rabbits and foxes meet. When α = 0 there are no encounters; the rabbits keep breeding and the foxes starve. For a specific α, the probability of encounter is proportional to the product of the numbers of foxes and rabbits. A reasonable choice for α is 0.01. – One way to solve this problem numerically is a simple *Euler* method, solving equations of the form:

$$x_{n+1} = x_n + \Delta t\; g(x_n)$$

using a … small increment of time *Δt*.

The equations become:

$$r_{n+1} = r_n + \Delta t\,(2\,r_n - \alpha\,r_n\,f_n) \quad \text{and} \quad f_{n+1} = f_n + \Delta t\,(-f_n + \alpha\,r_n\,f_n)$$

The features of the *WP43* make it ideally suited for problems of this type.

```
LBL 'Foxes'
  REM 'Initialization taking input from the stack'
  STO 'n.fox'
  R↓
  STO 'n.rabb'
```

[237] This is quoted from the *Hewlett Packard Journal* of 1975-11. Some parts of it look arbitrary but it is for demonstration purposes only. I modified the original *HP-25* program putting its quantities in *named variables* instead of *registers* and added comments for better readability and understanding.

```
  FIX 5
LBL 01
  REM 'Begin of time loop'
  RCL 'n.fox'
  ENTER↑
  ENTER↑
  RCL 'n.rabb'
  RCL 'meet'
  ×
  ×                 number of meetings between rabbits and foxes
  STO 'n.meet'      store it for later use in the other equation
  x≷y
  -                 change in number of foxes
  RCL 'Δt'
  ×
  +
  x<0.?
  0
  STO 'n.fox'       new number of foxes
  RCL 'n.rabb'
  2
  ×
  RCL 'n.meet'
  -                 change in number of rabbits
  RCL 'Δt'
  ×
  RCL 'n.rabb'
  +
  x<0.?
  0
  STO 'n.rabb'      new number of rabbits
  IP
  RCL 'n.fox'
  IP
  E 5
  /
  +                 merge numbers of rabbits and foxes into 1 output number
  PAUSE 20          pause for 2 seconds for data recording
  GTO 01            end of (infinite) time loop, return to its begin.
```

Example:

$r_0 = 300,\ f_0 = 150,\ \alpha = 0.01,\ \Delta t = 0.02$. Start with

0.01 [STO] **meet** **0.02** [STO] **Δt** **300** [ENTER↑] **150** [XEQ] **PROG** [Foxes]

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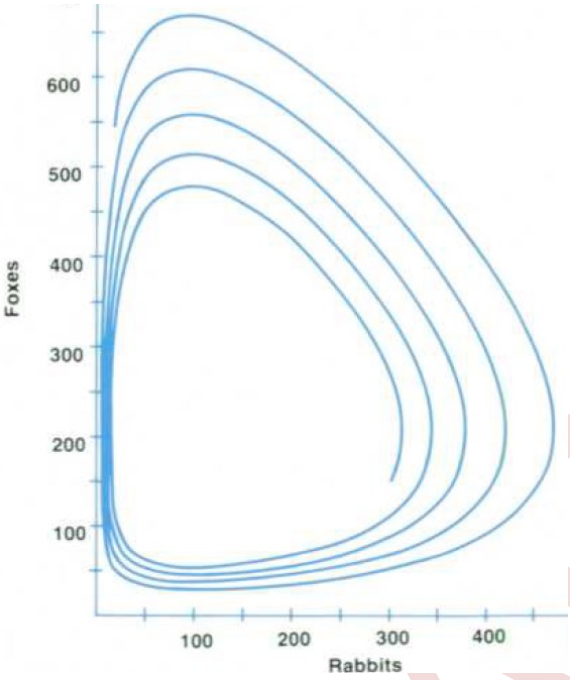

The results can be plotted by hand on a graph of foxes versus rabbits. Note that the calculations give … floating-point numbers, but the display is always truncated to an integer.

A plot of foxes versus rabbits is a circular loop with a period of about five time units (250 iterations). The rabbit minimum is 14, the maximum is 342 on the first loop. The fox minimum is 54 and the maximum is 478 on the first loop.

$r_0 = 100,\ f_0 = 200$ is a constant solution.

The following method solves ordinary 2nd order differential equations, a type frequently occurring in physics.[238] In a first **example**, we will solve the equation of motion for the fall of a parachutist (or skydiver):

$$\frac{d^2f}{dt^2} = g - b\left(\frac{df}{dt}\right)^2$$

with Earth's gravity acceleration and taking care of drag. Proceeding in small constant time steps Δt again, the following set of equations controls the vertical motion of the skydiver:

$$\left(\frac{df}{dt}\right)_{1/2} \approx \left(\frac{df}{dt}\right)_0 + \left[g - b\left(\frac{df}{dt}\right)_0^2\right]\frac{\Delta t}{2} = \left(\frac{df}{dt}\right)_0 + A_0\,\frac{\Delta t}{2}$$

$$f_1 \approx f_0 + \left(\frac{df}{dt}\right)_{1/2}\Delta t \quad \text{and} \quad \left(\frac{df}{dt}\right)_{3/2} \approx \left(\frac{df}{dt}\right)_{1/2} + A_{1/2}\,\Delta t\,,$$

$$f_2 \approx f_1 + \left({}^{df}/_{dt}\right)_{3/2}\Delta t \quad \text{etc.}^{237}$$

Assume start height at time zero ($t = 0$) is 1000 m and vertical velocity is zero

[238] See *ReMA I* for more information about this method.

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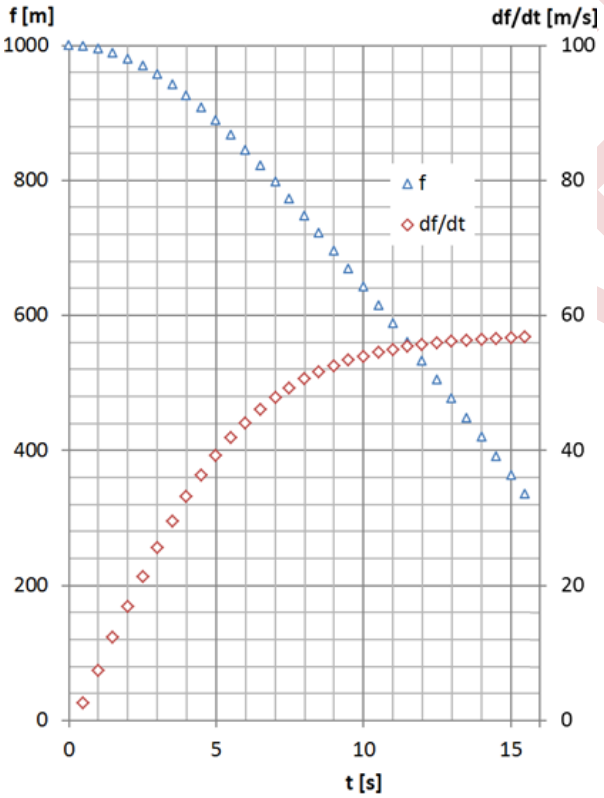
(i.e. $f(t=0) = f(t_0) = f_0 = 1000$ and $\left(\frac{df}{dt}\right)_0 = 0$). Using *named variables* ***Δt***, ***b***, ***t***, ***f***, and ***dfbydt*** (note a **/** must not appear in *names*), the following routine will compute height and velocity of the skydiver as functions of time:

```
LBL 'PFall'
   .5                  initialize all variables used
   STO 'Δt'
   .003                assumed realistic drag value for a body (with closed
   STO 'b'                                          parachute) falling in air
   1000                start height
   STO 'f'
   0                   start time and velocity
   STO 't'
   STO 'dfbydt'        end of initialization
LBL 01
   REM 'Begin of time loop'
   # g⊕                recall g⊕ from CONST
   RCL 'b'
   RCL 'dfbydt'
   x²
   ×                   b × (df/dt)²
   -                   g – b × (df/dt)² = A
   RCL 't'
   x>0.?               check time – it will be zero in 1st run
   GTO 02              from 2nd run on go to local label 02
   REM 'for the first run only'
   DROP↓               forget t
   2
   /                   A / 2
   GTO 03              go to common part
LBL 02
   REM 'from second run on'
   DROP↓               forget t
LBL 03
   REM 'common part of time loop resumes here'
   STO+ 'dfbydt'       calculate the new df/dt
   RCL 'Δt'
   STO+ 't'            calculate the new time
```

[239] Note ***A*** must be ≥ 0 always. Thus, this routine will work for velocities $< \sqrt{g/b}$ only. Furthermore, it will not work for abruptly decelerating fast initial movements (e.g., by opening a parachute in free fall).

```
  RCL× 'dfbydt'     df/dt × Δt
  STO- 'f'          calculate the new f
  VIEW 't'          show new time for plotting the next data points
  STOP              --------
  VIEW 'f'          show new height
  STOP              --------
  VIEW 'dfbydt'     show new velocity
  STOP              --------
 GTO 01             continue ad infinitum
END
```

Now, leave *PEM* and start the program: XEQ **PROG** (PFall).



Plotting the points calculated for ***f*** and ***df/dt*** will result in a diagram like this. Height decreases following a parabola over time at beginning but becomes linear later. Vertical velocity does not increase much after some 12 s here, approaching some 57 m/s while skydiving with closed parachute.

For comparison: The velocity limit with an open parachute ($b \approx 0.3$) will be < 6 m/s, so touchdown will be like falling from a wall 1.65 m high. But open it in time!

In a 2nd **example**, we will look at a horizontal harmonic oscillator with spring rate ***r*** and mass ***m***. Oscillating freely, its equation of motion is simply

$$\boldsymbol{m}\,\frac{d^2 f}{dt^2} = F = -\boldsymbol{r}\,f$$

Adding an external stimulating force and velocity-dependent damping, the

equation will change to

$$\boldsymbol{m}\frac{d^2f}{dt^2} = -\boldsymbol{r}\,f - \boldsymbol{b}\frac{df}{dt} + \boldsymbol{s}\sin\omega t \;\Leftrightarrow\; \frac{d^2f}{dt^2} = -\alpha\,f - \beta\,\frac{df}{dt} + \gamma\sin\omega t = A$$

with $\alpha = {}^{r}/_{m}$, $\beta = {}^{b}/_{m}$, and $\gamma = {}^{s}/_{m}$. Then the following set of equations controls the oscillator motion for small constant time steps $\boldsymbol{\Delta t}$:

$$\left(\frac{df}{dt}\right)_{1/2} \approx \left(\frac{df}{dt}\right)_0 + \left[-\alpha f_0 - \beta\left(\frac{df}{dt}\right)_0 + \gamma\sin\omega t_0\right]\frac{\Delta t}{2} = \left(\frac{df}{dt}\right)_0 + A_0\,\frac{\Delta t}{2}$$

$$f_1 \approx f_0 + \left(\frac{df}{dt}\right)_{1/2}\Delta t \quad \text{and} \quad \left(\frac{df}{dt}\right)_{3/2} \approx \left(\frac{df}{dt}\right)_{1/2} + A_{1/2}\,\Delta t$$

$$f_2 \approx f_1 + \left(\frac{df}{dt}\right)_{3/2}\Delta t$$

Now, all you have to do is rewriting the 1st part of previous program:

```
LBL 'Osci'
   .2                initialize all variables used
   STO 'Δt'
   1                 assumed relative spring rate
   STO 'α'
   .5                assumed relative damping
   STO 'β'
   1                 size factor for stimulation
   STO 'γ'
   RAD
   .3                frequency of stimulation
   STO 'ω'
   1.5               start position
   STO 'f'
   0                 start time and velocity
   STO 't'
   STO 'dfbydt'      end of initialization
LBL 01
   REM 'Begin of time loop'
   RCL 't'
   RCL 'ω'
   ×
   sin               sin(ω t)
   RCL 'γ'
```

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`×`	$\gamma \sin(\omega t)$
`RCL 'β'`	
`RCL 'dfbydt'`	
`×`	
`-`	$\gamma \sin(\omega t) - \beta\, df/dt$
`RCL 'α'`	
`RCL 'f'`	
`×`	
`-`	$\gamma \sin(\omega t) - \beta\, df/dt - \alpha f = A$
`RCL 't'`	
`x>0.?`	check time – it will be zero in 1st run

The remaining code can be taken over from the 1st routine as is (feel free to delete the output of *df/dt* if you are not interested in it). Play with the parameters and observe the results.

Very similar equations apply to vertical oscillators, actually to almost all vibrating, swinging, or bouncing objects and parts. Enjoy!

Consult *ReMA I* for more about solving differential equations, also in *2D*.

The Programmable Menu (MENU)

Your *WP43* features a *programmable menu* you can use to cause program branching. By this, you can create *menu*-driven programs. The command MENU selects this *programmable menu*; it will be displayed when the program stops. You can define each *item* in this *menu* so that when the respective key is pressed, a particular GTO or XEQ instruction will be executed. You can even redefine ▲, ▼, and EXIT.[240]

To define a softkey in the programmable menu:

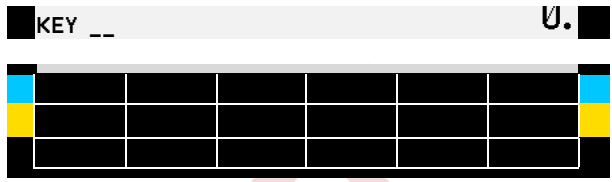
1. Store a *text string* of up to 7 characters in *register* **X**. This text will appear in the *menu space* for the *softkey* specified (if free space doesn't suffice, only the first characters of ***x*** will be displayed). **X** is neither used nor dropped when defining ▲, ▼, or EXIT.

[240] See *HP-42 OM*, pp. 145 - 148, and its *Programming Examples and Techniques*, pp. 29 - 39, 92 - 99, 158 - 160, and 184 - 192, for examples using the *programmable menu*.

2. Call KEYG (i.e. on key, go to)
   or KEYX (i.e. on key, execute). You find them in P.FN.

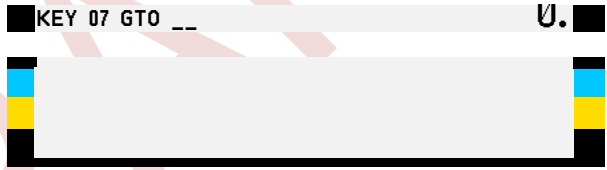
3. The *menu view* changes. The black fields indicate you shall specify which *softkey* you want to define:

   Call one of the 18 *softkeys* available (including 🟧 or 🟦, if applicable), ▲, ▼, or EXIT. Alternatively, enter the respective key number, 1 through 21 (the unshifted leftmost *softkey* carries #1, the 🟦-shifted leftmost *softkey* #13, ▲ #19, etc.).



   Let's assume KEYG here and the 🟧-shifted leftmost *softkey* being pressed, for instance.

4. The command echo changes:



5. Specify a program label using one out of the following four methods (the *menu view* changes to the one shown on p. 222):
   a. Key in a single-letter local label out of **A** – **E**.
   b. Key in a two-digit local label,
   c. Select a global label by pressing the corresponding *softkey* in PROG.
   d. Key in a global label character by character using α and *AIM*.

   Unless you have just defined ▲, ▼, or EXIT, x will be dropped. Repeat this procedure for each *softkey* in the *programmable menu* you want to define. A new definition replaces any previous definition that may exist for this *softkey*.

To display the programmable menu:

Execute the MENU function, e.g., by entering P.FN P.FN2 MENU.

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To clear all softkey definitions in the programmable menu:

Press [CLR] CLMENU (clear the *programmable menu*).

Basic Kinds of Program Steps

You have seen various program steps so far. Each step takes a single place in *program memory*, and each step is numbered automatically. Basically, these steps fall into five categories – each may contain either ...

1. a global or local label (like LBL 'Satell' or LBL 02 above) or
2. an entire command (like - or y^x or $\log_x y$ or STO× →'Prod2' or REM 'Note ξ varies acc. to Fēi Cháng-hǎo et al.') or
3. a *numeric constant*, either keyed in like -1.902×10^{-16}, $123\,45_{16}$, $1.7+i\times0.4$, $-23\,{}^{4}/_{5}$, or recalled from CONST like # λ_c;[241] or
4. a fixed *date* (like 2023-09-10) [241] or
5. a fixed *time* (like 21:45:17.654) [241] or
6. an *alphanumeric constant* (like This is the 2nd iteration output:); such a string may be up to 196 characters or 256 bytes or 4 lines long, whatever will be reached first. It will be displayed enclosed in ' (else some special *strings* might be misinterpreted as numbers or commands in program listings), though automatically stored in **X** without these delimiters.[242]

Since each constant takes one step in a routine, there is no necessity for separating them by ENTER↑ here.

Example:
Think of calculating 12.3 + 45.67 in a routine. Then keying in
12.3 [ENTER↑] ⬅ **45.67** [+] will result in a program snippet

[241] The respective value will be automatically stored in **X**.

[242] The character ' is not stored in any menu.

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```
12.3
45.678
+
```

which will do for returning 57.978. The missing `ENTER↑` saves one *byte* of program space and makes the routine a tiny bit faster. This may not really be crucial but you should know.

Constant *vectors* and *matrices* can't be entered directly in a program; instead, you can create them in advance and store them in *registers* or *variables* and manipulate these stored *items* (as described in *Section 2*) in routines as well, of course.

Each program step requires at least one *byte* of memory (see *ReMA B*). Use MEM? to learn about the free space remaining in the pool. We think you will hardly ever run out of program space – but it is possible. If you do while trying to enter a new program step, you will read the message `RAM does not suffice` (see *ReMA C* and *B* for ways to escape from such a situation).

You can use most operations provided on your *WP43* to solve repetitive and iterative problems of almost any kind you can imagine, with a speed exceeding the possibilities of earlier *HP* pocket calculators by far.

Keystroke programs may be more difficult to read than routines written in a high level language since inline comments and indenting are not supported (but feel free to use REM frequently to comment wherever beneficial). Also obvious control structures are missing. Loops in particular may need some habituation.

Let's look at an exemplary *PASCAL* structure and its *WP43* equivalent doing the same:

DRAFT

Pascal	Keystrokes	Comments for the keystroke program steps
.....	.....	
.....	.....	
.....	19	
.....	STO J	Load the comparative value.
.....	1.03403	Initialize loop control (begin with **1**,
.....	STO I	end at **34**, increment by **3**).
for I = 1 to 34 by 3 do;	LBL 01	Begin of the big loop.
begin;	REM 'xxx'	Write something meaningful here.
.....	.....	
.....	.....	
.....	.....	
.....	.....	
.....	RCL I	Recall the loop counter.
if I < 19 then do;	x≥? J	If ***i*** ≥ **19**
begin;	GTO 02	then jump to local label 02
.....	REM 'yyy'	else (for ***i*** < **19**) proceed here.
.....	.....	
.....	.....	
.....	.....	
.....	.....	
end;	GTO 03	When ready, skip next section.
else do;	LBL 02	Do what shall be done for ***i*** ≥ **19**.
begin;	REM 'zzz'	Write something meaningful here.
.....	.....	
.....	.....	
.....	.....	
.....	.....	
end;	LBL 03	End of if-condition.
.....	.....	
.....	.....	
.....	.....	
.....	.....	
.....	.....	
	ISG I	Eventually increment ***i*** and check.
end;	GTO 01	If ***i*** ≤ **34** then run the big loop again
.....	.....	else it is done now.
.....	.....	Continue.

Once you have learned the basics (like "do if true, skip if false", sometimes reverting the logic of a test, and checking the loop counter at the end of the loop), you will enjoy the freedom you have.

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Deleting Programs

To delete some steps of a program, do as explained on pp. 222f. Repeat as often as necessary.

To delete an entire program quickly, press [CLR] CLP, then

- PROG to choose the program, or
- [α] to enter the *program name*, or alternatively
- [ENTER↑] to delete the *current program*.

> **ATTENTION:** CLP will remove the entire program without further ado, not just the current routine. And CLP can't be undone! The space freed by CLP will be returned to the pool your *WP43* offers for data and programs.

To delete all programs stored in *RAM*, press [CLR] CLPall, think (!), and confirm your intention or deny. CLPALL will completely wipe out *PM*, i.e. you may get up to some 45 000 *bytes* for new programs and data (depending on the space your data in *RoV* are occupying). Note that also CLPALL, once confirmed, cannot be undone.

Flash Memory (*FM*)

In addition to the *RAM* provided, your *WP43* also allows you accessing its large *FM.* The major part of it is allocated to a so-called *FAT disk* (*FD*) of 6 MB (see a sketch in next chapter). You can use it for voltage-fail-safe storage of your programs and data, e.g., for backups.

Use [SAVE] for creating an on-board backup of your entire work (your calculator *configuration*, programs, *variables*, *register* contents, flag status, etc.).[243] Alternatively, SAVEST saves the *calculator state* in a named file on the *FD*. Such a *state* encompasses the same data as a backup but you may store various *states* under different filenames, so you can return to various calculator *states* depending on your actual tasks and requirements.[244]

[243] There will be only one backup file in *FM* – calling SAVE again will overwrite the previous backup.

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The different flavors of LOAD… (in I/O) are for recalling saved data from the backup or an arbitrary *state file*:

- **LoadΣ** recalls the saved statistical *matrices* STATS and HISTO as well as the *summation registers*,
- **LoadR** recalls the contents of all *general purpose registers* saved (this includes the entire *stack* and **L**), and
- **LoadSS** recovers the saved system settings of your *WP43* (incl. its *configuration*),
- **LoadV** recalls all user-defined *variables* saved,
- **LoadEq** all equations,[245]
- **LoadPr** all programs,[246] and
- **LoadAll** recalls the entire backup or state file at once, overwriting your present *calculator state*.

Turn to the *IOI* for more information about these nine commands.

While *FM* is a safe place, hence ideal for long-living data, it shall not be used for frequently changing temporary storage like in programmed loops.[247] Conversely, *registers* and standard user *program memory* residing in *RAM* are designed for data changing with high frequency – but they will not hold any data when the batteries are removed for longer than some *seconds*. So both *RAM* and *FM* have their specific advantages and disadvantages you should take into account for optimum benefit and longevity of your *WP43*.

---

[244] Instead, STOCFG stores the *WP43 configuration* in an *RoV*, i.e. in *RAM*. Note neither *system state* nor `.p43` files (see p. 256) nor *configurations* are easily user-readable.

[245] LOADEQ will add the recalled equations not listed in RAM before. Identical equations will not be touched.

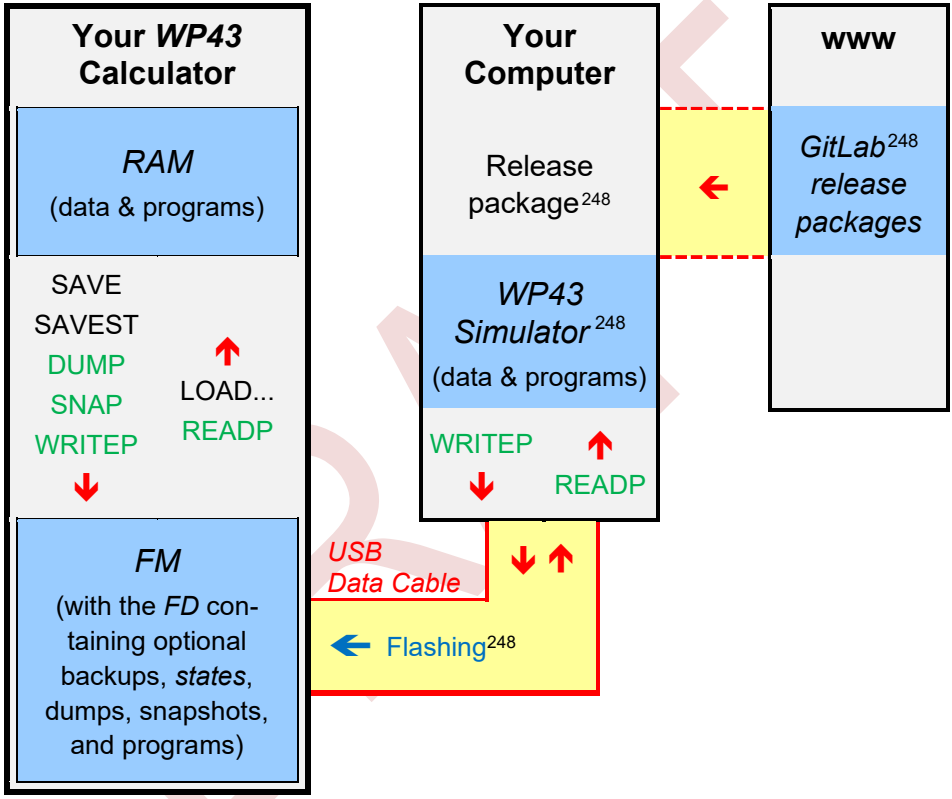
[246] LOADPR will add (append) the recalled programs to those present already. Since *program names* (i.e. their starting global labels) may be reused, programs carrying identical *names* may appear in *RAM* more than once, especially after LOADPR. This may fill the *RAM* faster than expected – your *WP43* will throw **RAM does not suffice** then; we recommend purging *program memory* before if appropriate.

[247] *FM* may not survive more than 100 000 flashes (i.e., e.g., >27 years with 10 flashes per day). Although this will be hardly reached in normal calculator use, we made commands writing to *FM* (SAVE, SAVEST, and WRITEP) non-programmable.

Communicating with Your Computer

Looking at I/O from your *WP43* calculator, this matter appears as pictured schematically below (not to scale).

For SAVE, SAVEST, and the different flavors of LOAD return to previous chapter. DUMP, SNAP, READP, and WRITEP are covered below.



Data exchange with your *PC*:

There are two ways you can exchange data:

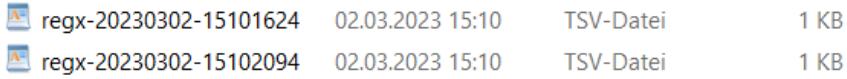
1. With a *USB* cable connecting your *WP43* with your *PC*, you can 'activate' the *FD* (a.k.a. *USB* disk) in *FM* via USB. Then said disk will

[248] Please see *ReMA E* and *F*.

be part of the address space of your *PC*, and you can use file management applications there (e.g., the *Explorer* on a *Windows PC*) to copy or move arbitrary files to and from this *FD*. Having done this, EXIT ENTER↑ will return the *FD* to your *WP43* exclusively, so you can continue calculating.

2. You can write programs from your *WP43* onto *FD*, save snapshots, backups, *system states*, or dump specific results (e.g., *matrices* or very long integers) in files there. You may then employ method 1 and deal with these files on your *PC*, enjoying more editing comfort than our little calculator can offer. We will elaborate on this way in the following:

   I/O **DUMP** dumps ***x*** with its full precision onto the *FD* in a file `regx-`***`yyyymmdd-hhmmssc`***`.tsv` (according to date and time of the dump; see two examples below) in directory `SAVFILES`. This is the way to permanently output all digits of very long integers ($>10^{44}$) – note you can SHOW a long integer up to 10^{296} easily but only its top 44 digits will stay on display after next keystroke.

   Use a text editor (e.g., *Notepad*) for dealing with `.tsv`-files.

   SNAP stores a snapshot (a.k.a. screenshot, hardcopy) of the entire calculator screen on the *FD* in a graphic file named ***yyyymmdd-hhmmssc***`.bmp` (according to date and time of the snapshot) in directory `SCREENS`. The same can be achieved by pressing ■ and E simultaneously – this will also work in *PEM* and with *temporary information* (like messages). Note the *WP43* display is too large to be printed on a vintage *HP* printer. See the *IOI* for more.

   I/O **WRITEP** writes your *current program* in a named `.p43` file on *FD* in the directory `PROGRAMS`.

   I/O **READP** reads a named `.p43` file from *FD* into *program memory*, so you can execute or edit this program.

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What works on your physical *WP43* shall work on the *Simulator* as well and vice versa – both platforms share the same set of functions. Just where your *WP43* will write *FD* files for DUMP and SNAP, the *Simulator* will write to the *clipboard*.

So you can write the current program (using WRITEP) from your *WP43* in a file on the *FD*, connect your *WP43* to your *PC*, read the program file (using READP) from the *FD* into *program memory* of the *Simulator*, then test, debug, and edit it there using your *PC* keyboard. When finished and satisfied with program execution, WRITEP from your *PC* on the connected *FD* again, disconnect, READP, and work with the refined file in *program memory* of your *WP43*. Find more about this procedure in *ReMA E*.

Printing:

If you have got a thermoprinter *HP82440 A* or *B*, you can use it to get immediate hardcopies of your results. There are fifteen print commands provided, all stored in 🖶 (a.k.a. PRINT, accessed via [blue] + [0]). They do the following:

🖶MODE *n*	sets the print mode (see the *IOI*).
🖶WIDTH	returns the number of print columns that x would take in the print mode set.
🖶TAB *n*	positions the printhead to column n.
🖶ADV	prints the current contents of the print buffer and a linefeed.
🖶DLAY *n*	sets a delay of n *ticks* to be used with each linefeed on the printer.
🖶# *n*	sends a single *byte* n to the printer, e.g., a control code.
🖶CHAR *n*	sends one character with code n to the printer.
🖶x	prints x.
🖶r *r*	prints the content of the *RoV* specified.
🖶Σ	prints the contents of all summation *registers*.
🖶STK	prints the contents of the entire *stack* as allocated.
🖶REGS	prints the contents of a set of *GP registers*.
🖶USER	prints all user-defined *variables* and *global program labels* in alphabetic order.

🖷PROG lists the current program.

We recommend consulting the *IOI* for details of these commands.

Local Data

After some time with your *WP43* you will have a number of routines stored, and keeping track of their resource requirements may become challenging. Most modern programming languages take care of this by declaring *local variables*, i.e. memory space allocated from general data memory and accessible for the *current (sub-) routine* only; when the routine is finished, this memory is released. On your *WP43*, mainly *registers* are used for data storage – so we offer *local registers* allocated to your current routine exclusively.

Example 1:

Let's assume you create a routine labeled P1 and need five *registers* (in addition to the *stack*) for your computations therein. Then all you have to do is just enter *PEM*, go into the routine P1, and key in (close to its top)

P.FN LocR 5 ENTER↑

specifying that you want five *local registers*. After this step, you can access these new *registers* using local addresses **.**00 − **.**04 throughout routine P1.

Now, if you call another routine P2 from P1, also P2 may request *local registers* via LOCR. Then these will carry *local register* addresses **.**00 etc. again, but, e.g., the *local register* **.**00 of P2 will be physically different from the *local register* **.**00 of P1, so no interference will occur (neither P2 can access *local registers* of P1 nor vice versa).

As soon as RTN or END is executed, the *local registers* of the respective (sub-) routine will be released and the memory they took will be returned to the pool of free space.

Local data holding allows for recursive programs, since every time such a routine is called again it will allocate a new set of *local registers* and *user flags* being different from the ones it got before. See the commands LOCR, LOCR?, and POPLR in the *IOI*.

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You may allocate up to 99 *local registers* per routine (as far as memory allows). In addition, you get 32 local *user flags* as soon as you request at least one *local register*.

Example 2::

The following subroutine acts as a built-in test preserving the *stack*: it checks if **X** is tagged as *decimal* or *sexagesimal degrees*, returning to the next instruction if true and skipping one instruction if false:[249]

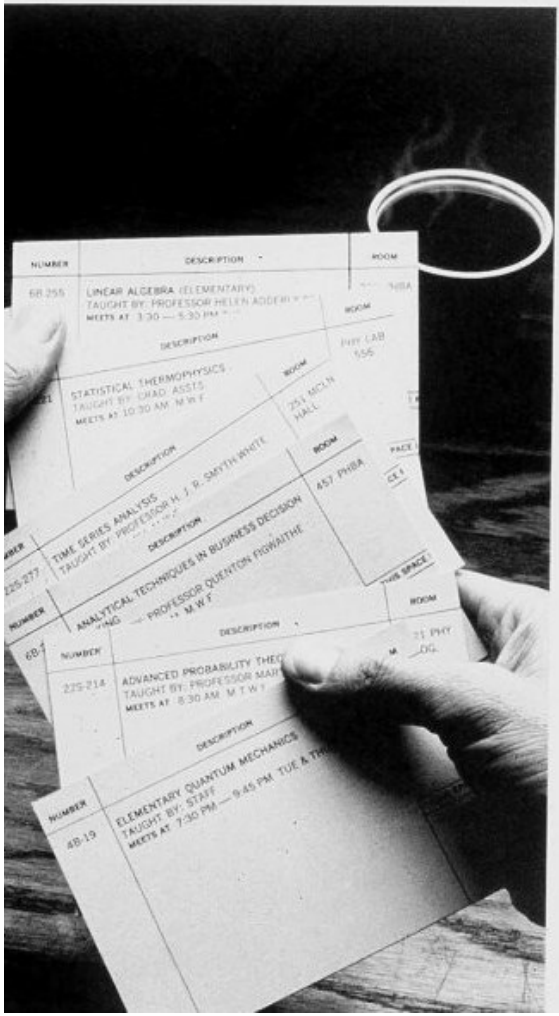
```
0001 LBL 'TAGGD?'
0002    LocR 08      allocate 8 local registers to the current routine
0003    STOSTK .00    for stack saving.
0004    ''           set up a minimum text string in X
0005    +             and append y (= previous x).
0006    176          176 = B0₁₆ = °
0007    x→α
0008    αPOS? Y      look for a ° in y.
0009    x< ? 0.       if there is none
0010    SKIP 002        then go to step 0013
0011    RCLSTK .00      else recall the entry stack
0012    RTN               and return to caller;
0013    RCLSTK .00      but if a ° was found then recall the entry stack
0014    RTN.SKP           and return to the caller but 1 step downstream.
```

Since you are free to nest subroutines (cf. p. 225), and each subroutine may allocate its own *local registers*, the total number of *registers* allocated may become quite large. The total number of accessible *registers* at one time, however, will never exceed 211 (cf. p. 59). Look up *ReMA B* for more information about the space required for various *data types*.

[249] Kudos to *Didier*.

DRAFT

If you're taking tough courses, you need all the help you can get.



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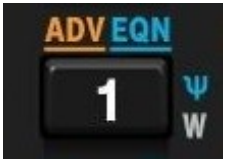
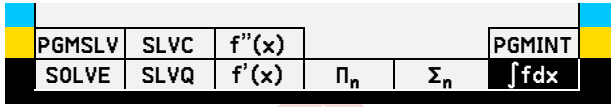
An advertisement of 1980 or 81. Compare f. 47 on p. 44.

SECTION 4: ADVANCED PROBLEM SOLVING

Beyond what you have seen so far, your *WP43* provides some powerful commands for computing programmable sums and products, definite integrals, 1st and 2nd derivatives as well as for solving equations. All these are provided in ADV or EQN. Pressing [ADV] in *RUM* results in this *view*:





The commands Σ_n, Π_n, SLVQ, SLVC, SOLVE, $\int$, f′(x), and f″(x) are explained below in this order. All of them are programmable. Also interactive integrating, deriving, and solving equations is possible and can be reached through [EQN]. See below for details and examples.

Programmable Sums

Do you need results of finite sums? Σ_n computes them for you. It is called with a loop control number (*LCN*) in **X** and a *label* trailing the command. Said *LCN* follows the format `ccccc.fffii` (cf. DSE etc.).

In its heart, Σ_n then works like this:

1. Σ_n sets the sum to **0** initially.
2. Σ_n fills all *stack registers* with ***c*** and calls the routine specified by ***label***. That routine returns a summand in **X**.
3. Σ_n adds this summand to said sum.
4. Σ_n decrements ***c*** by ***i*** (or by **1** if ***i*** = 0); if ***c*** ≥ ***f*** then Σ_n goes back to step 2, else it returns the final sum in **X**.

Example:

$$\sum_{k=0}^{100} \sqrt{k} = ?$$

DRAFT

Solution:

1. Write a little program for the internal calculation of the summands:

```
LBL 'ΣV'
  √x
  RTN
```

.

2. Enter

   100

   ADV Σ_n α [blue] S V ENTER↑ (or pick **ΣV** from PROG, cf. p. 222)

   and get 671.462 9 returned if FIX **4** is set.

Σ deliberately sums from the last term to the 1st, on the assumption that summations will often be of convergent series so this summing sequence should generally increase accuracy of result.

Programmable Products

Finite products can be computed as well as sums. The command Π_n is called with a loop control number in **X** and a *label* trailing the command – its parameters are the same as those required for Σ_n above).

In its heart, Π_n works almost as Σ_n :

1. Π_n sets the product to **1** initially.
2. Π_n fills all *stack registers* with ***c*** and calls the routine specified by the label. That routine returns a factor in **X**.
3. Π_n multiplies this factor with said product.
4. Π_n decrements ***c*** by ***i*** (or by **1** if ***i*** = 0); if ***c*** ≥ ***f*** then Π_n goes back to step 2, else it returns the final product in **X**.

Example (with SCI **4**):

$$\prod_{k=1}^{90} \sqrt{\frac{1}{\sqrt{1+k^2}}} = ?$$

Solution:

1. Write a little program for the internal calculation of the factors:

```
LBL 'PROD'
   √(1+x²)
   1/x
   √x
   RTN
```

2. Instead of calling PROD via ADV Π_n you can call it from another program and measure the time it takes. Thus write a second little program:

```
LBL 'P2'
   TICKS          Start time
   STO 00         Store the start time since Πn will fill the entire stack.
   90             Start value. Default step size is 1, final value 0.
   Πn 'PROD'
   TICKS          Stop time
   RCL- 00        Get the time difference in ticks.
   RTN
```

XEQ PROG P2 and get $5.941\,4\times10^{-70}$ and 9 *ticks* (0.9 s) returned when powered by battery, 0.3 s when powered by *USB*.

Solving Quadratic Equations

SLVQ finds the *real* and *complex* roots of an arbitrary quadratic equation $ax^2 + bx + c = 0$ with its parameters on the input *stack* [***c***, ***b***, ***a***, …] :

- If $r := b^2 - 4ac \geq 0$ then SLVQ will return two *real* roots $(-b \pm \sqrt{r}\,)/\,2a$ in $\mathbf{Y}$ and $\mathbf{X}$. If called in a routine then the step after SLVQ will be executed.
- Else, SLVQ will return the 1st *complex* root in $\mathbf{X}$ and the 2nd (*complex conjugate* of the 1st) in $\mathbf{Y}$. If called in a routine then the step after SLVQ will be skipped.

So actually SLVQ tests for *real* roots at its very end. In either case, it returns ***r*** in $\mathbf{Z}$. Higher *stack registers* are kept unchanged. $\mathbf{L}$ will contain parameter ***c***.

DRAFT

Example:
Find the roots of $4x^2 - 3x - 2 = 0$.

Solution (with FIX **4** chosen):

4 ENTER↑ **3** +/- ENTER↑ **2** +/-

ADV SLVQ returns $x = 1.175\,4$, $y = -0.425\,4$, $z = 41.000\,0$. Since z is positive, x and y are the two *real* roots of this equation here.

Check: Store x in $\mathbf{J}$ and y in $\mathbf{K}$. Then enter

RCL J FILL **4** × **3** − × **2** − returning $2.000\,0\times10^{-33}$ and

RCL K FILL **4** × **3** − × **2** − returning $0.000\,0$.

Remember your *WP43* calculates with 34 digits precision, so any result within $\pm\,3\cdot10^{-33}$ is equal to zero in this matter.

Solving Cubic Equations

SLVC finds the *real* and *complex* roots of an arbitrary cubic equation $ax^3 + bx^2 + cx + d = 0$ with its parameters on the input *stack* [***d***, ***c***, ***b***, ***a***, …] .[250] For real parameters and the *discriminant*

$$\Delta = b^2c^2 - 4ac^3 - 4b^3d - 27a^2d^2 + 18abcd\,,$$

there are three cases:

1. $\Delta > 0$ leads to three distinct *real* roots,
2. $\Delta = 0$ means a double or triple root,
3. $\Delta < 0$ leads to one *real* root and a pair of *complex conjugate* roots.

SLVC will return the three roots in $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$, plus ***Δ*** in $\mathbf{T}$. If called in a routine, SLVC will act as a test returning `true` for $\Delta \geq 0$, so the step after SLVC will be executed – for $\Delta < 0$, this step will be skipped.

[250] For finding roots of higher grade polynomials, feel free to apply the *Lin-Bairstow* method (see https://en.wikipedia.org/wiki/Bairstow%27s_method or in German https://de.wikipedia.org/wiki/Bairstowverfahren).

DRAFT

Arbitrary Algebraic Equations

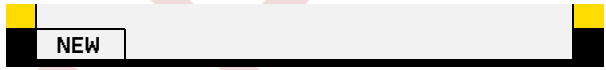
EQN lets you create, store, select, edit, and delete arbitrary *algebraic* equations. You may use each such equation for …

- solving it interactively for any *variable* it comprises,
- plotting it,
- integrating or differentiating it.

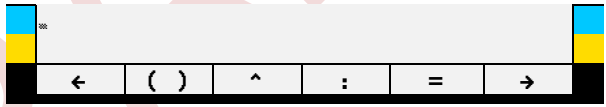
The total number of equations and *variables* storable is limited only by the amount of free memory available.

Example:

Press [EQN]. If there is no equation in memory yet, your *WP43* will return:



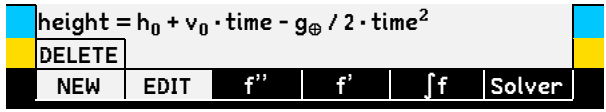
Press **NEW** to enter a new equation. You will get:



with *AIM* turned on (cf. pp. 205ff). Key in your equation, e.g.,

height = h [R↓] [0] [+] **v** [R↓] [0] [×] **time** [−] **g⊕** [/] [2] [×] **time** [^] [2] [251]

for the height of, e.g., a ball thrown vertically upwards with velocity v_0 starting at height h_0 (e.g., from the *Eiffel Tower* or a *Zeppelin* – well, rather drop it from the latter). Press [ENTER↑] for closing the *Equation Editor* and see:



[251] Type what you need. P. 271 shows what can be picked from the keyboard directly. The *Equation Editor* starts in lower case for good reasons.

The index ⊕ is stored in αMATH, and accessed via [▒] [−] [▼] … Picking a character from αMATH will insert this character into the equation and close αMATH, so you can continue editing. This applies also to the other alpha *menus* when writing an equation.

Even easier: You can pick $g_{\oplus}$ from CATALOG'CONST.

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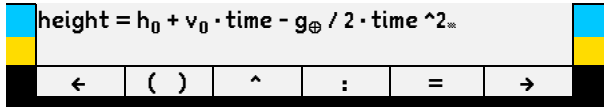
You will get such a *view*[252] whenever one or more equations are stored. The equation displayed in the blue row is called the *current equation*.[253]

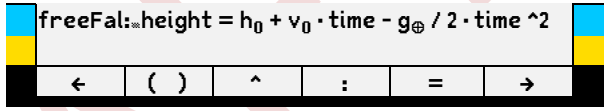
Pressing EDIT opens the *Equation Editor* for this equation:



You may modify this equation at any position by moving the edit cursor to the location behind the symbol(s) to be changed and pressing ⬅ as required followed by the new symbol(s) to be inserted here.

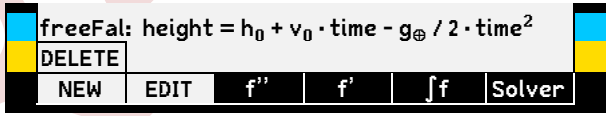
For labeling this equation, move the cursor left to its very begin using ←, and key in up to 7 characters (cf. p. 60) plus a colon there:

F R E E ▲ F ▼ A L :



(If an equation becomes wider than the screen, ellipses will be displayed at its end(s); then use ← and → for scrolling.)

ENTER↑



ENTER↑ terminates the *Equation Editor* and stores the modifications you made. Note editing an equation clears all its *variables*.

You are free to create and baptize *variables* in your equations to your liking, but remember *variable names* must be unique (cf. p. 60). Furthermore, you may employ any constant specified in your *WP43* by just calling its *name* (see pp. 293ff). You must not, however, reuse any *name* of such a constant for any of your *variables* – else your *WP43*

[252] Here, your *WP43* uses multiplication dots for sake of better readability (you could hardly distinguish ✖ and ✖ in an equation). For the same reason, spaces are inserted automatically. Implicit multiplications are not supported in equations.

[253] A dashed line will appear over the current equation when there are more. To select another equation, press ▲ or ▼ then until the requested one appears.

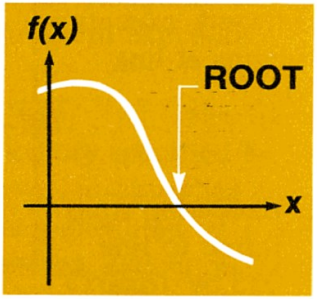
may well misunderstand your intentions. Actually, do not reuse any *item name* already defined for your *variables*.[254]

And watch proper spelling of functions and *variables*. Your *WP43* will definitely not decode `TAN(α)` as you may have meant it, but as you wrote it! How shall it know any better? Please see p. 265 and the last chapter of *ReMS 3*.

Evaluation of equations follows *PFEMDAS* (i.e. parentheses first, then functions, exponentiations, multiplications, divisions, additions, and eventually subtractions[255]). *Unary minus* is interpreted as a multiplication times −1. And the number of pending operations as well as the number of parameters in each such algebraic equation must not exceed nine.

The *Solver*

Interactively Solving Arbitrary Algebraic Equations



The built-in *Solver* application of your *WP43* is a special root finder that enables you to solve an equation for any of its *variables*. It allows solving for an arbitrary unknown as well as for finding the (real or complex) root(s) of an arbitrary equation.[256]

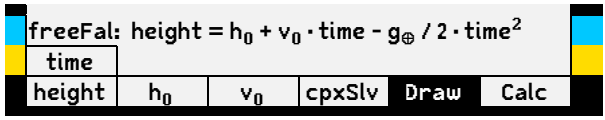
Press EQN, make the equation you want to solve the *current equation* (see previous chapter), and press Solver. Your *WP43* will check this equation for syntax errors (missing operators, misspelled functions, illegal *variable names*, etc.). It will then return a *menu* of all applicable *variables*, like the one in our **example**:

---

[254] Thus, the following hypothetical one-letter *variables* must not appear in any equation: ***A***, ***a***, ***B***, ***C***, ***c***, ***D***, ***e***, ***F***, ***G***, ***h***, ***I***, ***i***, ***J***, ***K***, ***k***, ***L***, ***R***, ***s***, ***T***, ***X***, ***Y***, and ***Z*** – but, e.g., ***j***, ***x***, ***y***, or ***z*** may. See the *IOI* for the *items* specified.

[255] *TNG*: Klammer vor Funktion vor Hoch vor Punkt vor Strich.

[256] *TNG*: Der eingebaute Gleichungslöser (*Solver*) Ihres *WP43* erlaubt Ihnen das Auflösen nach einer beliebigen Unbekannten bzw. das Finden reeller oder komplexer Nullstellen einer beliebigen Gleichung.



Your *WP43* recognized $g_\oplus$ being defined in CONST (see pp. 293ff). It will detect other constants as well, if applicable. Now you can enter values for each known *variable* by pressing the respective *softkey*, e.g.,

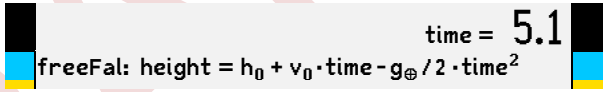
0 height **50** h_0 **15** v_0 corresponding to a target height 0, a start point on a 50 m platform, and a velocity of 15 m/s upwards at time zero. Only one unknown variable remains.

(Optionally, enter one or two initial guesses for the unknown like

5 time **10** time.)

Then press f F1 for the unknown …

time (now without any numeric input heading – the *Solver* takes what is in **X** and **Y** as initial guesses for *time*) and your *WP43* will solve the equation for this *variable* and return its value in **X** (for FIX 1 set):



i.e. 5.1 s until the ball hits the ground.[257]

If you want to recall your inputs or results later again, use RCL for the respective *variables*.

Another **example** (from the *HP-27S OM*):

Carbon-14 Dating. Wood on the outer surface of a giant sequoia tree exchanges carbon with its environment. The radioactivity of this wood is 15.3 counts per minute per gram of carbon. A sample of wood from the center of the tree yields 10.9 counts per minute per gram of carbon. The rate constant

---

[257] The equation $h_0 + v_0\, t - \frac{1}{2}\, g\, t^2 = 0$ was solved here actually. Its exact physical solution can be determined being $t = \left(v_0 + \sqrt{v_0^2 + 2\, g\, h_0}\right) / g$. Take care that $(v_0^2 + 2\, g\, h_0)$ stays positive here – thrown from the bottom of a deep pit, the ball may never reach the ground outside.

See the *IOI* for the contents of the other *stack registers*. Note you can as well use SLVQ for solving **freeFal**.

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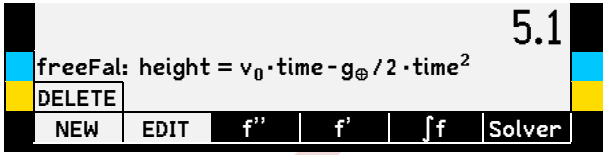
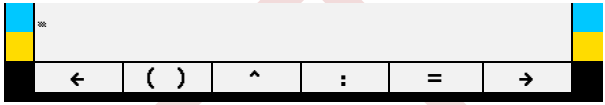
for the radioactive form of carbon, ^{14}C, is 1.20×10^{-4}. How old is the tree? What is the half-life of ^{14}C?

Solution (assuming you continue directly after solving previous problem):
Exit previous equation and enter a new one for radioactive decay:

EXIT



NEW



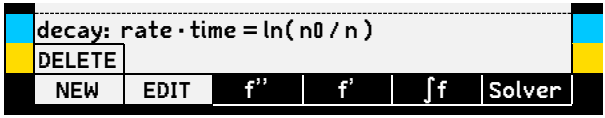
DECAY : RATE × TIME = LN( ) N0/N

Press [ ] [e^x] to enter e^( ), [ ] [√x] to get √( ), and [|x|] to insert | |. All other functions like ln, lg, sin, arccos, tanh, etc. shall be either

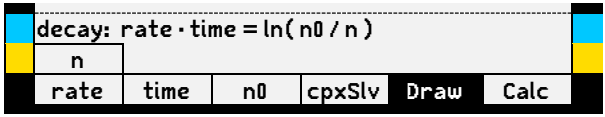
- picked from CATALOG'FCNS or
- called by typing their *names* as spelled in the *IOI*, trailed by ( ). **Take care:** e.g., SIN or LN will not be accepted as a function for obvious reasons.

Specify the argument within the parentheses (or bars) like above. This applies to dyadic operations by analogy – their arguments shall be separated by a colon (like COMB(x:y)). Leave the parentheses by → , if applicable.

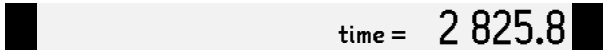
ENTER↑



Solver



1.2 [E] [+/-] **4** rate **15.3** n0 **10.9** n time



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This is the computed age of this tree in years.

Now, calculate the half-life of ^{14}C; this is the time needed for half the material present to decay; thus, enter

2 nO 1 n time

time = 5 776.2

Since the rate constant was given with 1% accuracy in the setting above, your calculatory result for the half-life can't be any better, i.e. (5776 ± 58) *years*; this is consistent with today's 'official' value of (5 730 ± 40) *years*.

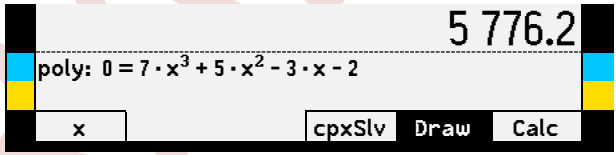
One more **example**:

Find the roots of $7x^3 + 5x^2 - 3x - 2 = 0$.[258]

Solution:

1. Enter the equation as demonstrated above.
2. Make this equation the *current equation* and press Solver.

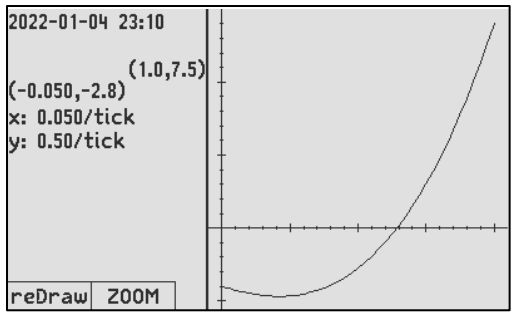
   You will see:



3. Optionally enter one or two initial guesses for the unknown like

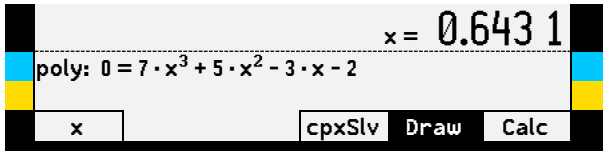
   0 x 1 x .

   Press Draw for drawing the equation between these two values. Counting the minor ticks you can estimate this function having a root at about 0.64. Let's see – press EXIT and you will get the previous screen again.

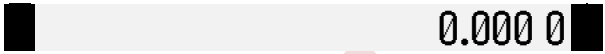


4. Set the display format to FIX **4**, then leave DISP via EXIT. Press F1 for the unknown x (now without any numeric input heading), and your *WP43* will solve the equation for this *variable* and return its value in X:[259]

[258] This problem could be solved using SLVC as well. Feel free to compare the results.



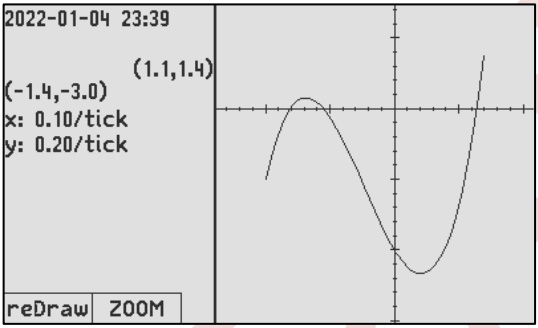
5. If you want to crosscheck you can press

   Calc returning 

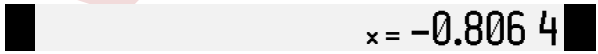
   for the function value at this location, confirming the result of the *Solver*.[260]

6. There may be up to two more roots – you guess they are on the left side:

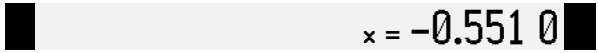
   a. Enter two new initial guesses for the unknown like **−1** **x** **.7** **x**, and check the function with **Draw**; use **ZOOM** to get a convenient tick size. You can read from the plot there will be one more root close to −0.55 and another one at −0.8. Then press **EXIT** to close the diagram.



   b. Overwrite the guesses with **−1** **x** **−.6** **x**, then press **x** once more, and you will get:



   c. Enter two new initial guesses like **−.7** **x** **−.4** **x**. Press **x** once more and you will get:



Feel free to look into *Section 5* of the *HP-27S OM* for more about interactive solving of algebraic equations.

---

[259] If you want the *Solver* looking for *complex* roots as well, press **cpxSlv** instead of the last **x** here. In any case, the *Solver* will also return values in **Y**, **Z**, and **T** – see SOLVE in the *IOI*.

[260] If you wish to compute the function at any other location – for whatever reason – you shall enter the location x using **x** before calling **Calc**.

DRAFT

Interactive Solving Expressions Stored in Programs

Instead of operating on an equation as described in previous chapters, your *WP43* can also solve an expression ***f*** stored in a program. Then,

1. Write a program for ***f***.
2. Press ADV SOLVE and select the program you want solved.
3. Enter values for all known *variables* of ***f***.
4. Let your *WP43* compute the unknown *variable*.
5. Leave the *Solver*.

We will go through this step by step:

1. Write a program for ***f***:
   - It shall begin with a global label.
   - It must define all *variables* required for calculating ***f***.
   - It shall be as efficient as possible since it is going to be executed many times.

   For interactive solving, we recommend proceeding as follows for this program: From its 2nd step on, *menu variables* shall be declared using MVAR instructions (cf. p. 236) covering all *variables* of ***f***; no trailing VARMNU here! The subsequent body of the routine shall then evaluate ***f*** recalling these *variables*. For a *Solver* routine, the original expression shall be rewritten in a way that ***f*** = **0** is fulfilled.

   Example (with FIX **1** chosen):
   Let's return to the equation we dealt with in the last two chapters:

   $$\mathrm{height} = h_0 + v_0 \cdot \mathrm{time} - g_\oplus / 2 \cdot \mathrm{time}^2$$

   This is easily rewritten: $h_0 + v_0 \cdot \mathrm{time} - g_\oplus / 2 \cdot \mathrm{time}^2 - \mathrm{height} = 0$

   So the required program might look like this:

```
LBL 'FreeF'
  MVAR 'height'
  MVAR 'h0'
  MVAR 'v0'
  MVAR 'time'
```

(no trailing VARMNU here!)

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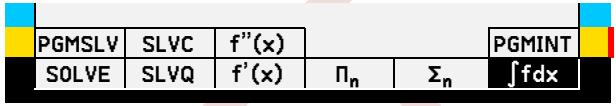
```
# g⊕
-2
/
RCL× 'time'
RCL+ 'v0'
RCL× 'time'
RCL+ 'h0'
RCL- 'height'
RTN
```

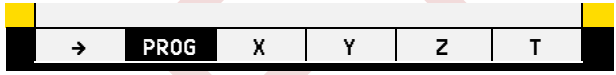
take this out of CONST.

now we have got *f*.

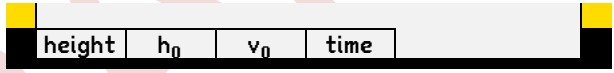
2. Press ADV :



Choose SOLVE :



Press PROG and pick the proper program for *f* (here FreeF). You will get the corresponding *menu* of *variables*, i.e. here:



3. Enter values for all known *variables* of *f* and (optionally) one or two guesses for the unknown.

   In our case, we may just take the values we know from above:

   0 height 50 h0 15 v0 5 time 10 time

4. Let your *WP43* compute the unknown *variable*.

   Press time once more but without a heading numeric entry now. Your *WP43* will return time = 5.1 as you have expected (cf. p. 264).

5. Leave the *Solver*.

   Pressing EXIT will return to the top *view* of ADV as known from step 2.

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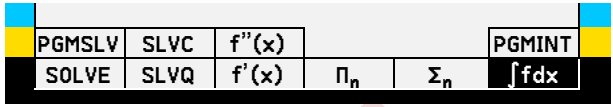
Using the *Solver* in a Program

For using the *Solver* in a programs, it has to be told what you did tell it interactively so far. Calling ADV in *PEM*, you will need PGMSLV not used before:



PGMSLV is to specify the program calculating ***f***. This step must be found in your program before SOLVE is called.

Furthermore, define the necessary *variables* in advance and load the known values, e.g., using STO. Eventually, the unknown *variable* must be specified calling SOLVE.

Example:

Let's return to the equation we dealt with in the last chapters. So the required program for *f* might look like this (like previous program but without the MVAR steps if the *variables* required were defined before):

```
LBL 'FreeFp'
  # g⊕
  -2
  /
  RCL× 'time'
  RCL+ 'v0'
  RCL× 'time'
  RCL- 'height'
  RTN
```

take this out of CONST.

now we have got ***f***.

The program one level above could contain a section looking like this:

```
...
PGMSLV 'FreeFp'
SOLVE 'time'
VIEW 'time'
...
```

specify the function to be solved.
solve for time.
display the solution.

Before starting this program (let's call it **L**), fill the *variables* of the equation to be solved, e.g., with the start values known from above:

0 STO VAR height **50** STO VAR h_0 **15** STO VAR v_0

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Optionally fill the unknown with a 1st guess, e.g., with 5 as we specified above (a 2nd guess will be taken from **X**):

5 STO VAR time

Alternatively, you can put these STO steps in **L** at any place before SOLVE is called. Or put in MVAR steps for all required *variables* as we did for **FreeF** above. Or mix both methods.

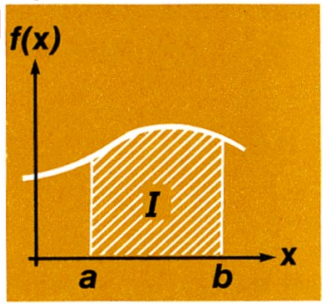
Call **L** via XEQ L and you will get time = 5.1 (cf. p. 264).

Eventually turn to *ReMA J* for more information about automatic root finding and some caveats.

Integration

Numeric Integration of Arbitrary Algebraic Expressions

Many problems in mathematics, science, and engineering require calculating the *definite integral* of a function. ... When an integral is difficult or impossible to evaluate by analytical methods, it can be calculated using numerical techniques frequently still.[261] You can compute (almost) arbitrary definite integrals numerically using the command ∫ on your *WP43*.



Example:

Certain problems in physics and engineering require calculating *Bessel functions*. The *Bessel function of the 1st kind and order 0* can be expressed as

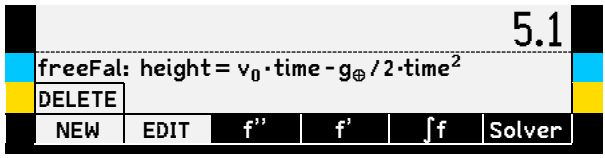
$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} cos[x\,\sin(t)]\,dt$$

Solution:

This is calculated in *radians*, thus enter MODE RAD and press EQN :

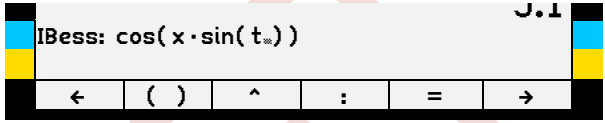
[261] Quoted from the *HP-34C OHPG*, pp. 202f (this was the 1st pocket calculator worldwide featuring numeric integration, thanks to professor *William Kahan* – see the *ReM*). If you know the analytical solution of an integral, however, you should apply this knowledge.

Above function is not in the equation list yet. So, press NEW and start entering the integrand:

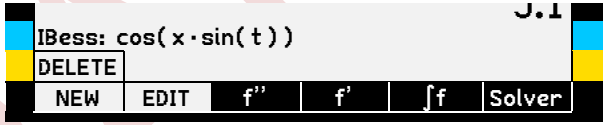


▲IB▼ESS :

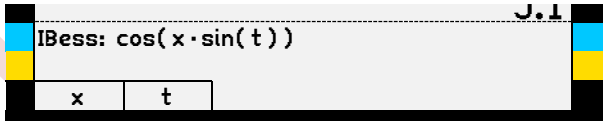
Continue with COS ( ) X ✕ SIN ( ) T



Note you did not have to step into any parentheses, your *WP43* put you at the right place automatically. Close and store this function by pressing ENTER↑. The *menu* will return to the previous one:

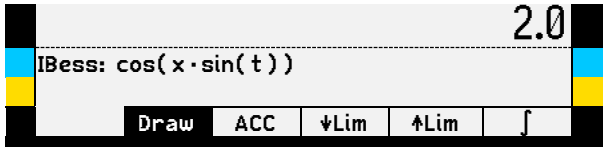


Press ∫f now. Your *WP43* will then check the current equation (cf. pp. 261ff) for syntax errors (missing operators, illegal *variable names*, misspelled functions, etc.).[262] It will then return a *menu* of all applicable *variables*:[263]



You can enter a value for each *variable* you know (i.e. for each *integration constant*) by pressing the respective *softkey* trailing the value, e.g., **2** x .[264]

Since there is only one variable left here now (***t***), this will be the *variable of integration*. Thus, the *menu* changed.



[262] You will have noticed that **IBess** is not an equation but just one side of it. To keep the system lean, such functions are listed under EQN nevertheless.

[263] Such a *menu* will not be displayed if just one *variable* is present in the integrand.

Even your *WP43* can't compute an integral exactly, it approximates its value numerically. The accuracy of this approximation depends on the accuracy of the integrand's function itself as calculated by your program. This is affected by round-off error in the calculator and also by the accuracies of the *integration constants* you specified.

ACC is a *real* that defines the relative error of the integration. With ***ACC*** = **0.001**, for instance, you can be sure that

$$\left|\frac{v_T - v_C}{v_C}\right| \leq 0.001$$

(with v_T being the true value and v_C the computed value of the integrand) at any point between ***↓Lim*** and ***↑Lim***.

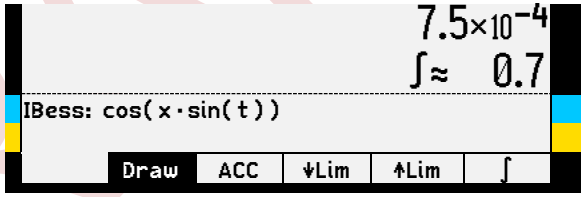
We want to see the result accurate to three decimals. Thus enter

.001 ACC for the accuracy of computation as specified above,

0 ↓Lim for the lower integration limit,

π ↑Lim for the upper integration limit,

and start integrating by pressing ∫. After a short while, your *WP43* will return:



Don't forget to divide this result[265] by π to get the correct value for $J_0(2)$:

DISP FIX 3 π / [266]

Enter other values for ***x*** and integrate again to get $J_0(x)$ at other locations.

[264] For recalling such an *integration constant*, just press RCL VAR and choose the respective *softkey*.

[265] The value ***y*** above is the estimated accuracy of the integral.

[266] ∫≈ vanishes with ▭. – We did not include the division by ***π*** in our function **IBess**. since this is evaluated many times during the integration process; thus, the fewer steps the integrand contains the faster the result can be returned.

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Interactive Integrating Expressions Stored in Programs

Instead of operating on an 'equation' as described in previous chapter, your *WP43* can also integrate an expression ***f*** stored in a program. Therefore, you shall

1. write a program for ***f***.
2. press ADV ∫fdx.
3. enter values for all known *variables* (*integration constants*) of ***f***, for ***ACC***, and for the integration limits. Select the *variable of integration*.
4. Let your *WP43* compute the definite integral specified.[268]

We will go through this step by step:

1. Write a program for ***f***:
   - It shall begin with a global label.
   - It shall define all *variables* required for calculating ***f***.
   - It shall be as efficient as possible since it is going to be executed many times.

   We recommend proceeding as follows: From the 2nd step of this program on, *menu variables* shall be declared using MVAR instructions (cf. p. 236) covering all *variables* of ***f***. The subsequent body of the routine shall then evaluate ***f*** recalling these *variables*.

   Example:
   Let's return to the integrand we dealt with in the last chapter. Then the required program for ***f*** might look like this:

```
LBL 'IBessI'
  MVAR 'x'
  MVAR 't'
  RCL 't'
  sin
```

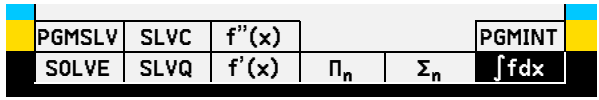
[267] You may have noticed a function $J_y(x)$ is implemented in your *WP43* directly returning the value of the *Bessel function of 1st kind and order* ***y*** at location ***x***. Feel free to compare the results.

[268] This follows closely the procedure as described for the *Solver* above.

```
RCL× 'x'
cos
RTN
```

now we have got ***f***.

2. Press ADV :



Choose ∫fdx :

Press PROG and pick the proper program for ***f*** (here IBessI). You will get the corresponding *menu* of *variables*, here:

3. Enter values for all known *variables* (*integration constants*) of ***f*** and select the *variable of integration*.

In our case, we may take the value we know from above: **2** x t. So ***t*** will be the *variable of integration*. The *menu* will change now:

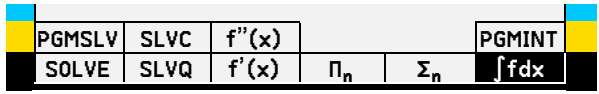
We enter (like in previous chapter) **.001** ACC **0** ↓Lim π ↑Lim

4. Let your *WP43* compute the definite integral specified.

Press ∫ to integrate with all the parameters as chosen, and your *WP43* will return ∫≈ 0.704 as you might have expected (cf. previous chapter). Divide by **π** to get the value for $J_0(2)$ as above.

Using the *Integrator* in a Program

For using the *Integrator* in programs, it has to be told what you sent to it interactively so far. Calling ADV in *PEM*, you will need PGMINT not used so far:



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It is for specifying the program calculating ***f*** before the integration is called. You shall define the necessary *variables* in advance. Use STO to load them with the known values before integrating. Then call the *menu* ∫fdx:

The integration limits as well as the requested accuracy shall be stored before integrating. Eventually, the *variable of integration* must be specified calling ∫.

Example:

Let's return to the integrand we dealt with in the last two chapters. So the required program for *f* might look like this:

```
LBL 'IBessP'
  RCL 't'
  sin
  RCL× 'x'
  cos
  RTN
```

now we have got ***f***.

The program one level above could contain a section looking like this:

```
LBL 'IntP'
  ...
  PGMINT 'IBessP'
  0
  STO '↓Lim'
  π
  STO '↑Lim'
  0.001
  STO 'ACC'
  ∫fd 't'
  VIEW X
  ...
```

specifying the function to be integrated.

integrate over time.
display the solution.

Before starting this program, load the *variables* staying constant under integration, e.g., with the start values known from above:

2 STO VAR (x).

XEQ PROG (IntP) and you will see ∫≈ 0.704 as you may have expected (cf. previous chapter). Divide by π to get $J_0(2)$ as above.

We recommend checking the integrand carefully for irregularities. Feel free

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to look up *ReMA J* for more information about automatic numeric integration, some caveats, and hints.

Differentiation

Differentiating Arbitrary Algebraic Equations

There are two commands provided returning the values of the first two derivatives of the function ***f*(*x*)** at position ***x***. This function ***f*(*x*)** can be specified in an equation.

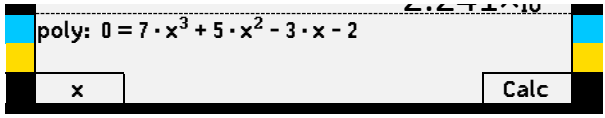
f'(x) returns the 1st derivative. For computing it, ...

1. f'(x) will first look for a user routine labeled '**δx**' (or '**δX**', '**Δx**', or '**ΔX**', in this order[269]), returning a fixed step size ***dx*** in **X**. If that routine is not defined, ***dx* = 0.1** is set for default.
2. f'(x) will fill all *stack registers* with ***x*** and call ***f*(*x*)** then. It will evaluate ***f*(*x*)** at ten points equally spaced in the interval ***x* ± 5 *dx*** (if you expect any irregularities therein, change ***dx*** to exclude them).
3. On return, the 1st derivative will be in *stack register* **X**, while **Y**, **Z**, and **T** will be clear and the position ***x*** will be in **L**.

Example (with SCI 3 set):
Take the equation **poly:** again (cf. pp. 266f). Instead of checking two function values left and right of the root you could check the slope at the root just once.

Solution:
You have got **poly:** in EQN already. For each of the three roots found, calculate the root first, then the 1st derivative of **poly:** at that point:

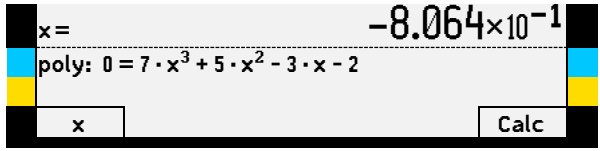
1. Press EQN, make **poly:** the *current equation*, and press **Solver**. You will see then:



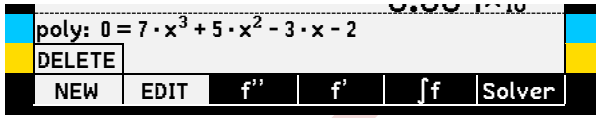
2. Find the 1st (leftmost) root as shown above:

[269] Note there is a function Δx but that doesn't interfere with this global label.

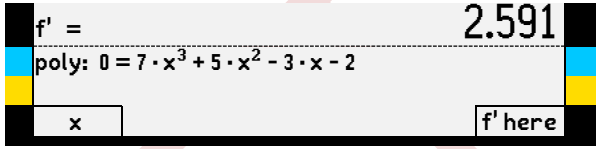
−2 x −1 x x



Press EXIT :



Press f' :



Note that f' returns the value of the 1st derivative at this location immediately since **poly:** features only one *variable*; else f' would have needed your input via the *softkeys* displayed and pressing **f' here** thereafter. So the slope of **poly:** at ***x*** = −0.8064 is 2.591. Get the slopes at the two other root positions the same way:

EXIT returns to the top *view* of EQN as above.

3. **Solver**

Find the 2nd root of **poly:** **−.7** x **0** x x

EXIT returns to the top *view* of EQN as above.

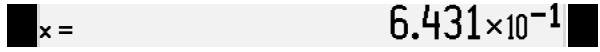
f' returns

EXIT returns to the top *view* of EQN as above.

4. **Solver**

Find the 3rd (rightmost) root of **poly:**

0 x **1** x x



EXIT returns to the top *view* of EQN as above.

f' returns

So the slope of **poly:** at ***x*** = −0.8064 is 2.591, at ***x*** = −0.5510 it is −2.134,

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and at $x = 0.6431$ it is 12.11; so the sequence of slopes is positive, negative, and positive as expected.

f"(x) works in the same way, computing the 2nd derivative of the function.

Interactive Differentiating Expressions Stored in Programs

Instead of operating on an equation as described in previous chapter, your *WP43* can also derive an expression $f(x)$ stored in a program. Then,

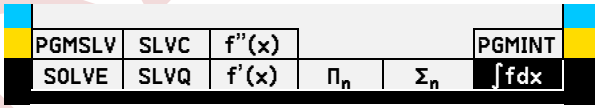
1. Write a program for $f(x)$. It must begin with a global label. For interactive derivation, we recommend proceeding as follows: [270]

   From the 2nd step of this program on, *menu variables* shall be declared using MVAR instructions (cf. p. 236) covering all *variables* of $f(x)$. The subsequent body of the routine shall then evaluate $f(x)$ recalling these *variables*.

2. Optionally, write another program labeled '**δx**' (cf. p. 277).

3. Enter values for all known *variables* (*derivation constants*) of $f(x)$. Put the location where you want to know the derivative into **X**.

4. Press ADV :



5. Press f'(x) or f"(x). You will get:

6. Choose PROG to pick the label of the program containing the function $f(x)$ (or enter its label directly as described on p. 221).

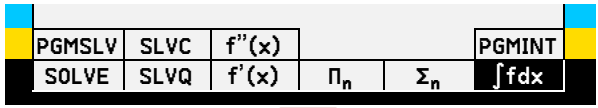
7. Let your *WP43* compute the 1st or 2nd derivative at location x.

[270] This follows loosely the procedures as described for the *Solver* and *Integrator*.

Computing Derivatives in a Program

For computing derivatives in programs, proceed as demonstrated in previous chapter. Just remember you should omit the MVAR instructions in your program calculating $f(x)$; instead, define the necessary *variables* in advance and load them with the known values using STO.

Call ADV in *PEM*:



Press **f'(x)** (or **f''(x)**): You will be asked for the label of your program calculating $f(x)$ – type it or pick it from the list as explained in steps 5 and 6 in previous chapter. Your *WP43* will compute the requested derivative for you in this program step.

Nesting Advanced Operations

You can nest SLV, ∫, f'(x), f''(x), Σ, and Π in routines to any depth as far as memory allows and your patience and power last. Solving nested advanced operations may require many, many calculations to be executed. When dealing with such problems, we recommend

a) checking the innermost routines for minimum execution time and

b) connecting your *WP43* to an *USB* outlet for supplying external power.

Example:

Light is observed to be diffracted when passing through very small holes, an effect most obvious when using laser light. Its intensity behind a circular hole is

$$I(r) = I_0 \times \left(\frac{J_1(2\pi r)}{\pi r}\right)^2 \quad \text{with} \quad J_1(x) = \frac{1}{\pi}\int_0^{\pi} cos[\,t - x\,sin(t)]\,dt$$

at distance ***r*** from the center of the beam.

$J_1(x)$ is the *Bessel function of the 1st kind and order 1* (cf. p. 271). Find the first three roots of the intensity (i.e. the radii where no light will be observed).

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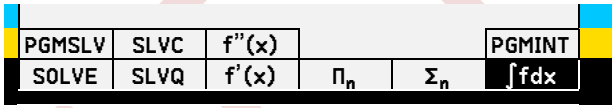
Solution:

1. Write a little program for the integrand $f(t) = cos[\, t - x\, sin(t)]$:

```
LBL 'J1'
   RCL 't'
   ENTER↑
   sin
   RCL× 'x'
   -
   cos
END
```

$sin(t)$
$x\, sin(t)$
$t - x\, sin(t)$
$cos[\, t - x\, sin(t)]$

2. Write another little program for calculating the intensity $I(r)$. Actually, to determine its roots you just need solving $J_1(2\pi r)$. (And remember ADV called in *PEM* displays:




```
LBL 'II'
   MVAR 'r'
   1.×10^-5
   STO 'ACC'
   CLX
   STO '↓LIM'
   π
   STO '↑LIM'
   RCL× 'r'
   STO+ X
   STO 'x'
   PGMINT 'J1'
   ∫fd 't'
END
```

cf. p. 273.

πr
$2\pi r$

specifies the program of the integrand (see the *menu*).
computes $\pi \times J_1(2\pi r)$ and returns it in **X**.

3. Enter DISP SCI 3
   MODE RAD
   ADV SOLVE

4. Enter PROG (II).
   You will see

5. Enter .1 r 1 r r . You will get 6.098×10^{-1} after some time.[271]

6. Enter **1** r **1.5** r r and you will get 1.117 .

7. Enter **1.5** r **2** r r and you will get 1.619 .

You will find further instructions and examples in *HP-42S RPN Scientific Programming Examples and Techniques.* Despite its title, this manual also comprises significant material about the *Solver*, *Integrator*, *matrices*, and statistics. Watch the differences between *HP-42S* and *WP43* though.

Just in case: If you find the *Solver* or *Integrator* consuming too much time for any computation whatsoever, feel free to abort it by pressing EXIT.

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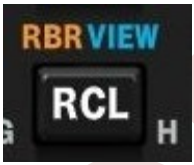
[271] The algorithm finds this root directly after some 15 s using the *WP43 Simulator* in *SPC* (cf. *ReMA E*) on a *PC* running *Windows 10* or some 12 s with FASTFN. This task will take 16 *minutes* on your *WP43* calculator in *SPC* powered by battery alone, 9.5 *minutes* with FASTFN, some 4 *minutes* with *USB* power supply but ~~FASTFN~~, and 3 *minutes* with *USB* and FASTFN. Look up FASTFN in *ReMS 1*.

SECTION 5: TWO BROWSERS, TWO APPLICATIONS, AND TWO SPECIAL MENUS

For monitoring free memory, used *registers* and *flags* easily, two *browsers* are provided (RBR and STATUS, see below). And there are two very useful applications: a TIMER (or stopwatch, see pp. 286f) and the "*Time Value of Money*" (*TVM*, see pp. 289ff). Furthermore, two special *menus* providing constants and unit conversions will ease your path in science and engineering (see pp. 293ff).

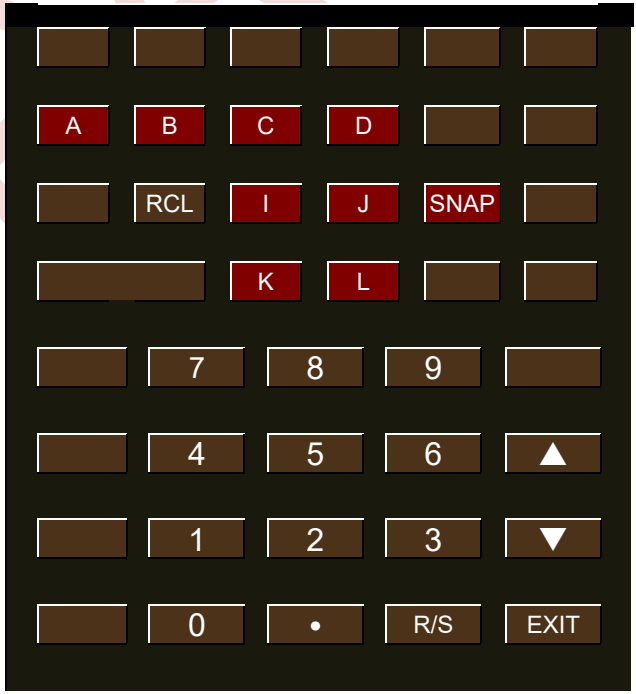
The Browsers RBR and STATUS

These two *browsers* may be called in all modes except *AIM*. They are not programmable. Some special keys and rules apply within these *browsers* as explained below.



RBR browses all currently allocated *registers* and shows their contents. See here its *virtual keyboard*.

It looks similar to the one in *TAM* (cf. pp. 60ff) but differs in detail.



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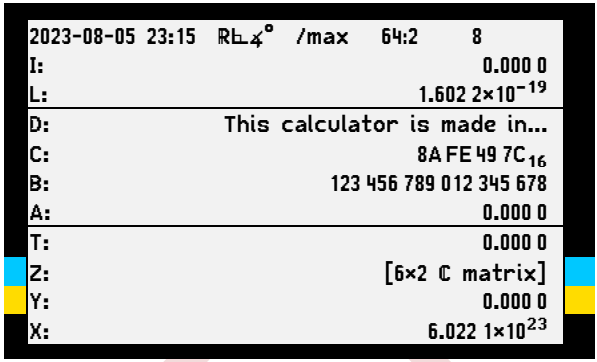
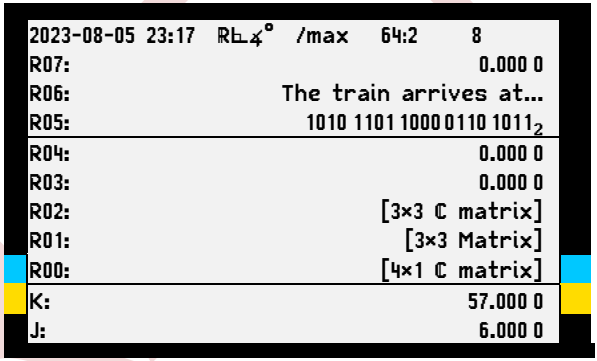
The 1st display you will see after calling RBR covers *registers* **X** … **I** (contents will deviate on your screen – individual numbers will be shown in the display format currently selected, while *text strings* may be abbreviated and *matrices* will be; fractions will be displayed with their decimal value):





▲ goes up the *stack*, continuing with the remaining lettered *registers*, then with **R00**, **R01**, etc. as shown below. For **R00** … **R99**, every fifth *register* is displayed overlined to guide the eye:

After **R99**, **X** will be shown again.

▼ browses the *registers* going down from **R99** (if starting with the screen above) to **R00**; then continues with **K**, **J**, down to **X**. After **X**, **R99** will be shown again.

. turns to *local registers* if any are allocated to the current routine, starting with **R.00**. Then, ▲ and ▼ browse *local registers* up and down until another . turns to *named variables* (incl. *system variables*), and one more . returns to the 1st screen of RBR as shown above. If no *local registers* are allocated, . directly turns to *named variables*.

R/S toggles display to show the *register* contents or the space allocated to them (see *ReMA B*).

A … D or I … L browse immediately to the corresponding *register*.

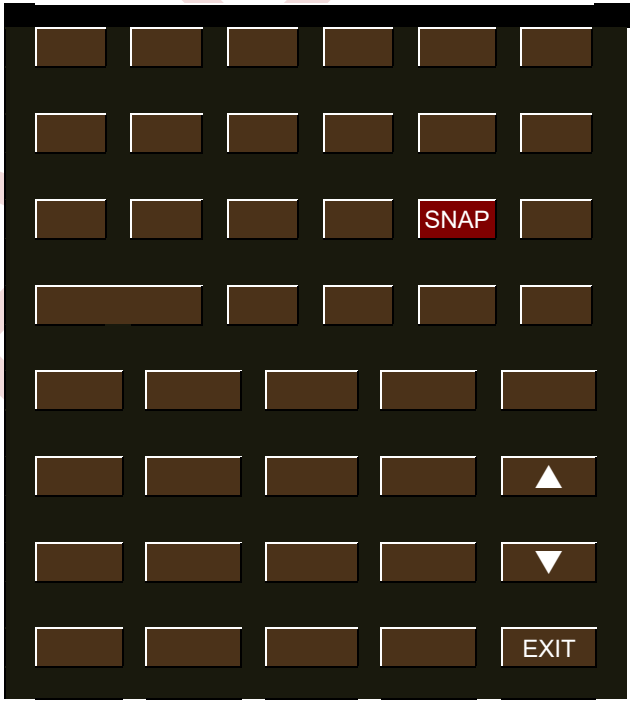
0 0 … 9 9 browse immediately to the corresponding (*global* or *local*) *register*. If no such *local register* is allocated, it returns to the 1st *global register*. If pressed while *named variables* are displayed, it jumps to the corresponding global *register*.

RCL in *RUM* recalls the content of the *register* displayed in the lowest row …
in *PEM* enters a corresponding step `RCL __` } and leaves RBR.

SNAP writes a snapshot of the present RBR display on the *FD* (cf. p. 252). Pressing 🟨 suffices for calling SNAP here.

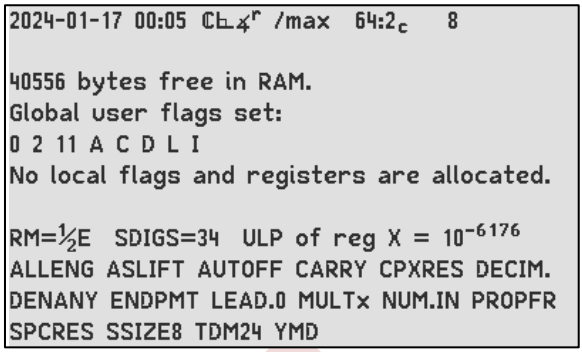
EXIT leaves the register browser RBR.

FLAG STATUS displays the amount of free memory available and the *flags* set.[272] See here its *virtual keyboard*. Local *user flags* will only be displayed if ≥ 1 *local registers* are allocated.



[272] … inspired by STATUS on *HP-16C* and *WP 34S*. – Note the font browser FBR shares the same virtual keyboard.

Some global settings and all *system flags* set are displayed in the bottom rows



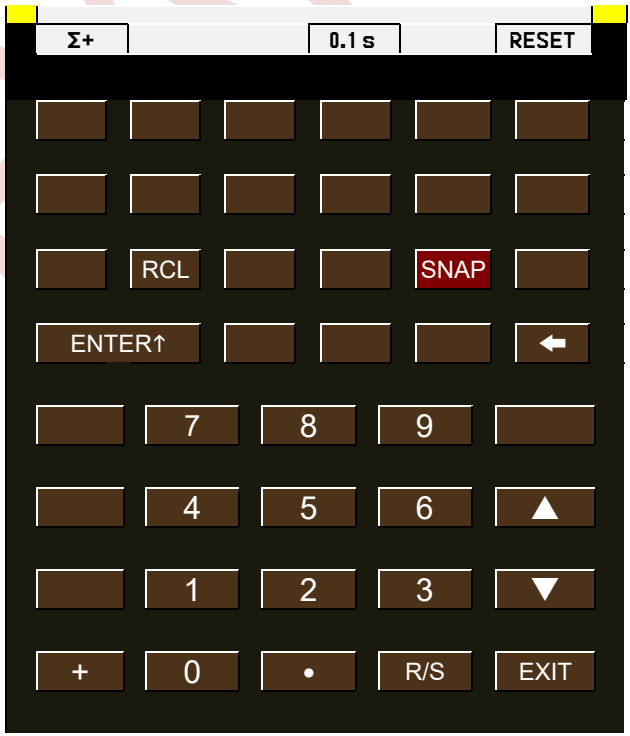
▲ and ▼ toggle between *views* if more *flags* are set.

SNAP writes a snapshot of the present STATUS display on the *FD* (cf. p. 252). Pressing 🟨 suffices for calling SNAP here.

EXIT leaves STATUS.

The Timer Application

Your *WP43* provides a timer, a.k.a. stopwatch. See its *virtual keyboard* here.



Press ⏱ to start it.

Then the top numeric row will be replaced by 0:00:00.0 [00], unless the timer was running before already (then the accumulated run time will replace zero here).

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Thereafter, pressing …

[R/S] starts or stops the timer without changing its value. [273]

[0.1 s] toggles displaying tenths of *seconds* (*startup default* = 'display').

[RESET] or [←] resets the timer to zero without changing its status (running or stopped). It deletes the *total time* if applicable (see [.]).

This is not the global RESET command, it just shares its *label*.

[Σ+] adds the present (total) time value (converted to a *real* ***y***) and the data point number (***x*** = 1, 2, 3, …) to STATS.

[0] [0] … [9] [9] sets the *current register address* (*CRA*). Its *startup default* is 00. The *CRA* is displayed between rectangular brackets like

```
27:31:55.6 [08]
```

You shall enter both digits always.[274]

Since the *CRA* may be incremented automatically, set it so there will be sufficient unused registers following for your records.

[ENTER↑] stores the present (lap) time value in the *CRA* without changing the timer status or value, then increments the *CRA* by 1.

[.] combines [ENTER↑] and [←] in one keystroke, resetting the (lap) time. Unless already displayed, the *Total time* counted since the last explicit press of [←] or [RESET] will be shown like

```
 2:04:15ᵀ    0:02:29   [06]
```

or

```
26:02:31.7ᵀ  10:00:49.6 [11]
```

and updated from then on.

---

[273] The *WP43* timer works as in the *WP 34S* but display differs. With respect to the timer of the vintage *HP-55*, there are two differences:

1. Your *WP43* will not take ***x*** (as it is when calling TIMER) as start time. Start times are supported by RCL here instead (see next page).
2. Your *WP43* will display tenths instead of hundredth of *seconds*. Reaction times of the hardware don't allow for more precision anyway.

[274] On the *HP-55*, a single digit sufficed since only 10 *registers* were featured for this purpose. And there was no automatic *CRA* increment.

[.] allows for automatic recording lap times in a series of *registers*. Note the *total time* is volatile – it will vanish and be reset when [←] or [RESET] is explicitly pressed (cf. above).

[+] combines the functionalities of [Σ+], [ENTER↑], and [←] in one keystroke. This allows for recording lap time as well as *total time* for later offline analysis, e.g., for computing the *arithmetic mean* and *standard deviation* of lap times after leaving TIMER.

[▲] (or [▼]) increments (or decrements) the *current register address* by 1. [▼] allows for overwriting the last (lap) time stored.

[RCL] ***nn*** recalls ***r nn*** and adds it to the lap time recorded without changing the timer status. The *total time* is not affected by this addition.

[SNAP] writes a snapshot of the present display on the *FD* (cf. p. 252). Pressing [ ] suffices for calling SNAP here.

[EXIT] leaves the timer without changing its status. So the time string in top numeric row will vanish but the timer may continue incrementing in background (indicated by ⏱ in the *status bar*) until …

a) you will return to TIMER and stop it explicitly by [R/S] or ...

b) you will turn off your *WP43* manually (it will not turn off automatically [275]) or ...

c) your *WP43* will shut down due to low battery (*USB* power supply is recommended for long timing).

No other keys will work in TIMER. The time counted can be indicated up to 99:59:59.9 (more than 4 days) – then the display will overflow. This applies to the *total time* as well.

For subtracting split times you have to leave TIMER.

TIMER is not programmable.

[275] … as long as the timer is running in foreground. Running in background, however, your *WP43* may turn 'off' but will continue incrementing nevertheless! Care for reliable power supply if you are up to do longer timings than 10 *minutes*, else you may easily deplete the battery.

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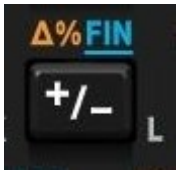
The Time Value of Money (*TVM*)



TVM is a financial application proven for decades, computing, e.g., the future value (***FV***) of

1. a repeated investment or
2. a regular down-payment for a credit

based on its present value (***PV***), its interest rate per annum in percent (***i%/a***), the required payment per period (***PMT***), the number of periods per annum (***per/a***), and the total number of payment periods (***n_{PER}***).[276] Similar financial problems will often occur also to technical people, so we included a *menu* FIN'TVM in your *WP43*.

For your information, the general formula for such problems reads

$$0 = PV + (1 + i\,p)\,\frac{PMT}{i}\,[1 - (1+i)^{-n_{PER}}] + FV\,(1+i)^{-n_{PER}}$$

with the deduced parameter $i = \frac{I\%/period}{100} = \frac{I\%/year}{100} \,/\, \frac{periods}{year}$ and the binary switch $\boldsymbol{p}$:[277] If payments occur at the…

- end of each period then $\boldsymbol{p = 0}$ (choose **End** in TVM).
- beginning of each period then $\boldsymbol{p = 1}$ (choose **Begin** in TVM).

TVM uses the cash flow convention that cash outlays (payments) are input as negative, and cash incomes are entered as positive.[278]

---

[276] *TVM* was launched with *HP*'s very 1st financial '*problem solver*', the *HP-80* of 1972, and was implemented on each and every *HP business* or *financial calculator* thereafter. An early advertising sheet promised 'you can improve and simplify your time-and-money management' applying *TVM* quickly. 'Random-entry financial keys let you key in problems in any order. And you can change any number at any time.'

All this was true for certain – remember it was a time before spreadsheet software became available. It may even hold nowadays to some extent since hardly any modern financial tool solving such problems is more compact and faster at hand than a pocket calculator featuring *TVM*. Note also the lasting success of the *HP-12C*, the 'gold standard' of financial calculators, made and sold for 43 years so far.

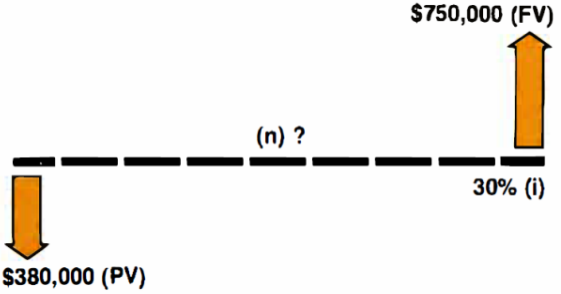
[277] For the notation of the *TVM* formula, cf. f. 53 on p. 50.

[278] This convention was introduced with the *HP-92*. The *HP-42S OM* states:

The correct *sign* (positive or negative) for *TVM* numbers is essential. The calculations will make sense only if you consistently show payments out as negative and

The present value ***PV*** always occurs at the beginning of the 1st period. It can also be an initial cash flow or a discounted value of a series of future cash flows. – The future value ***FV*** is always meant to occur at the end of the n_{PER}th period. It can also be a final cash flow or a compounded value of a series of cash flows.

Example for calculating the number of periods: [279]



A potential development site currently appraised at $380 000 appreciates at 30% per year. If this rate continues, how many years will it be before this land is worth $750 000?

Solution:

DISP FIX 2 will suffice for all financial problems here. Then all you have to do is keying in the known parameters and boundary conditions:

FIN TVM

Begin					End
RCL n_P	i%/a	per/a	PV	PMT	RCL FV
n_{PER}	i%/a	per/a	PV	PMT	FV

Begin

−380000 PV	−380 000.00	present value paid,
30 i%/a	30.00	% interest rate per year,
0 PMT	0.00	no payments, [280]
1 per/a	1.00	default.
750000 FV	750 000.00	Now, how long does it take to reach this?
n_{PER}	2.59	*years*.

payments in (receipts) as positive. Perform a calculation from the point of view of either the lender (investor) or the borrower, but not both!

[279] The examples in this chapter are quoted from the *HP-27 OH*. Enjoy the typical amounts of money and interest rates of a time long passed. And get used to an amount of 1234.56 US$ reading $1234.56 in the *USA* (even crying with dB95 … mmh … 95 dB will not help). Note not every unit is a unit there. – Experience some financial English, too.

[280] ATTENTION: Such a calculation might return wrong results for ***PMT*** ≠ 0.

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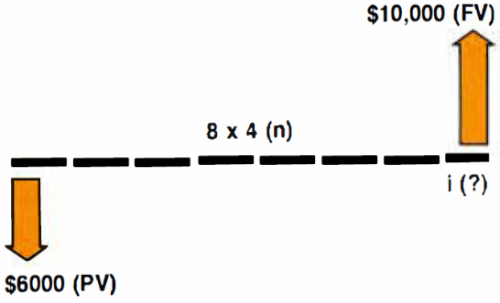
Example for finding the necessary interest rate for compounded amounts:

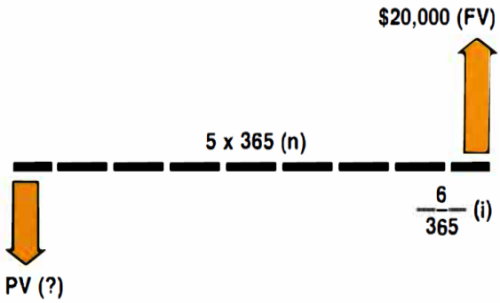


What annual interest rate must be obtained to amass $10 000 in 8 years on an investment of $6 000, with quarterly compounding? (Continue with the favorable settings of previous case.[281])

Solution:

−6000 PV	−6 000.00	present value paid,
4 per/a	4.00	quarters,
10000 FV	10 000.00	future value,
8 ENTER↑ 4 × n_{PER}	32.00	periods. Now, we need...
i%/a	6.44	% interest rate per year to reach this

Example for finding the present value of a compounded amount:



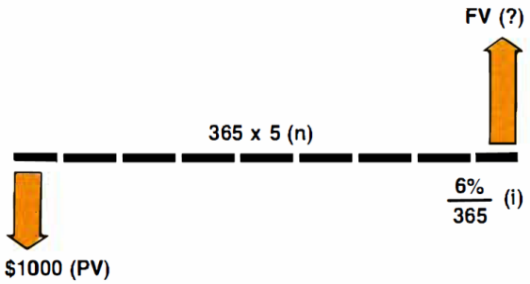
In 5 years when your son starts college, you will need $20 000. You deposit a lump sum in a certificate account that earns 6% compounded daily. How much do you need to deposit today to reach that goal? (Continue with above settings.[281])

Solution:

20000 FV	20 000.00	future value,
6 i%/a	6.00	% interest rate per year,
365 per/a	365.00	days per year,
5 ENTER↑ 365 × n_{PER}	1 825.00	periods. Now, we need...
PV	−14 816.73	to be deposited.

[281] Else you shall load all six *TVM variables* except the unknown before solving. (Background: Solving the *TVM* equation doesn't change any of the five *variables* given.)

Example for finding the future value of a compounded amount:



The local trading post manager opened up a savings operation 5 years ago, offering 6% compounded daily. Gold miner *Yellowstone Sam* deposited $1000 at that time, and now wants to know his present balance and the total accrued interest after all this time. (Continue ...)

Solution:

−1000 PV	−1 000.00	original deposit,
6 i%/a	6.00	% interest rate per year,
365 per/a	365.00	days per year,
5 × n_{PER}	1 825.00	periods. Now, Sam has...
FV	1 349.83	present balance meaning...
PV +	349.83	accrued interest.

The irrational adding of PV here is due to the cash flow convention of p. 289 (cf. the 2nd paragraph of f. 164 on p. 134).

Nominal interest rate converted to effective rate:

Example for finding the effective annual interest rate:
What is the effective annual rate of interest if the annual nominal rate of 12% is compounded quarterly? (Continue ...)

Solution:

−100 PV	−100.00	base value,
12 i%/a	12.00	% nominal rate per year,
4 per/a	4.00	quarters per year,
4 n_{PER}	4.00	compound periods in total;
FV	112.55	
PV +	12.55	% effective interest rate.

Turn to *App. 3* (pp. 339ff) for more applications of *TVM* (annuities etc.).

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The Catalog of Constants

Your *WP43* contains 79 physical, astronomical, surveying, and mathematical constants, sorted alphabetically. Pressing CONST the first time, you will get:

G	G_0	G_C	g_e	$GM_\oplus$	$g_\oplus$
c_2	e	e	F	F_α	F_δ
a	a_0	a_{Moon}	$a_\oplus$	c	c_1

Besides by browsing with ▲ and ▼, you can reach the constants most easily using the alphabetical search method demonstrated in *ReMS 2*.

The *names* of the 15 astronomical and surveying and 10 mathematical constants are printed on colored background in the table below, as well as the six fundamental physical constants. The unit of each constant is listed here as well.[282]

Name	Unit	Remarks
a	*day*	*Gregorian* year
a_0	m	*Bohr* radius
a_{Moon}	m	Semi-major axis of the Moon's orbit around the Earth.
$a_\oplus$	m	Semi-major axis of the Earth's orbit around the Sun. Within its uncertainty, $a_\oplus$ equals 1 AU (*astronomic unit*).
c	m/s	Speed of light in vacuum
c_1	$m^2 W$	1st radiation constant
c_2	$m \cdot K$	2nd radiation (*Wien*) constant

[282] Find the numeric values of the constants and their uncertainties as well as all units used here explained in *ReMS 2*. – The photograph of *M31* (the *Andromeda* galaxy) above was taken by *Boris Štromar*; compare the vintage painting overleaf.

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Name	Unit	Remarks
e	C	Elementary charge
e	1	*Euler's* e
F	C/mol	*Faraday* constant
F_α	1	*Feigenbaum's* α and δ
F_δ		
G	$\mathrm{m}^3/\mathrm{kg\ s}^2$	*Newtonian* constant of gravitation; also known as γ from other authors. See also $GM_\oplus$ below.
G_0	$1/\Omega$	Conductance quantum
G_C	1	*Catalan's* constant
g_e	1	*Landé's* electron g-factor
$GM_\oplus$	$\mathrm{m}^3/\mathrm{s}^2$	Geocentric gravitational constant (according to the Earth model *WGS84* – see *ReMS 2*)
$g_\oplus$	m/s^2	Standard Earth gravity acceleration [283]
h	J s	*Planck* constant, a.k.a. elementary quantum of action
$\hbar$	J s	So-called '*Dirac* constant', actually just $h/2\pi$
i	1	$\sqrt{-1}$
k	J/K	*Boltzmann* constant
K_J	Hz/V	*Josephson* constant
l_p	m	*Planck* length
m_e	kg	Electron mass
M_{Moon}	kg	Mass of the Earth's Moon
m_n	kg	Neutron mass
m_p	kg	Proton mass
M_p	kg	*Planck* mass
m_p/m_e	1	Proton to electron mass ratio

[283] Constants defining standard conditions are printed on orange.

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Name	Unit	Remarks
m_u	kg	Atomic mass constant
$m_u c^2$	J	Energy equivalent of atomic mass constant
m_μ	kg	Muon mass
$M_\odot$	kg	Mass of the Sun
$M_\oplus$	kg	Mass of the Earth. See also $GM_\oplus$ above for more precision
N_A	$1/\text{mol}$	*Avogadro's* number
NaN		NaN means "Not a Number" (e.g., $0/0$ or $ln(x)$ for $x < 0$ or $tan(90°)$ unless in *complex* domain). NaN covers poles as well as regions where a function result is not defined at all. Note that infinities, on the other hand, are considered numeric in your *WP43* (see the end of this table). Any such non-numeric result of a calculation will throw an error message unless SPCRES (or *flag* D) is set; NaN allows that functions written by you can return NaN instead.
p_0	Pa	Standard atmospheric pressure
R	$\text{J}/\text{mol K}$	Molar gas constant
r_e	m	Classical electron radius
R_K	Ω	*Von Klitzing* constant
R_{Moon}	m	Mean radius of the Moon
R_∞	$1/\text{m}$	*Rydberg* constant
$R_\odot$	m	Mean radius of the Sun
$R_\oplus$		Mean radius of the Earth
Sa	m	Semi-major axis … according to the Earth model *WGS84* (see *ReMS 2*)
Sb		Semi-minor axis
Se^2	1	1st eccentricity squared
Se'^2		2nd eccentricity squared
Sf^{-1}		Flattening parameter

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Name	Unit	Remarks
T_0	K	= 0°C, standard temperature
t_P	s	*Planck* time
T_P	K	*Planck* temperature
V_m	m^3/mol	Molar volume of an ideal gas at standard conditions
Z_0	Ω	Characteristic impedance of vacuum
α	1	Fine-structure constant
γ	$m^3/kg\ s^2$	*Newtonian* constant of gravitation; also known as **G** from other authors. See also $GM_\oplus$ above.
γ_{EM}	1	*Euler-Mascheroni* constant
γ_P	Hz/T	Proton gyromagnetic ratio
$\Delta\nu_{Cs}$	Hz	Hyperfine transition frequency of ^{133}Cs
ε_0	F/m	Electric constant or vacuum permittivity
λ_C	m	*Compton* wavelengths of the electron, neutron, and proton
λ_{Cn}		
λ_{Cp}		
μ_0	H/m	Magnetic constant or vacuum permeability
μ_B	J/T	*Bohr* magneton
μ_e		Electron magnetic moment
μ_e/μ_B	1	Ratio of electron magnetic moment to *Bohr's* magneton
μ_n	J/T	Neutron and proton magnetic moment
μ_p		
μ_u		Nuclear magneton
μ_μ		Muon magnetic moment
σ_B	W/m^2K^4	*Stefan-Boltzmann* constant
Φ	1	Golden ratio

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Name	Unit	Remarks
Φ_0	Wb	Magnetic flux quantum
ω	rad/s	Angular velocity of Earth according to the Earth model *WGS84* (see *ReMS 2*)
-∞	1	These infinities may result from *real number* calculations. They are counted as special numeric values (cf. NaN) and can be transferred if SPCRES (or *flag* D) is set, else an error message will be thrown.
∞		

Employ these constants for further useful equivalences, e.g.:

- express *joules* in *electron-volts* ($1\ \mathrm{eV} \approx 1.602 \times 10^{-19}\ \mathrm{A\,s\,V} \Rightarrow 1\ \mathrm{J} \approx 6.24 \times 10^{18}\ \mathrm{eV} = 6.24 \times 10^{6}\ \mathrm{TeV}$),
- calculate the wavelength from the frequency of electromagnetic radiation via $\lambda = c/\nu$ (so 100 THz correspond to 3.0 μm),
- determine the energy of electromagnetic radiation from its frequency via $E = h\nu$ (thus, $1\ \mathrm{THz} \times h = 6.63 \times 10^{-22}\ \mathrm{J} = 4.14\ \mathrm{meV}$). $1\ \mathrm{eV}$ corresponds to $242\ \mathrm{THz}$ (i.e. $\nu^{-1} = 4.1\ \mathrm{fs}$, $\lambda = 1.2\ \mu\mathrm{m}$).

Another **example**:
Remember *Albert Einstein's* most popular formula $E = m\,c^2$? So if you want to see the energy equivalent (in *electron-volts*) of one of the small particle masses given in *kilograms* above, multiply its mass by

$$c^2/e \approx 5.610 \times 10^{35}\ \mathrm{m^2/A\,s^3}$$

and you are done: $m_e\ c^2/e = 510\,999\ \mathrm{eV} \approx 511\ \mathrm{keV}$, for instance, m_p corresponds to 938 MeV, etc.

One final **example**:

Assume American advanced scientists will succeed in producing a tiny bit of anti-matter in one of their high-tech laboratories one day – let's say 0.1 μg of anti-hydrogen, carefully stored isolated in ultra-high vacuum (*UHV*).

Although in future, most probably *US* power transmission lines will still look like they do today since this is a well-tried American standard.[284] Thus, under slightly extreme weather conditions, an accidental blackout may easily

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happen for some days[285] – the electric vacuum pumps will run down, and a subsequent vacuum breakdown will let atmospheric gas leak into the shiny vacuum vessel where it will interact with the precious anti-matter and annihilate immediately. How much energy is going to be released then?

Solution:

You only need the same tiny amount of (common) matter, so $0.2\,\mu g$ will annihilate in total within the vessel. $1\,\mu g = 10^{-6}\,g = 10^{-9}\,kg$. Thus enter:

DISP ENG 3	
.2 E +/- 9	0.2×10^{-9}
CONST c	299.8×10^{6}
ENTER↑ ×	89.88×10^{15}
×	17.98×10^{6}

… resulting in 18 MJ set free. The odds are frightening high this lab will need no cleaning in next weeks.[286]

On the other hand, $0.1\,\mu g$ of anti-matter require, e.g., $N_A/10^7$ atoms of anti-hydrogen (with N_A being *Avogadro's* number). This means 6×10^{16} atoms or 3×10^{16} molecules of this gas (i.e. 30 000 million millions of anti-hydrogen molecules). Luckily, this amount is far from being produced in any lab for the time being.[287]

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[284] This kind of open cable routing can be observed in India in the extreme.

[285] Think of a thunderstorm, blizzard, hurricane or alike, maybe even fostered by anthropogenic climatic change. Though don't be afraid, this is all fake news created by insane minds according to the greatest president of that blessed nation (feel free to swap adjectives).

[286] For comparison, 1 kg of *TNT* releases 4.6 MJ. So 18 MJ will cause a great blast.

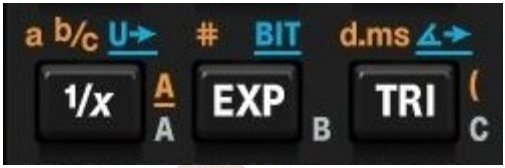
[287] … and proper *UHV* vessels show very low leak rates as well, so the annihilation energy may be released in little quantums (you can actually determine their size) over a longer time interval – power supply may be reestablished in time and vacuum pumps operating again.

For crucial applications, however, uninterruptible power sources based on batteries and/or independent generators are sincerely recommended to be installed locally wherever supplies are threatened by the actual state of public infrastructure being significantly less than great.

Furthermore, be aware that ordering antipasti and pasta together in an Italian ristorante is strictly at your own risk. You have been warned.

Unit Conversions

Your *WP43* features 22 angular conversions stored in ∠→ (cf. p. 140) and 142 conversions more in U→. The latter *menu* mainly provides means to convert local units to *SI* and vice versa.[288]



Also T_0 found in CONST is useful for converting *centigrade* temperatures to *kelvin* and vice versa. It is not repeated in U→ since it is only added or subtracted.

U→ is structured like a tree. Press U→ and you will see this *view*:

containing the labels of eight branches for converting units of Energy, Power, Time, Force & pressure, mass, length (x), Area, and Volume. The entire structure of U→ is shown below (with the *menu* rows printed top down following common reading habits). Some *conversion labels* require more than 6 characters due to their width – then extra high *menu* rows will be displayed:

---

[288] In U→, all conversions come as pairs. Select the proper pair, then press the softkey for the target unit. *SI* units are always on the left.

The *SI* system of coherent units of measurement is agreed on internationally and adopted by almost all countries on this planet for long. Advantage: You don't need any conversions while working within *SI*. Thus, most of the material appearing in U→ will look quirky to obsolete for the overwhelming majority of mankind.

Old units die hard, however, in some corners of this world (English is spoken in all of those). Thus, U→ will also help when you get caught in a time loop and happen to be thrown back into such an obstinate environment. U→ may also give you a slight idea of the mess we had in the world of measuring before going metric following the French Revolution over 220 years ago.

For symmetry reasons, we included also 9 (refined) traditional Chinese units in U→.

Without *Imperial*, US-American, and Chinese units, U→ would feature 24 entries only.

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	F1	F2	F3	F4	F5	F6	Remarks
U→:	E:	P:	Time:	F&p:	m:	x:	*submenu* headers, units of temperature, torque, and ratios – the latter in an extra-high row
	°C ↔	°F			V:	A:	
	dB ↔	power ratio	dB ↔	field ratio	Nm ↔	lbf·ft	
E:	J ↔	cal	J ↔	Btu	J ↔	Wh	units of energy
P:	W ↔	hp_E	W ↔	hp_{UK}	W ↔	hp_M	units of power
Time:	s ↔	day			s ↔	year	units of time
F&p:	N ↔	lbf	Pa ↔	bar	Pa ↔	psi	units of force and pressure
	Pa ↔	in.Hg	Pa ↔	torr	Pa ↔	atm	
			Pa ↔	mmHg			
m:	kg ↔	lb.	kg ↔	cwt	kg ↔	oz.	units of mass[289]
	kg ↔	stone	kg ↔	short cwt	kg ↔	troy oz.	
	kg ↔	ton	kg ↔	short ton	kg ↔	carat	
	kg ↔	liǎng			kg ↔	jīn	
	g ↔	oz.	g ↔	troy oz.	g ↔	carat	kitchen units
x:	m ↔	au	m ↔	l.y.	m ↔	pc	units of length
	m ↔	mile	m ↔	nmi.	m ↔	foot	
	m ↔	inch	m ↔	point	m ↔	yard	
	m ↔	lǐ	m ↔	yǐn	m ↔	zhàng	
	m ↔	chǐ	m ↔	cùn	m ↔	fēn	
	m ↔	fathm					
	mm ↔	inch	mm ↔	point	m ↔	surv. $foot_{US}$	'kitchen' units
	km ↔	mile	km ↔	nmi.			

[289] Generally, consecutive calculations in science and engineering are executed the easiest (and safest) way using *SI* base or derived units without any prefixes.

Some *British Imperial* project team members, however, have asked for 'kitchen units' (they meant *millimeter*, *kilometer*, *gram*, *hectare*, *litre*, and *millilitre*) appearing explicitly on the *SI* side of some conversions. In consequence, now you have to watch out that you don't lose powers of 10 when using those conversions (see Example 4 below, for instance). Feel free to turn to your old cookbooks again.

	F1	F2	F3	F4	F5	F6	Remarks
A:	m^2 ↔	acre	m^2 ↔	mǔ	m^2 ↔	$acre_{US}$	units of area
	ha ↔	acre	m^2 ↔	ha	ha ↔	$acre_{US}$	'kitchen' units
V:	m^3 ↔	gal_{UK}	m^3 ↔	barrel	m^3 ↔	gal_{US}	units of volume
	m^3 ↔	$floz_{UK}$			m^3 ↔	$floz_{US}$	
	mℓ ↔	tsp_{UK}	mℓ ↔	$tbsp_{UK}$	mℓ ↔	cup_{UK}	*Imperial* kitchen units
	mℓ ↔	$floz_{UK}$	mℓ ↔	$pint_{UK}$	ℓ ↔	gal_{UK}	
	mℓ ↔	tea $spoon_I$	mℓ ↔	table $spoon_I$	mℓ ↔	cup_I	international kitchen units
	mℓ ↔	tsp_{US}	mℓ ↔	$Tbsp_{US}$	mℓ ↔	cup_{US}	*US* kitchen units
	mℓ ↔	$floz_{US}$	mℓ ↔	$pint_{US}$			
			ℓ ↔	quart	ℓ ↔	gal_{US}	

In *ReMS 2*, you will find comprehensive explanations of the units used in these conversions.

Combine conversions as you like. Units are matched carefully. ENG **2** will do for all examples following here:

Example 1:

For filling your tires with a maximum pressure of 30 psi the following will help you at gas stations in Europe and beyond:

30 U→ F&p: Pa ← psi	returns	$207.\times10^3$ Pa.
Pa → bar		2.07 bar.

Now you can set the filler and will not blow your tires.

Example 2:

Your friend tells you she has got 10 *cubic feet* of debris on her veranda after flooding (the dams in the *Mississippi* delta turned out being of less use than once thought). What does this mean in real units?

1 U→ x: m ← foot	returns	$305.\times10^{-3}$
3 EXP y^x		28.3×10^{-3}
10 ×		$283.\times10^{-3}$ m^3.

This is less than $^1/_3$ *cubic meter*. OK, some work – but manageable.

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Example 3:

A network switch is specified for 3 320 Btu/h. What?!?

3320 U→ E: J ← Btu returns 3.50×10^6 J/h.

Since $1J = c_c Wh \iff 1\,{}^{J}/_{h} = c_c W$ applies, you can use

J → Wh for converting and get 973. W, i.e. almost 1 kW.

Now you know what will be going on there.

Example 4:

In *Section 2*, there was a problem resulting in a box featuring a volume of 19 $^{11}/_{16}$ *cubic inches*. So, what does this volume mean in real units instead? And how much water can such a box contain in areas where people are condemned to deal with *British Imperial* units nowadays still?

1 U→ x: ▼ mm ← inch returns 25.4

3 EXP y^x 16.4×10^3

19 . 11 . 16 x 323.×10^3 mm^3.

Since 1 mm^3 = 10^{-3} cm^3 and 1 cm^3 = 1 ml,

1000 / returns 323. ml ≈ $^{1}/_{3}$ *litre*. And

U→ V: ▼ ml → floz$_{UK}$ returns 11.4 *fluid ounces*.

But is this true? We actually don't know, maybe the result is

RCL L ml → floz$_{US}$ 10.9 *fluid ounces* instead?

You have to ask for the target location first! Medieval times …

Though you could solve this *British Imperial* problem 7 keystrokes faster and easier as follows:

1 U→ x: m ← inch returns 25.4×10^-3

3 EXP y^x 16.4×10^-6

19 . 11 . 16 x 323.×10^-6

U→ V: m^3 → floz$_{UK}$ 11.4 *fluid ounces*.

Example 5:

A celestial object moves with a velocity of 0.1 *parsec per year*. What does this mean in standard units? What does it mean in relation to the velocity of light? And how does this translate for air pilots?

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.1 U→ x: m ← pc	returns	3.09×10^15	m.
EXIT	returns to the top *view* of U→.		
1 Time: s ← year /	returns	97.8×10^6	m/s.
CONST c /	returns	326.×10^−3	= 32.6% of *c*.

Since 1 h = 3600 s ⇔ 1 $^{m}/_{s} = 3.6\ ^{km}/_{h}$.

Thus, RCL L x		97.8×10^6	m/s.
3600 x	returns	352.×10^9	m/h.
… corresponding to U→ x: m → nmi		190.×10^6	nmi/h

or 190 *mega-knots*.[290]

Example 6:

What is the average velocity of the Earth orbiting the Sun?

One full Earth orbit is approximately $2\pi r$ with $r = a_{\oplus}$:

2 π x CONST $a_{\oplus}$ x	returns	940.×10^9	m.

It needs one entire year for running this distance. This is:

1 U→ Time: s ← year		31.6×10^6	s.
Hence, /	returns	29.8×10^3	m/s
or 3.6 x		107.×10^3	km/h.

Example 7:

What on earth means 'one *furlong per fortnight*'?

1 *furlong* equals $^{1}/_{8}$ *mile*. And 1 *fortnight* is 14 *days*. Thus,

8 1/x U→ x: ▼ m ← mile	returns	201.	m and
14 EXIT Time: s ← day	returns	1.21×10^6	s.
Hence, /	returns	166.×10^−6	m/s
or 3600 x		599.×10^−3	m/h ≈ 0.6 m/h.

Many snails are faster.

[290] Sounds like a unit created for the great *Alexander* visiting *Gordion* in Asia in 333 BC.

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Supported by your *WP43*, you will find further easy ways to produce whatever conversion you may need.

In emergencies of a particular kind,[291] it may be helpful knowing that …

- *becquerel* (Bq) equals *hertz* in your *Geiger-Müller* counter,
- *gray* (Gy) is the unit for deposited or absorbed energy (1 Gy = 1 $^{\mathrm{J}}/_{\mathrm{kg}}$; see the *ReM*),
- and *sievert* (Sv) is *gray* times a radiation dependent dose conversion factor (≥1) for the damage caused in biological material including human bodies (remember the problem on pp. 98f).

Also in this field, some outdated units may still be found in literature:

- For those admiring the very first *Nobel* laureate in physics (in 1901), *Wilhelm Conrad Röntgen*, for discovering the X-rays, the charge generated by radiation in matter was measured by the unit *roentgen* with $1\,R = 2{,}58 \cdot 10^{-4}\ ^{\mathrm{A\,s}}/_{\mathrm{kg}}$.[292]
- Pour les fidèles amis et admirateurs de Madame *Marie Skłodowska Curie* (1903 *Nobel* laureate in physics and 1911 in chemistry), there was a unit *curie* with $1\,\mathrm{Ci} = 3{,}7 \cdot 10^{10}\,\mathrm{Bq} = 3{,}7 \cdot 10^{10}\ decays/s$. You can deduct from the size of this unit that lumps of radioactive

[291] Such emergencies would never happen, of course, the responsible parties affirmed.

Those emergencies, however, did not know that – hence they simply occurred against (almost) all official experts' statements. In consequence, mankind gathered (and still gathers) experience with radioactivity.

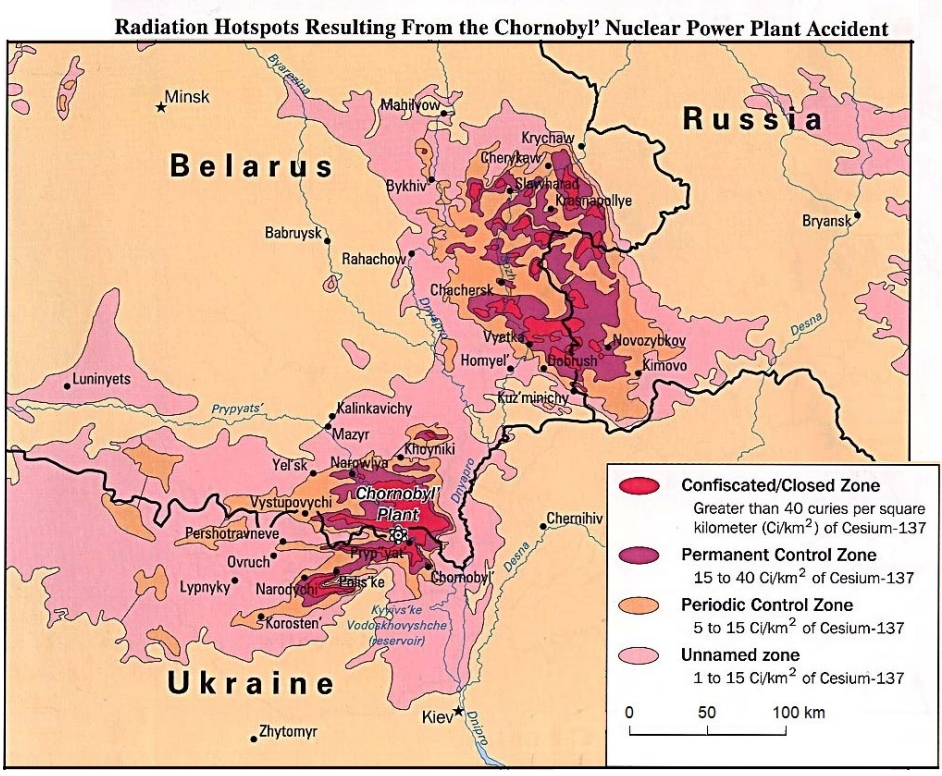
In this matter, our warmest regards go to Algeria*, Argentina, Armenia, Australia*, Austria, Belarus*, Belgium, Brazil, Bulgaria, Canada, China, Czech Republic, Finland, France, Germany, Hungary, India, Iran, Israel, Japan, Kazakhstan*, Kiribati*, Lithuania*, the Marshall Islands* (incl. Bikini and Eniwetok), Mexico, Mururoa*, the Netherlands, North and South Korea, Pakistan, the Philippines, Romania, Russia, Slovakia, Slovenia, South Africa, Spain, Sweden, Switzerland, Taiwan, Tibet*, the Ukraine*, United Arab Emirates, United Kingdom, *USA*, and Xinjiang-Uighur* (in alphabetical order).

The countries marked with a star above suffer from actions of their so-called 'motherlands' between 1945 and today – spotting those de-facto colonialists is left as an exercise for the reader. The other countries compiled controlled their industry or military in a way that activated areas on their own territory could happen (see, e.g., the two maps below, though the latter not showing an emergency but an experiment on humans).

material were ‘absolutely no problem’ for the pioneers in this field. How could they know better at that time? [293]

Example:

In the confiscated/closed zone around Chernobyl (some 4300 km^2!), ≥ 40 Ci/km^2 of ^{137}Cs were measured in 1996 (note the distance scale in the map). How many *decays/s* (a.k.a. Bq) did that mean per *square foot*? How many were these in 1986, how many are these in 2025, how many will these be in 2086 and 2186? [294]



[292] *Conrad Röntgen* (1845 – 1923) ruined his hands in those experiments – he could not know better yet. He died from carcinoma of the intestine, aged 78.

[293] Pioneering is certainly exciting but may carry risks. *Marie Curie* (1867 – 1934) died from aplastic anemia, aged 66. Seems that *Conrad*, although earlier, was more cautious or just luckier with respect to collateral health damages from his experiments.

Solution (with ENG **2**; the half-life of ^{137}Cs is 30.2 *years*, cf. p. 99):

40 ENTER↑ **3.7** E **10** × E **6** /	1.48×10^{6} Bq/m²,
1 U→ x: **m ← foot** EXP x^2	92.9×10^{-3} m²/ft²,
×	$137.\times10^{3}$ Bq/ft² in 1996,
96 ENTER↑ **86** − **30.2** STO K / 2^x ×	$173.\times10^{3}$ Bq/ft² in 1986,
1986 ENTER↑ **2025** − RCL K / 2^x ×	70.7×10^{3} Bq/ft² in 2025,
25 ENTER↑ **86** − RCL K / 2^x ×	17.4×10^{3} Bq/ft² in 2086,
100 +/− RCL K / 2^x ×	1.76×10^{3} Bq/ft² in 2186.

- A few decades ago, *rem* (i.e. *roentgen equivalent in men*[295]) measured what *sievert* does today (1 rem = 10 mSv).
- And 1 rad (*radiation absorbed dose*) equals 0.01 *gray*; this is still a lot since values greater than *millirad* are seldom found in literature (but see the map overleaf showing iodine-131 fallout exposure in America[296]):

[294] This map was found at *https://en.wikipedia.org/wiki/Chernobyl_exclusion_zone#Post-1991:_Independent_Ukraine*, taken from the *CIA Handbook of International Economic Statistics* (1996).

[295] That unit must be outdated – it is not regarded gender equitable nowadays anymore.

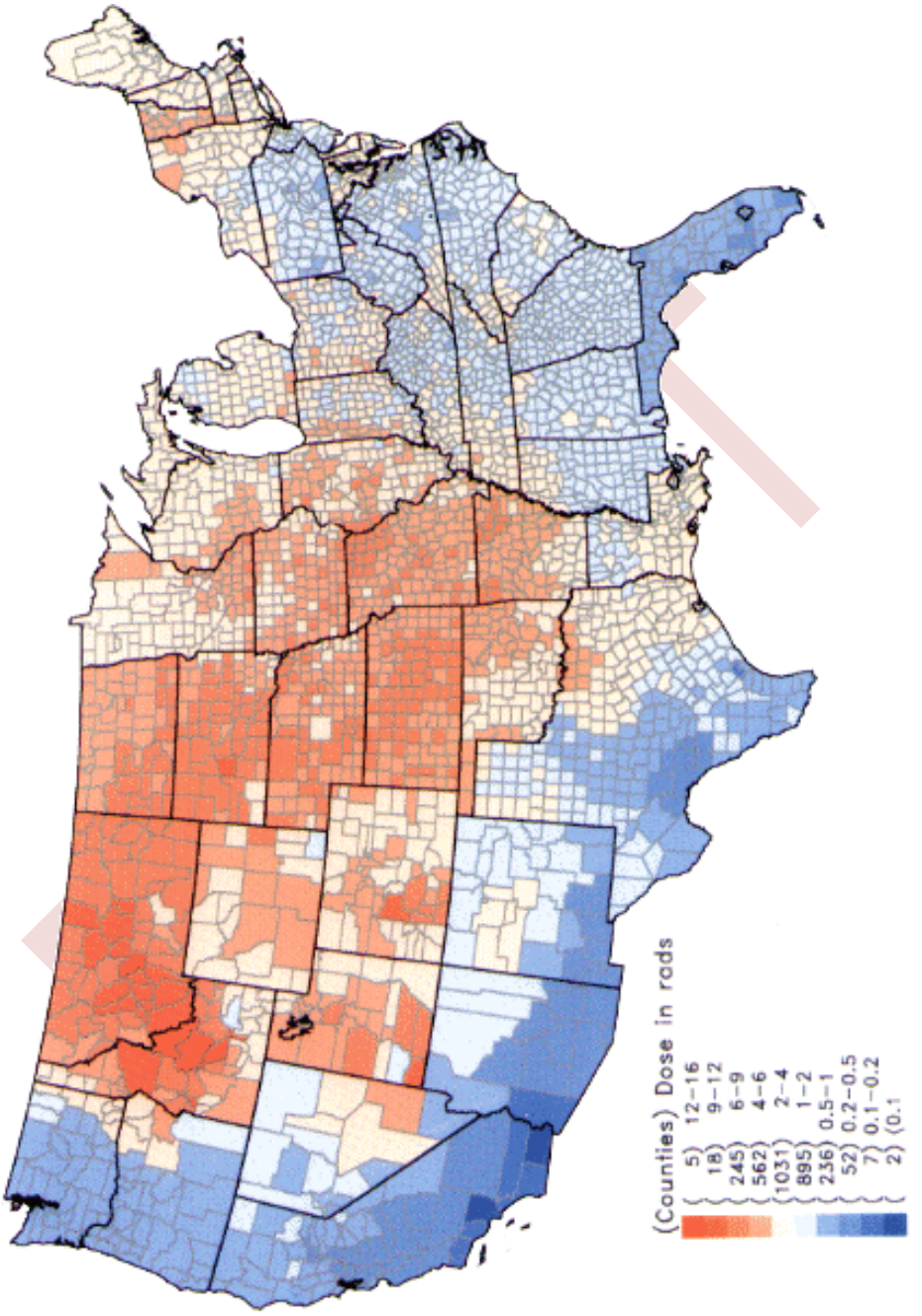
[296] Source: *Study Estimating Thyroid Doses of I-131 Received by Americans From Nevada Atmospheric Nuclear Bomb Test, National Cancer Institute* (1997). See *https://commons.wikimedia.org/w/index.php?curid=524198*. Not everything on this map is plausible though – data look inconsistent from state to state, and north of Montana seem to have lived no *Americans*. Transborder responsibility? Not here! Also note the huge time delay between the bomb test and the publication of this study, certainly not attributable to the half-life of the iodine isotope investigated (cf. p. 99).

Anyway, the *USA* should have stayed east of the Mississippi as they solemnly promised in treaties with the 'first nations' around 1800. Though actually fulfilling such treaties is another story, as well as observing human rights (obviously not only the most decorous and law-abiding Europeans had emigrated and created the new population in America). Formally apologizing two centuries later doesn't make those breaches undone. Neither does hushing them now.

Also younger history shows sad examples alike (e.g., USSR 1941, Tibet since 1950). And some alleged big powers think they can act this way nowadays still: compare

- the *Sino-British Joint Declaration* of 1984 with real life in Hongkong since 2014 and
- the *Budapest Memorandum* of 1991 with real life in the Ukraine since 2014.

PACTA SVNT SERVANDA as the Romans stated 2000 years ago already.


(Counties) Dose in rods
5) 12–16
18) 9–12
245) 6–9
562) 4–6
1031) 2–4
895) 1–2
236) 0.5–1
52) 0.2–0.5
7) 0.1–0.2
2) (0.1

SECTION 6: CREATING YOUR VERY PERSONAL *WP43*

Your *WP43* is the 1st calculator worldwide allowing for fully customizing its user interface: You may assign an arbitrary function to almost any location, unshifted or shifted, on the keyboard or in a *menu*. *User mode* will then bring your assignments to the front, so you can interact with your *WP43* via an interface you designed yourself.

Use ASSIGN for storing your personal favorite assignments: [ASN] allows for reassigning the entire keyboard except [f], [g], [α], and [F1] – [F6]. We recommend keeping also other basic functions accessible (see pp. 317f for some caveats).

In the explanations starting overleaf, …

▭ stands for the *softkey* applicable (i.e. [F1] – [F6], optionally preceded by [f] or [g]),

[***key***] represents an arbitrary labelled key of your *WP43* or a [f]- or [g]-shifted label on its keyboard,[297] and

name is the *name* of an arbitrary *item* (be it an operation, function, digit, symbol, routine (i.e. global label), *variable*, or *system flag*). A new *name* must follow the rules stated on p. 60.

Such a *name* may be…

a) keyed in spelling it via [α] … [ENTER↑] [298] or

b) picked from CATALOG or any other *menu* or

c) called by accessing the respective key/label on the keyboard.[297]

Just pressing [ENTER↑] where a *name* is expected will be interpreted as an *empty name* (echoed NULL). This will clear the assignment at the location specified returning to its *startup default.*

[297] See p. 319 for assigning [ENTER↑], [EXIT], [←], [▲], and [▼].

[298] Here, upper and lower case letters will be checked, so your *WP43* will regard **Var1** and **VAR1** as being different *name*s. Superscripts and subscripts are not discriminated from normal symbols, so **data1T** and **data$_1^T$** are seen as equal *names* (but the first may ease typing and the latter reading for you).

Assigning Your Favorite Functions

Here is how you can tailor the surface of your *WP43* according to your individual preferences:

ASN ***name*** [***key***] will assign this named *item* to [***key***] in *user mode*. It will throw an error if a *name* keyed in doesn't exist.

ASN ***name*** ▢ in particular will assign this *item* to the respective *softkey* of the *user menu* displayed at the time you press this ▢ (preceded by f or g, if applicable), replacing the label shown there before. It will throw an error if the target is not within a *user menu*.

Each user assignment will hold until it is either overwritten by a new assignment or an *empty name* is entered (see previous page).

All user assignments will be visible and accessible in *user mode* only (see pp. 317ff) – except the *items* assigned to the three *user menus* provided (MyMENU , MyPFN, and Myα, see below).

Example 1 (showing also all prefixes to be pressed):

Let's assign the statistical sample standard error to g + CC (this location is labelled $\bar{x}$ on the keyboard). There are three ways to do this as specified below (printing all the keystrokes necessary here):

a) f ASN f α ▼ S M ENTER↑ g $\bar{x}$

b) f ASN f CATALOG FCNS S M s_m g $\bar{x}$

This way will work always – even if you remember neither where the target function is stored nor how it is spelled exactly. It will be demonstrated step by step starting overleaf.

c) f ASN g STAT s_m g $\bar{x}$

On the other hand,

f ASN ENTER↑ g $\bar{x}$

will reset g-shifted CC to factory default $\bar{x}$ as explained at the bottom of previous page.

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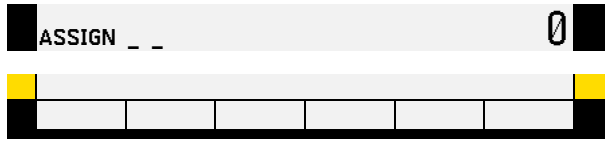
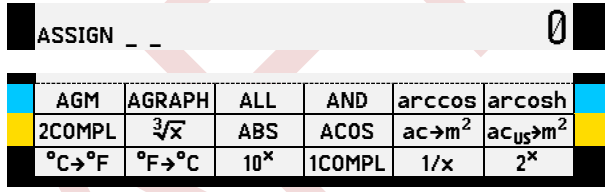
We will walk you through solution b) here, starting with a clear *stack* (press **0** FILL if necessary). Only the *menu section* and the command echo row will be shown in the following since all action will take place there:

ASN



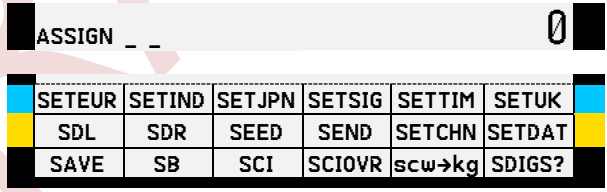
CATALOG

FCNS



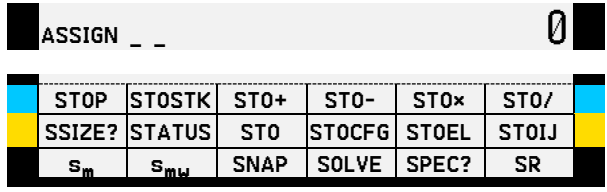
This is the top *view* of CATALOG'FCNS. Now enter the 1st letter of the requested command:

S



Quickly entering the 2nd letter helps substantially:

M (if you find you waited too long before pressing M, just wait another few seconds, then key in S M quickly here instead)



Now, press F1 for the function to be assigned:

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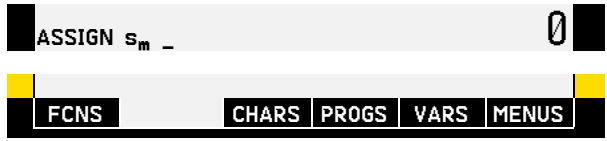
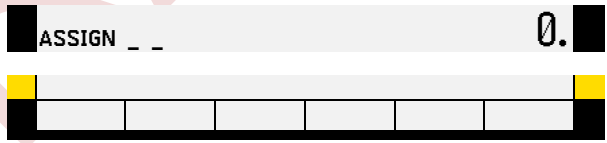
s_m



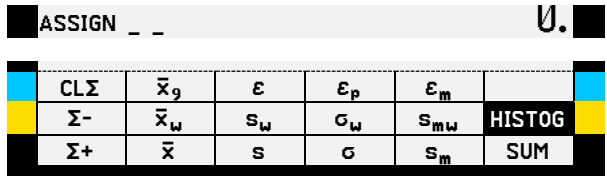
… and this assignment is done. This last *menu view* will stay on screen until it is EXITed explicitly or another *view* or *menu* is called. You can verify your assignment by setting *user mode* via USER and pressing which will be echoed s_m now – the function will always stay accessible, however, via CATALOG FCNS .

Example 2 (assuming *SDC*):
Assign the weighted arithmetic mean to the 1st key in MyMENU:

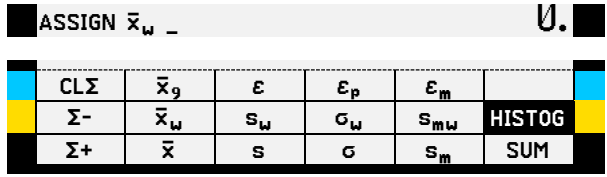
ASN



STAT

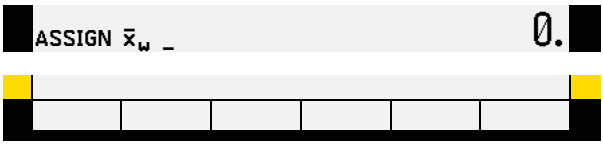


$\bar{x}_w$



DRAFT

USER



Pressing USER exited all *menus* being open and brought MyMENU on the screen (which is empty still – note the grid indicating an empty *menu* in display). Now, press

F1

… and $\bar{x}_w$ is where we want it to be.

Summarizing, ASN STAT $\bar{x}_w$ USER F1 did this assignment.

MyMENU will appear whenever all other *menus* are exited,[299] i.e. …

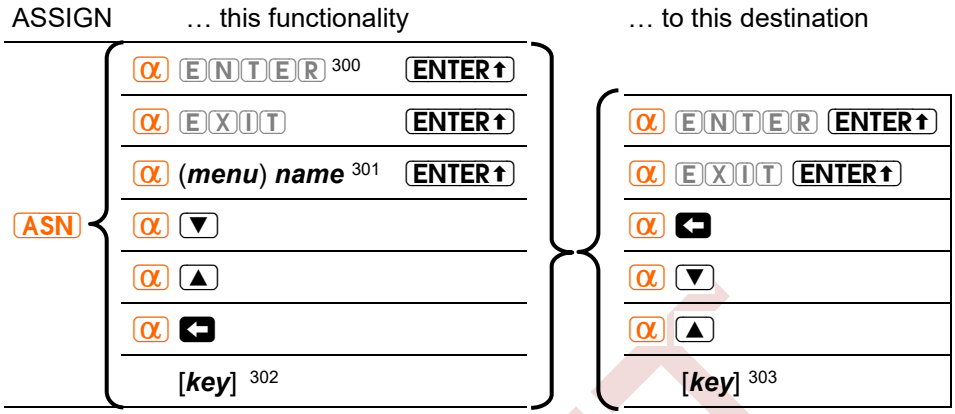
- either after you left all applicable *submenus* and pressed EXIT in the parent *menu*
- or after you pressed EXIT longer than 1 *second* (calling EXITALL).

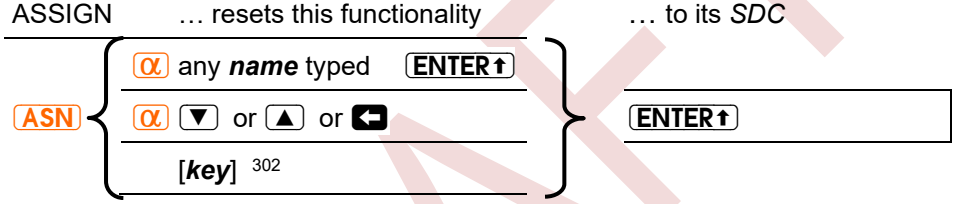
MyMENU will then stay displayed until another *menu* is called. This holds for your *WP43* being in *user mode* or not (see below).

Thus, filling MyMENU may well be the very 1st step of customizing your *WP43*. You may, for instance, put the six trigonometric functions into the unshifted row of MyMENU and will have them almost always at hand. In analogy, MyPFN will appear in *PEM*, and Myα in *AIM*.

As mentioned above, some keys and the *menus* need special treatment in assigning. Thus, the general rule for ASSIGN reads as listed in the table overleaf (the last row of either column contains the standard case dealt with so far):

[299] In *alpha input mode*, however, Myα will appear instead of MyMENU, and it will stay displayed until another *menu* is called. This also applies to *PEM* and MyPFN by analogy.





Creating Your Own Menus

ASN USER ***new_menu_name*** ENTER↑ will define a new *user menu*. ASN USER turns on *AIM* so you can immediately enter the new *menu name* (following the rules of p. 60).

Example:
To create a *menu* **Favfun** for your favorite functions, just enter:

---

300 This is to assign the functionality of the ENTER↑ key to another location. ASN α turns on *AIM*. Spelling just ENTER (without ↑) is sufficient here.

301 The *name* (or *menu name*) must be spelled.

302 This may be also 🟨 or 🟦 plus a key; or it may open a (possibly *multi-view*) *menu* containing the functionality to be assigned. In the latter case, press the *softkey* pointing to this source.

303 This may also be 🟨 or 🟦 plus a key; or it may open a *user menu* containing the destination to be assigned. In the latter case, press the *softkey* pointing to this destination.

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ASN USER F ▼ A V F U N ENTER↑

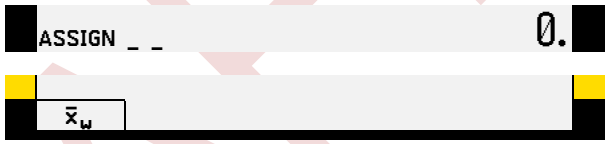
The new *menu name* will be inserted in CATALOG'MENUS (note ASN will throw an error if the *menu name* specified turns out being defined already). The new *menu* itself will be created with 18 blank entries – its size is fixed, no new *submenus* are allowed in it.[304] You may fill it now.

Example:

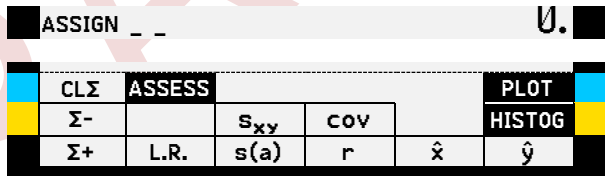
Assign the y-forecasting function to the 4th key in **Favfun** (assuming you did not define any other *menu* starting with 'Fa' before).

Also the solution of this case will be shown step by step: It starts with the last display of last paragraph since MyMENU stays on screen as long as no other *menu* is called, and we assigned one function to it just in previous chapter.

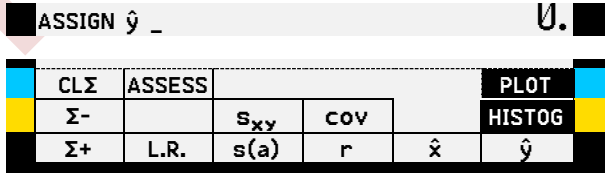
ASN



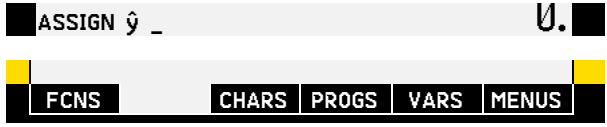
FIT



ŷ



CATALOG



[304] You must not refer to any *menu* being undefined at the time you enter its *name*. You are free to refer to any existing *menu* though.

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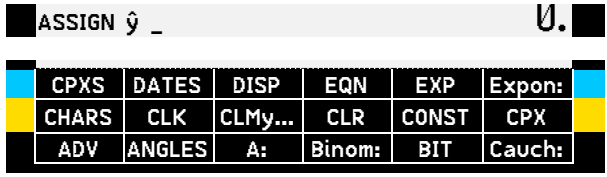
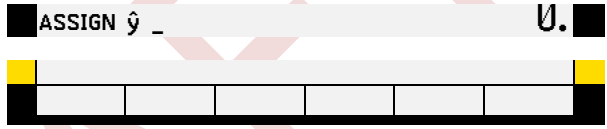
MENUS



Here you see the 1st *view* on all the *menus* defined on your *WP43*. Now, enter F and the *view* jumps to the corresponding position in this *submenu*:

Press F1 now

Favfun



Since Favfun was just created above there is nothing to be seen in the *menu section* of the display yet. Pressing F4, however, you will get

Summarizing, ASN STAT ▲ ŷ CATALOG MENUS F Favfun F4 did the job here. Favfun will remain on screen until it is EXITed explicitly or another *menu* or USER is called.

Assigning Your Favorite Characters

You must be in *alpha input mode* (*AIM*) to do the following. Then,

ASN ***character*** [***key***] will assign the character specified to [***key***]. You can pick the character to be assigned from the alpha keyboard or an arbitrary alpha *menu* as introduced on p. 206.

[***key***] may be any legal label location except f, g, α, F1 – F6, ENTER↑, or EXIT. The assignment will become valid when *AIM* is called in *user mode* or when *user mode* is called in *AIM*.

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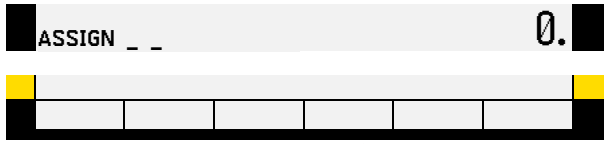
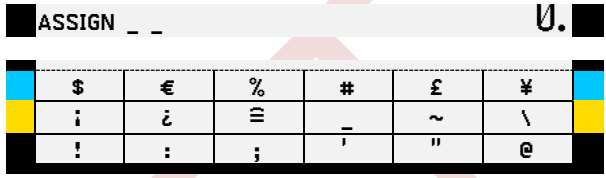
Example (showing all keys to be pressed, incl. prefixes):

Assign the *Yuan* or *Yen* symbol ¥ (contained in α•) to the 3rd key in Myα (assume *SDC* once again):

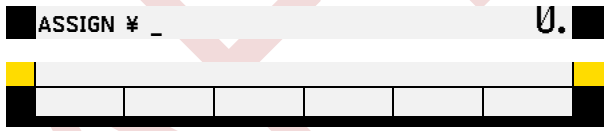
α
ASN



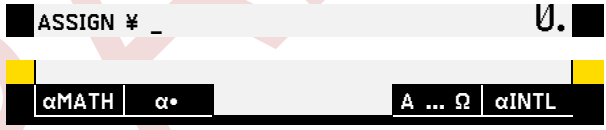
.



¥ (press F6)



USER



Pressing USER exited all *menus* being open at that time so Myα could slip on the screen with its *startup default* content.

F3

Summarizing, ASN . ¥ USER F3 did this assignment.

Purging User-Defined Items

Also user-defined *items* (*menus*, *variables*, or programs) are contained in CATALOG. Needing memory space, you can delete such *items* via DELITM easily. It lives in CLR. Calling it will show this *menu*:

CLR DELITM PROGS … ▢ allows for deleting the program (starting with the *global label*) selected; cf. CLP.

CLR DELITM VARS … ▢ allows for deleting the *variable* selected.

CLR DELITM MENUS … ▢ allows for deleting the *menu* selected.

Predefined *labels*, *variables*, or *menus* can't be deleted.

Launching Your Personal User Interface (*User Mode*)

USER toggles *user mode*. Therein, your (user) assignments become valid wherever applicable. Compare previous chapters. *User mode* gives you unexcelled freedom for creating and applying your personal calculator user interface. Enjoy and play with the opportunities you have: remember you may reassign everything except 🟧, 🟦, α, and F1 … F6 (you might call ASSIGN also in *user mode*).

We recommend leaving also ENTER↑, EXIT / ON,[305] CATALOG, and OFF untouched.

WARNING: Do not remove a necessary function from the user keyboard by overwriting it (but you may move it).

Example:

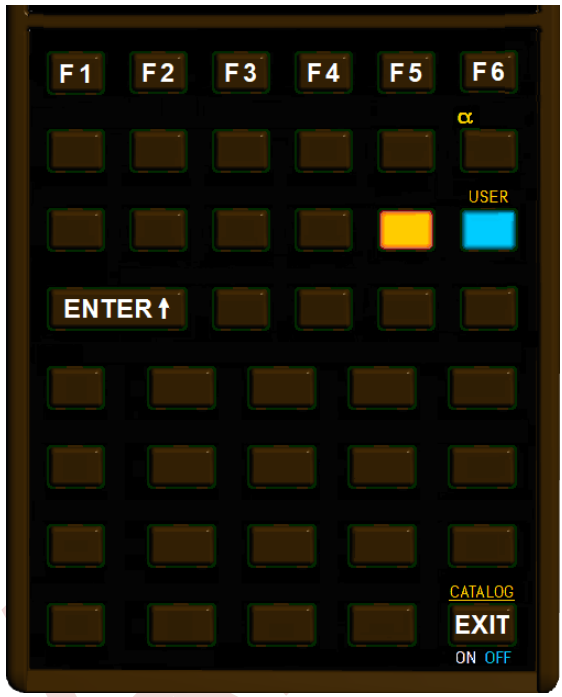
We recommend assigning USER to another location before assigning anything to 🟧-shifted 🟦 – else you will block your return from *user mode*.

In case of emergency, a *hard reset* (using the **RESET** hole) may be your only escape – returning you from *user mode*, but erasing all your precious programs, data, and settings in *RAM.* Only what you saved in *FM* (incl. the data on the *FD*) will survive. Thus, we recommend

[305] Note EXIT and ON are permanently linked.

checking the consequences meticulously prior to assigning any function.

All your assignments are strictly at your own risk.



Pressing any function key (optionally preceded by [f] or [g]) displays a preview of the operation currently assigned to this location, left in T numeric row – if you realize you have chosen the wrong key, simply keep it depressed until the display falls back to NOP after 1 *second*. This *preview and fallback* applies to all key functionalities except [0] … [9], [.], ([+/-] in numeric entry,) [E], [←], and [F1] … [F6].[306] Pressing [EXIT] will fall 'back' to EXITALL exiting all menus (consult the *IOI*). [f] and [g] are echoed but will not fall back. *Preview and fallback* are particularly helpful in *user mode*, when the function called at a particular location may differ from the label on the keyboard.

Once you have reached a steady user layout, store it in a *RoV* using STOCFG (or save it in a *state file* using SAVEST) together with the other settings mentioned on p. 86. Printing keyboard overlays for your personal layouts may pay well, especially if you reassign just functions appearing on the faceplate (so you can place all of them on one overlay).

Overlays cover the faceplate entirely and are fixed in 3 slots provided on either side of the keyboard rim (see *ReMA F* for their locations and dimensions). STOCFG, SAVEST, and overlays are especially beneficial

[306] On *HP-42S*, *preview and fallback* were absent also for PRGM, ASSIGN, STO, RCL, XEQ, SHOW, and OFF.

if you plan having further alternative layouts – you can load any of them via RCLCFG or LOAD… when you need it.[307] Think of creating a dedicated *configuration,* e.g., for working with *short integers,* bringing *Boole's* operations on the faceplate. This way you may easily make expenses for other calculators obsolete.

If, however, you should get lost in your various assignments and *configurations*, look for U top right in the *status bar* – and press USER (or wherever you have assigned this functionality meanwhile) which will return immediately to the *SDC* of your *WP43* as you know it from your very beginning. (And if you want to get rid of any outdated user layout and free the memory allocated to it, simply clear the *register* or delete the *variable* where it is stored – cf. pp. 316f.)

We sign off wishing you long lasting joy and benefit working with your very own, personalized *WP43.*

[307] We recommend keeping suitable records what you stored where. – RCLCFG will throw an error if you attempt recalling something unless a *configuration*.

APPENDIX 1: KEY RESPONSE TABLE

Here you find all direct inputs explained per keystroke, top left to bottom right of the keyboard of your *WP43*. For each key in keyboard row ***R***, its unshifted function is shown first, then its 🟨- and its 🟦-shifted function, if applicable. Most keys change functionality in *AIM* (cf. pp. 205ff), hence the 'alpha' meanings are listed thereafter with labels printed in darker fields and descriptions on light grey. Remember *AIM* is also used for entry of equations.

Generally, see the *IOI* for more about any operation mentioned below.

R	Keystrokes	Meaning	
1		Such an **unshifted *softkey*** (actually F1 - F6) calls the *item* displayed in the bottom *menu row* of the display …	… in the corresponding column 1 - 6 and executes it. It will do nothing if no *item* is displayed at this position (cf. pp. 32f).
	🟨	A **shifted *softkey*** calls the *item* displayed in the golden or blue *menu row* …	
	🟦		

2	$1/x$	Inverts the number or the *matrix* x (cf. pp. 34 and 186).
	a b/c	Enters *fraction display mode*, showing *reals* as *proper fractions*. For active *fraction display mode*, toggles between *proper* and *improper fractions* (cf. pp. 136ff).
	U→	Opens the *menu* of unit conversions (cf. pp. 299ff).
	A	Enters the Latin letter **A** or **a**.
	Ä	Opens the *catalog* of all the Latin letters provided, also accented ones.
	🟦 A	Enters the Greek letter Α or α.

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R	Keystrokes	Meaning
2	EXP	Opens the *menu* containing y^x, x^2, x^3, roots, logarithms, and hyperbolic functions (cf. p. 32).
	#	If pressed trailing integer input, defines its base. Else converts a closed x into a *short integer* of the base specified (cf. pp. 151ff), or extracts the integer or fractional part of x.
	BIT	Opens a *menu* containing *Boole's* operations (AND, OR, NOT, etc.) as well as further bit manipulating commands.
	B	Enters the Latin letter **B** or **b**.
	#	Enters the character **#**.
	▭ B	Enters the Greek letter **Β** or **β**.

2	TRI	Opens the *menu* containing trigonometric and hyperbolic functions and their inverses (cf. p. 29).
	d.ms	If pressed trailing numeric input, enters an *angle* in *degrees*, *minutes*, and *seconds* (i.e. sexagesimal notation). Else sets *angular display mode* (*ADM*) to sexagesimal angles (cf. pp. 139ff).
	∠→	Opens the *menu* of angular conversions (cf. p. 140).
	C	Enters the Latin letter **C** or **c**.
	(	Enters an opening parenthesis.
	▭ C	Enters the Greek letter **Γ** or **γ**.

2	ln	Returns the natural logarithm of x (cf. pp. 34 and 94).[308]
	.d	If pressed trailing numeric input, enters a *date* (cf. p. 199). Else either leaves fraction display mode (see a b/c above), if applicable, or converts … • an *integer* to a *real* (cf. p. 148), • a sexagesimal *angle* to a decimal number (cf. p. 142), • a sexagesimal *time* to a decimal number (cf. p. 204).

[308] I.e. either of the number x or of all *elements* of the *matrix* x.

R	Keystrokes	Meaning
	lg	Returns the (common) decadic logarithm of x (cf. ln). [308]
	D	Enters the Latin letter **D** or **d**.
	)	Enters a closing parenthesis.
	🟦 D	Enters the Greek letter Δ or δ (delta).

2	e^x	Raises **e** to the power of x (cf. pp. 34 and 94). [308]
	h.ms	If pressed trailing numeric input, enters a sexagesimal *time*. Else converts x to such a *time* (cf. pp. 199f).
	10^x	Raises **10** to the power of x (cf. pp. 34 and 94). [308]
	E	Enters the Latin letter **E** or **e**.
	🟨 e^x	Enters *e^(* in an equation or *e^* in a text string.
	🟦 E	Enters the Greek letter Ε or ε (e-psilon).

2	$\sqrt{x}$	Returns the square root of x (cf. pp. 34 and 88). [308]
	α	Sets *AIM* for entering characters (cf. pp. 205ff).
	a.FN	Opens the *menu* of functions operating on *text strings* (cf. pp. 210f).
	F	Enters the Latin letter **F** or **f**.
	🟨 $\sqrt{x}$	Enters √(in an equation or √ in a text string.
	🟦 F	Enters the Greek letter Φ or φ (phi).

3	STO	Stores x in the destination specified (cf. pp. 57ff).
	ASN	Assigns an *item* to a key or *softkey*, allowing you to create your very personal user interface (cf. pp. 308ff).
	SAVE	Saves all your data in a backup file (cf. p. 249) on the *FD* from where you may recover them entirely or partially.

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R	Keystrokes	Meaning
	G	Enters the Latin letter **G** or **g**.
	🔵 G	Enters the Greek letter Γ or γ (gamma).

3	RCL	Recalls a stored object into **X** (cf. pp. 57ff). If pressed in RBR, recalls the object at the bottom display row or enters a corresponding program step (cf. pp. 283f) and leaves RBR.
	RBR	Calls the *register browser* (cf. pp. 283f).
	VIEW	Views the destination, i.e. displays its address and contents directly below the *status bar* until next keystroke (cf. p. 62).
	H	Enters the Latin letter **H** or **h**.
	CH	Enters the Latin letters **CH** or **ch**.
	🔵 H	Enters the Greek letter **X** or χ (chi).

3	R↓	Rolls the *stack* contents one *level* down (cf. p. 42).
	R↑	Rolls the *stack* contents one *level* up
	CPX	Opens the *menu* of commands operating on *complex numbers* like CONJ, CROSS, DOT, and Re≷Im (cf. pp. 166ff).
	I	Enters the Latin letter **I** or **i**.
	↓	Makes next character a subscript (if applicable).
	🔵 I	Enters the Greek letter **I** or ι (iota).

3	CC	Closes, composes, cuts, or converts a complex x, depending on context (cf. pp. 166ff and see p. 333).
	|x|	Returns the absolute (unsigned) value of x (cf. p. 34) or the magnitude of x. [308]
	∠	Returns the phase of x. [308]

R	Keystrokes	Meaning
	J	Enters the Latin letter **J** or **j**.
	|x|	Enters $\left\|\blacksquare\right\|$ in an equation or | in a text string.
	η	Enters the Greek letter **H** or **η** (ēta).

3		*Prefix* to reach a golden function label. Pressing twice will cancel it.
	SNAP	Dumps the current screen into a file on the *FD*, i.e. takes a snapshot or screenshot. See p. 252 and the *IOI* for more.

3		*Prefix* to reach a blue function label. Pressing twice will cancel it.
	USER	Toggles *user mode* (cf. pp. 317f).

4	ENTER↑	Context sensitive key, see p. 333
	FILL	Fills all *stack registers* with x
	DROP↓	Drops x from the *stack* (cf. p. 42).

4	x≷y	Swaps the contents of **X** and **Y** (cf. p. 42).
	x≷	Swaps x and the contents of the destination specified.
	STK	Opens the *menu* of *stack* related operations (drops, swaps, and shuffles, cf. pp. 38ff).
	K	Enters the Latin letter **K** or **k**.
	x≷y	Enters the character ≷.
	K	Enters the Greek letter **K** or **κ** (kappa).

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R	Keystrokes	Meaning
4	+/−	If pressed during input of mantissa or exponent, changes its sign (cf. p. 25). Else multiplies x times −1.
	Δ%	Returns $100\ {}^{(x-y)}/_{y}$. Leaves y unchanged; cf. pp. 92f.
	FIN	Opens the *menu* of financial functions (i.e. various % functions and *TVM* – cf. pp. 289ff and 336ff).
	L	Enters the Latin letter **L** or **l**.
	+/−	Enters the character **±**.
	L	Enters the Greek letter **Λ** or **λ** (lambda).
4	E	Allows entering an exponent of 10 for convenient entry of very big or very small numbers (cf. p. 25).
	DSP	Changes the number of digits displayed while keeping the format selected (FIX, SCI, ENG, or ALL).
	DISP	Opens a *menu* of commands for numeric display formatting like FIX, SCI, ENG, and ALL (cf. pp. 86ff).
	M	Enters the Latin letter **M** or **m**.
	↑	Makes next character a superscript (if applicable).
	M	Enters the Greek letter **M** or **μ** (mȳ).
4	←	Context sensitive key, see p. 335.
	↶	Undoes the last command executed (cf. p. 55).
	CLR	Calls a *menu* containing commands for clearing (cf. pp. 70f).

R	Keystrokes	Meaning
5	/	If there is a pending question like `Are you sure?`, enters **N** for 'no'. Else divides y by x. For *matrices*, multiplies y times x^{-1}.
	||	Returns $\left(1/x + 1/y\right)^{-1}$.
	MOD	Returns y modulo x. Cf. pp. 157f.
	N	Enters the Latin letter **N** or **n**.
	f /	Enters the character /.
	g N	Enters the Greek letter N or ν (nȳ) .

5	7	Enters the digit **7**.
	EDIT	Opens x for editing.
	FIT	Opens the *menu* of operations for *2D* sample statistics: Σ+, Σ−, CLΣ, various covariances, as well as functions for curve fitting, and *MSA* (cf. pp. 107ff).
	O	Enters the Latin letter **O** or **o** (looking like Greek o-mikron).
	f 7	Enters the character **7**.
	g O	Enters the Greek letter Ω or ω (o-mega).

5	8	Enters the digit **8**.
	USB	Activates the *USB* line to your *PC*.[309]
	MODE	Opens the *menu* of operations for setting *ADM* (cf. p. 139) or rounding mode and maximum denominator (cf. pp. 136ff). Contains also the link to *DMCP*.
	P	Enters the Latin letter **P** or **p**.
	f 8	Enters the character **8**.
	g P	Enters the Greek letter Π or π (pi).

[309] Works on the calculator exclusively, thus this label is invisible on the *Simulator*.

R	Keystrokes	Meaning
5	9	Enters the digit **9**.
	LBL	Enters a label for the *current step* in *program memory*.
	RTN	Returns to the caller (cf. pp. 214ff).
	Q	Enters the Latin letter **Q** or **q**.
	f 9	Enters the character **9**.
	g Q	Works like unshifted O above.

5	XEQ	If there is an open question like `Are you sure?`, confirms it; else – if in *PEM* – inserts a call to the subroutine with the label specified; else (i.e. in *RUM*) calls the routine with the label specified and starts executing it (cf. pp. 214ff).
	GTO	Goes to the specified location in *program memory*.
	FLAG	Opens the *menu* of *flag* commands, being most useful in *PEM* (cf. pp. 214ff).
	f .	Enters a colon (**:**).
	QU	Enters the Latin letters **QU** or **qu**.

6	×	Multiplies y times x.
	x!	Returns the factorial of x (or $\Gamma(x+1)$ for non-integer x). Cf. pp. 20, 34, and 103.
	PROB	Opens the *menu* containing combinations, permutations, the *gamma function*, random number generators, and all the probability distributions supported (cf. pp. 103ff).
	R	Enters the Latin letter **R** or **r**.
	f ×	Enters the character **×** or **·**.
	g R	Enters the Greek letter **P** or **ρ** (rho).

R	Keystrokes	Meaning
6	4	Enters the digit **4**.
	Σ	Opens the *menu* of statistical sums accumulated (cf. p. 132).
	STAT	Opens the *menu* of operations for *1D* and *2D* sample statistics: Σ+, Σ−, CLΣ, various means and measures for scattering, as well as functions for histograms (cf. pp. 107ff). Contains also a link to FIT (cf. above).
	S	Enters the Latin letter **S** or **s**.
	4	Enters the character **4**.
	S	Enters the Greek letter **Σ** or **σ** (sigma).

6	5	Enters the digit **5**.
	R←	Calls →REC, converting polar coordinates ***r*** (in **X**) and ***ϑ*** (in **Y**) to rectangular coordinates ***x*** and ***y*** (cf. pp. 142ff).
	→P	Calls →POL, converting rectangular coordinates ***x*** and ***y*** to polar coordinates ***r*** (in **X**) and ***ϑ*** (in **Y**). Cf. pp. 21f and 142ff.
	T	Enters the Latin letter **T** or **t**.
	5	Enters the character **5**.
	T	Enters the Greek letter **T** or **τ** (tau).

6	6	Enters the digit **6**.
	⏱	Calls the timer application (a.k.a. stopwatch, cf. pp. 286ff).
	CLK	Opens the *menu* of *time* and *date* commands (cf. pp. 199ff).
	U	Enters the Latin letter **U** or **u**.
	6	Enters the character **6**.
	ϑ	Enters the Greek letter **Θ** or **ϑ** (theta).

R	Keystrokes	Meaning
6	▲	Context sensitive key, see p. 335.
	≡△	Moves the program pointer one step back (cf. pp. 214ff). Will repeat with $2Hz$ when pressed longer than $0.5s$.
	SF	Sets the flag specified.

7	−	Subtracts x from y.
	INTS	Opens the *menu* providing the digits **A** … **F**, operations on *integers*, and *integer sign mode* settings. This *menu* is most useful with *short integers* (cf. pp. 151, 154ff, and also BIT above).
	PART	Opens the *menu* containing FP, IP, SIGN, DECOMP, etc.
	V	Enters the Latin letter **V** or **v**.
	⇧ −	Enters a minus sign.
	≈	Opens the *menu* of mathematical symbols.

7	1	Enters the digit **1**.
	ADV	Opens the *menu* of advanced operations for solving arbitrary programmed equations, finding roots, integrating, differentiating, computing sums and products (cf. pp. 257ff).
	EQN	Opens the *menu* of all equations currently defined (cf. pp. 260ff). Allows for editing as well as for interactive solving, integrating, and differentiating.
	W	Enters the Latin letter **W** or **w**.
	⇧ 1	Enters the character **1**.
	ψ	Enters the Greek letter Ψ or ψ (psi).

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R	Keystrokes	Meaning
7	2	Enters the digit **2**.
	MATX	Opens the *menu* of *matrix* operations (incl. $[M]^{-1}$, $\|M\|$, $[M]^T$, CROSS, DOT, and the *Matrix Editor*, cf. pp. 176ff).
	X.FN	Opens the *menu* of advanced mathematical (extra) functions. See the *ReM*.
	X	Enters the Latin letter **X** or **x**.
	2	Enters the character **2**.
	X	Enters the Greek letter Ξ or ξ (xi).

7	3	If there is a pending question like `Are you sure?`, enters **Y** for 'yes'. Else enters the digit **3**.
	π	Recalls the first 34 digits of the number **π**.
	CONST	Opens the *catalog* of fundamental physical, astronomical, mathematical, and surveying constants (cf. pp. 293ff).
	Y	Enters the Latin letter **Y** or **y**.
	3	Enters the character **3**.
	Y	Enters the Greek letter Υ or υ (y-psilon).

7	▼	Context sensitive key, see p. 335.
	≡∇	Moves the program pointer one step forward (cf. pp. 214ff). Will repeat with $2Hz$ when pressed longer than $0.5s$.
	CF	Clears the flag specified.

R	Keystrokes	Meaning
8	[+]	Adds x to y.
	LOOP	Opens the *menu* containing INC, DEC, and the related loop control commands ISG, DSE, etc. (cf. pp. 230f).
	TEST	Opens the *menu* of binary tests (cf. pp. 226ff).
	[Z]	Enters the Latin letter **Z** or **z**.
	[+]	Enters the character **+**.
	[Z]	Enters the Greek letter Z or ζ (zeta) .

8	[0]	Enters the digit **0**.
	I/O	Opens the *menu* of I/O-related operations (cf. pp. 249f).
	[⎙]	Opens the *menu* of commands for printing.
	[?]	Enters a question mark.
	[0]	Enters the character **0**.
	[⎙]	Enters the printer character.

8	[.]	Usually enters a decimal radix mark (cf. p. 25). If pressed twice in numeric input, allows for entering a fraction directly (cf. pp. 73f and 136ff). In *register* or *flag* addressing, [.] heads a local address (cf. pp. 60ff).
	SHOW	Shows the number (or string) x with its maximum displayable precision (or length) until next keystroke.
	INFO	Opens the *menu* of commands returning system information (cf. p. 229), being most useful in *PEM* (cf. pp. 214ff).
	[,]	Enters a comma.
	[,]	Enters a point.
	[•]	Opens a character *menu* of punctuation marks etc.

R	Keystrokes	Meaning
8	R/S	Context sensitive key, see p. 334.
	P/R	Toggles *PEM* and *RUM* (cf. pp. 23 and 214ff).
	P.FN	Opens the *menu* of dedicated programming functions, being of most use in *PEM* (cf. pp. 214ff).
	␣	Enters a blank space character.
	!	Enters an exclamation mark.
	␣	Free xxx

8	EXIT	/ ON : Context sensitive key, see p. 334.
	CATALOG	Opens the *catalog* of everything, be it provided or created (functions, *variables*, *menus*, programs, etc.). See *ReMS 2* for its structure and contents.
	OFF	If in *PEM*, inserts the command `OFF` behind the *current step* (cf. p. 215). Else turns your *WP43* off.

Context Sensitive Keys

Seven context sensitive keys need longer explanations – find them in the table starting overleaf, sorted alphabetically. If any of these keys is pressed, your *WP43* will run top down through a sequence of key-specific tests – for the first test becoming true, your *WP43* will execute the corresponding operation and return, waiting for your next input.

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Key	Condition(s)	Meaning
CC	**X** contains an open (input) number, cf. p. 25	For ~~POLAR~~, CC closes input, checks, and saves it as *real* part of a forthcoming *complex number*, then waits for your input of its *imaginary* part. Else CC closes input, checks, and saves it as magnitude, and waits for your phase input. Cf. pp. 166ff.
	X contains a closed *complex number*, *vector*, or *matrix*	For ~~POLAR~~, CC splits ('cuts') x into its *real* and *imaginary* parts, returning the *real* part in **Y** and the *imaginary* part in **X**. Else CC splits x into its magnitude r and phase ϑ, returning r in **Y** and ϑ in **X**.
	X and **Y** contain two closed *reals*	Interprets y and x either (for POLAR set) as magnitude and phase, or (for ~~POLAR~~) as *real* and *imaginary* parts. CC combines y and x like a dyadic function, composing one *complex number* x.
	X and **Y** contain two closed *real matrices* of identical dimension	Returns one *complex matrix* x, working in analogy to previous row.
	Else	Throws an error.
ENTER↑	Waiting for parameter input	Closes pending command input and executes said command (cf. p. 66).
	Asking for confirmation	Confirms the question.
	In TIMER	Honored as described on pp. 286f.
	In numeric input or *AIM*	Closes input and pushes it either … • on the *stack* (cf. pp. 36f and 42) or • in the current matrix element (cf. p. 182) or • in the pending program step.
	In *PEM*	Inserts the command ENTER↑ behind the current program step.
	Else	Copies x into **Y**.

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Key	Condition(s)	Meaning
EXIT / ON	*WP43* turned off	Turns your *WP43* on.
	Waiting for numeric, text, or parameter input	If an input *menu* is open, closes this *menu*. Else if waiting for parameter input, cancels the pending command. Else closes input.
	TI displayed (cf. pp. 72f)	Clears *TI* returning to the calculator state as was before this *TI* was thrown.
	Asking for confirmation	Denies the question.
	In RBR, STATUS, TIMER	Leaves the application (cf. pp. 283ff).
	In a *sub-menu*	Leaves the current *sub-menu* returning to its parent *menu*.
	In a *menu* or *browser*	Leaves the current *menu* or *browser* without executing anything, returning to the status of your *WP43* as it was before.
	⌛ after special commands	Aborts the execution of a running FACTOR, NEXTP, SOLVE, or ∫.
		Stops executing the running program after completing the current program step. ❗ will be lit until next keystroke.
	In *AIM*	Closes ***x*** and leaves *alpha input mode*.
	In *PEM*	Leaves *program entry mode* like P/R.
	Else	Does nothing.
R/S	In TIMER	Starts or stops the timer without changing its value (cf. pp. 286f).
		Acts like EXIT does above.
	In *AIM*	Inserts a blank character.
	In *PEM*	Enters STOP after the current program step.
	Else	Runs the *current routine* (cf. pp. 214ff) or resumes its execution starting with the step after the *current step*.

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Key	Condition(s)	Meaning
[▲] or [▼]	After STO or RCL	Honored as described on pp. 61ff.
	In RBR, STATUS, TIMER	Honored as described on pp. 283ff.
	In ASSESS	Honored as described in the *IOI*.
	In ASSIGN	Honored as described on pp. 308ff.
	A	[▼] sets lower case and continues testing.
	α	[▲] sets upper case and continues testing.
	In EQN	[▲] goes to next or … [▼] to previous equation, if applicable.
	In a *multi-view menu*	[▲] goes to next or … [▼] to previous *view* in the current *menu*.
	In *PEM*	[▲] goes to previous or … [▼] to next program step. Will repeat with $2Hz$ when pressed longer than $0.5s$.
	In *RUM*	Browses the *current routine* with… [▲] going to previous program step or … [▼] executing the *current* step and proceeding to next step (cf. p. 225).
[←]	Open command parameter input	Deletes the last character entered. If none is left, [←] cancels the pending … …command like [EXIT].
	Open (alpha-) numeric input	… input (cf. p. 25).
	TI displayed	Acts like [EXIT] does above.
	Asking for confirmation	Denies the question.
	In TIMER	Resets the timer (cf. pp. 286f).
	In *PEM*	Deletes the *current* program step.
	Else	Calls the command CLX.

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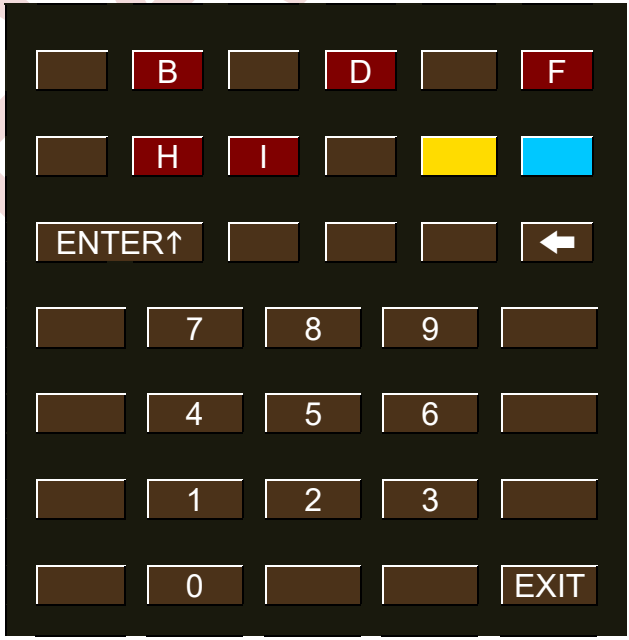
Virtual Keyboards

Virtual keyboard in *TAM* for comparisons, STO and RCL, addressing *flags*, etc. (cf. pp. 60ff).

Virtual keyboard in *TAM* for the commands # and →INT (cf. pp. 149f).



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Virtual keyboard in *TAM* for addressing *labels* (with GTO, LBL, XEQ, etc.; cf. p. 222).

Virtual keyboard for inserting characters in *AIM*, e.g., for texts and equations (cf. p. 206, especially for explanation of the three alpha menus and the Greek letters supported by your *WP43*, and 260ff).

There are further *virtual keyboard*s effective in FBR and RBR (cf. pp. 283ff), in STATUS (cf. pp. 285f), and in TIMER (cf. pp. 286ff).

APPENDIX 2: OPERATOR PRECEDENCE

Since your *WP43* will always execute just one operation at a time (cf. p. 51) it doesn't have to care for operator precedence. Hence it is your job to control the sequence of operations you present to your *WP43.* There are common rules and conventions in mathematics dealing with that – you have learned them in school. Here is just one **example** for affirmation and/or reminding:

$$1 - 2 \cdot 3^4 : 5 + \sin 2\left(6 - \sqrt[3]{7^2}\right) \cdot 8! + ln\left(-9^{2^3} \cdot 45^{(6/7)}\right)^2$$

or, written for another part of this world needing more space:

$$1 - 2 \times 3^4 \div 5 + \sin 2\left(6 - \sqrt[3]{7^2}\right) \times 8! + ln\left(-9^{2^3} \times 45^{(6/7)}\right)^2$$

This expression may be solved the following way, for instance, using your *WP43* in *SDC*:

9 ENTER↑ 2 ENTER↑ 3 EXP y^x	
y^x +/−	returns -9^{2^3}; note the arguments automatically fill in correctly.
6 ENTER↑ 7 / 45 $x\gtrless y$ y^x	calculates $45^{(6/7)}$.
× x^2 ln	solves the rightmost (4th) term.
7 x^2 $\sqrt[3]{x}$ 6 $x\gtrless y$ − 2 × TRI sin	solves the sine.
8 x! ×	solves the 3rd term.
+	adds it to the 4th.
3 ENTER↑ 4 EXP y^x 2 × 5 /	solves the 2nd term.
−	subtracts it from said sum.
1 +	returns the overall solution 3 300.988 666 858 076

The colors indicate the three *stack registers* employed for this solution (cf. pp. 41ff). Note $x\gtrless y$ is used twice to swap arguments.

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APPENDIX 3: FURTHER APPLICATIONS OF TVM

First, let's use *TVM* to solve a very old problem, before pocket calculators and cash-flows were even thought of:

Example:

The inventor of chess was asked by his king for a wish as reward. He is said to have requested one grain of rice on the 1st field, two grains on the 2nd, four on the 3rd, eight on the 4th, etc. until the 64th field. The king is reported to have laughed about that modest wish. How much rice had the king to lay out in total after all?

Solution:

FIN	TVM End		since grains are laid out at the end of each field,
1	PV	1.00	grain in the 1st field,
64	n_{PER}	64.00	fields in total
100	i%/a	100.00	% interest rate per field,
1	per/a	1.00	for obvious reasons.
0	PMT	0.00	no rice before field 64.
FV		$-1.844\,674\,407\,370\,96\times10^{19}$	grains had to be laid out. How much is that?

Nowadays, one grain of rice weighs about 65 mg. At the time of this legend, rice was smaller and lighter, let's assume 20 mg per grain. So

$$18.4 \times 10^{18} \times 20 \times 10^{-6}\ \text{kg} \approx 370 \times 10^{12}\ \text{kg} = 370\ \text{Gt}\ (gigatons),$$

meaning enough for the inventor, his wife, their kids and grandkids … including their families … and their villages … for all their lifes by far (the total amount is a major multiple of the global rice production of today).

It is told the king never again asked an inventor for a wish. Other sources report he forced the inventor to count his reward.[310]

[310] There are easier ways to calculate said total amount of rice, but this is the easiest using *TVM*. Calculating exactly, the inventor should get 18 446 744 073 709 551 615 grains of rice. Enjoy counting! Assume counting 5 grains per *second*, how long will it take to count them all?

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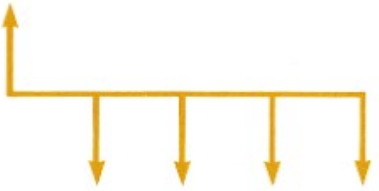
Consonant with the definition on p. 289, throughout *TVM* pictures, amounts received are represented by arrows pointing up, money laid out (paid, invested) by arrows pointing down. Various types of financial problems can be sketched like this then:[311]

Generalized Net Cash Flow Diagrams and Terminology

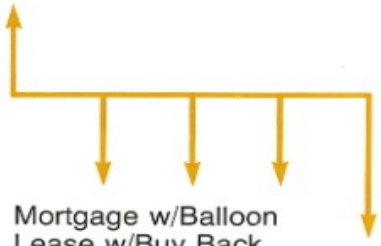
(Note that diagrams involving payments may be represented with payments at the beginning or end of the period.)

Compound Growth
Savings Account
Appreciation

Savings Plan
Sinking Fund
Pension Fund
Annuity (series of payments)



Mortgage
Lease
Direct Reduction Loan
Direct Reduction Installment
Amortization
Annuity



Mortgage w/Balloon
Lease w/Buy Back
Lease w/Residual
Annuity

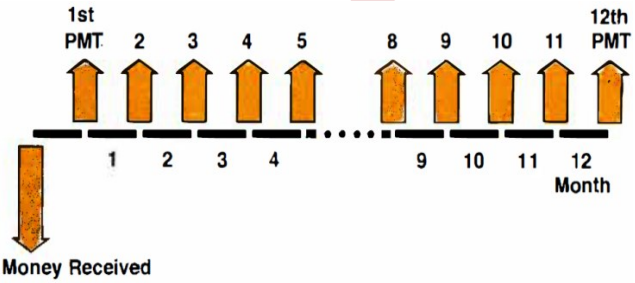
All pictures as well as the text printed in blue throughout this appendix are quoted from the *HP-27 OH*. All calculations are executed in FIX **2**. Enjoy the boundary conditions of that time gone for long – those were the days ...

[311] *TN*: You can use this picture as a tiny dictionary of some financial terms in American English. Here, the word "with" is abbreviated by "w/" although this doesn't save any space at all. Abbreviamania …

Ordinary Annuities (a.k.a. Payments in Arrears)

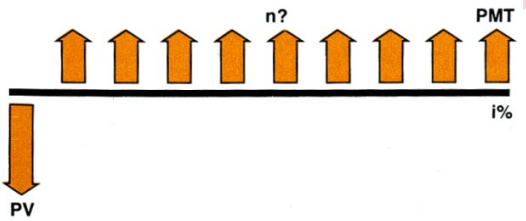
An *annuity* is a series of equal payments made at regular intervals. The time between annuity payments is called the payment interval or payment period. If your payment is due at the end of each payment period, it's called an *ordinary annuity* or *payment in arrears*. Examples of ordinary annuities are a car loan (where you drive away now and pay later) or a mortgage (where the payments start one month after you get your loan).

The time/money relationship for an ordinary annuity with monthly payments for a year would look like this → [312]



Example for finding the number of periods for an ordinary annuity:

Through an insurance fund, you have accumulated $50 000 for your retirement. How long can you withdraw $3 000 every 6 months (starting 6 months from now) if the fund earns 5% per annum compounded semiannually?



Solution:

FIN TVM End		withdrawals are due at the end of each period,
0 FV	0.00	value at the end,
−50000 PV	−50 000.00	principal (capital) for your …
3000 PMT	3 000.00	payments,
5 ENTER↑ 2 / i%/a	2.50	% semiannual interest rate,
1 per/a	1.00	since you want to know the number of semiannual withdrawals.
n_{PER}	21.83	so your savings will last for almost 11 years.

[312] *TN*: The label 'Money received' in this picture is misleading – it is the money paid to the bank or insurance – while the payments (PMT) go to you. Seems *HP* took the point of view of the bank or insurance here. Cf. f. 278 on p. 290.

Example 1 for finding the interest rate for an ordinary annuity:

What is the annual interest rate (a.k.a. *APR* for *annual percentage rate*) on a 2-year, $1 775 loan with $83.65 monthly payments? (Continue with the settings of previous case):

Solution:

12 **per/a**	12.00	months per year,
2 **x** n_{PER}	24.00	periods in total,
−1775 **PV**	−1 775.00	principal (capital),
83.65 **PMT**	83.65	payment;
i%/a	12.11	% *APR*.

Borrowers are sometimes charged fees related to the issuance of a mortgage, which effectively raises the interest rate. Given the basis of the fee charge, the true annual percentage rate may be calculated.

Example 2 for finding the interest rate for an ordinary annuity:

A borrower is charged 2 points for the issuance of his mortgage. If the mortgage amount is $50 000 for 30 years, and the interest rate is 9% per year, with monthly payments, what annual percentage rate is the borrower paying? (1 point is equal to 1 % of the mortgage amount.) (Continue with ***FV*** = 0 …)

Solution:

First, compute the payment amount which is based on $50 000

9 **i%/a**	9.00	annual interest rate,
12 **per/a**	12.00	months per year,
30 **x** n_{PER}	360.00	periods in total,
50000 **PV**	50 000.00	principal (capital);
PMT	−402.31	payment (will be reused).
RCL PV **.98** **x** **PV**	49 000.00	effective amount received,
i%/a	9.23	% effective *APR*.

What's really happening? For a mortgage with fees, the borrower is making payments on the original loan amount, which corresponds with the initial calculation of the payment amount. If you borrow $10 000, but are

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immediately charged $500 in fees, you really only receive $9 500. But, your payments are based on $10 000. With fees, then, you're really paying the same for less money, which generates the need to compute the true *APR*.

Example for finding the payment amount for an ordinary annuity:

Find the monthly payment amount on a 30-year, $52 000 mortgage at 9.75% annual interest rate. (Continue with the settings of previous case, so ***per/a*** and n_{per} are specified correctly already.)

Solution:

52000 PV	52 000.00	mortgage,
9.75 i%/a	9.75	% annual interest rate;
PMT	−446.76	monthly payment.

A common financial occurrence is an annuity that has a large payment at the end. The last payment – usually considerably larger although it could also be smaller than the others – is called a *balloon payment* or *balloon*.

By subtracting the present value of the balloon payment from the loan amount, the problem effectively becomes "What is the monthly payment on a direct reduction loan?"

Example (finding the payment for an ordinary annuity with *balloon*):

Yellowstone Sam is heading north, and will invest in an $8 000 dog sled and team. His loan specifies 60 monthly payments at 10% with a *balloon payment* in the 60th month of $3 000. What will his monthly payments be?

Solution:

12 per/a	12.00	months per year,
60 n_{PER}	60.00	payment periods in total,
10 i%/a	10.00	% annual interest rate;
−3000 FV	−3 000.00	future value of *balloon*,
0 PMT	0.00	no payments first hand,
PV	1 823.37	present value of *balloon*;
8000	8 000.00	gross value of loan amount,

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x⇄y − PV	6 176.63	net present value of loan amount less *balloon*,
0 FV PMT	−131.24	monthly payment.

Example for finding the present value of an ordinary annuity:

Yellowstone Sam decides to purchase a snowmobile. He plans to pay $80 per month for 3 years, and he's willing to pay 10% annual interest. How much can he afford to pay for the snowmobile? (Continue…)

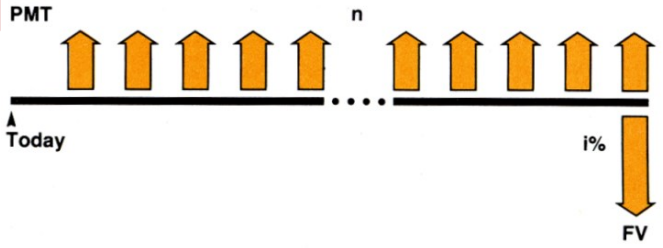
Solution:

12 per/a	12.00	months per year,
3 × n_{PER}	36.00	payment periods in total,
10 i%/a	10.00	% annual interest rate;
−80 PMT	−80.00	monthly payment,
PV	2 479.30	price he can pay for the snowmobile.

With loan calculations, you generally solve for *n*, *i*, *PMT*, or *PV*. There is another type of ordinary annuity called a "*sinking fund*", where you make payments at regular intervals into a fund to discharge a debt (for example, to pay off a bond issue at maturity). With *sinking fund* calculations, you solve for *n*, *i*, *PMT*, or *FV* (how much you will have in the fund at a future date).

Sinking fund payments start at the end of the first period, like in the sketch here.

This is different from opening a savings account with a starting deposit today. Savings are annuity due calculations and will be described later in this section.



Example for finding the future value of an ordinary annuity:

A $100 000 bond is to be discharged by the sinking fund method. If, starting 6 months from now, you deposit $3 914.75 twice a year into a sinking fund

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that pays 5% compounded semiannually, will you be able to pay off the bond in 10 years?

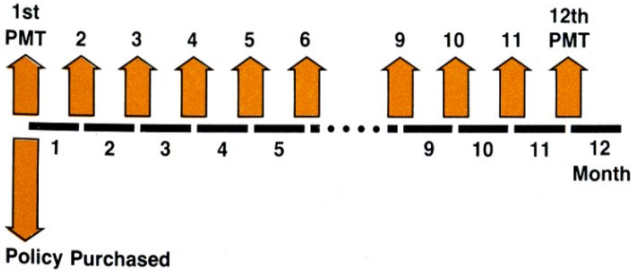
Solution:

2 per/a	2.00	halves per year,
10 ⓧ n_{PER}	20.00	payment periods in total,
5 i%/a	5.00	% annual interest rate;
0 PV	0.00	start value,
−3914.75 PMT	3 914.75	semiannual deposit,
FV	100 000.95	balance of the fund after 10 years – it will just make it!

Annuities Due (a.k.a. Payments in Advance)

With some annuities – like insurance premiums or a lease – the payment is due at the beginning of the month. This is called an *annuity due* because the payment falls at the beginning of the payment period. Other terms are *payments in advance* or *anticipated payments*.

An *annuity due* with monthly payments for a year – say, a car insurance policy[313] – looks like the sketch here.



Notice that with an annuity due, you have a payment right away at the beginning of the first interval (with an ordinary annuity, your payment is not due until the end of the first period, but you also have a payment at the end of the entire term).

[313] *TNG*: *Policy* entspricht hier einer *Police*.

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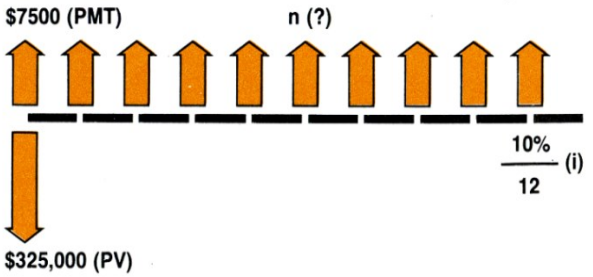
The following calculations all deal with *annuity due* problems, e.g. savings, insurance, leases, and rents.

Example 1 for finding the number of periods for an annuity due:



Given an investment possibility of $325 000 that will immediately produce rental income of $7500 per month, how long must the investment be held to yield 10% per annum?

Solution:

FIN TVM Begin		payments are due at begin of each period,
12 per/a	12.00	months per year,
10 i%/a	10.00	% annual interest rate,
−325000 PV	−325 000.00	investment;
0 FV	0.00	final balance;
7500 PMT	7500.00	monthly rental income;
n_{PER}	53.43	months.

Example 2 for finding the number of periods for an annuity due:

If you deposit $50 a month in a savings account that pays 6% interest, how long will it take to reach $1 000? (Continue with the settings…)

Solution:

6 i%/a	6.00	% annual interest rate,
0 PV	0.00	start balance,
1000 FV	1 000.00	future value;
−50 PMT	−50.00	monthly payments to the bank.
n_{PER}	19.02	months.

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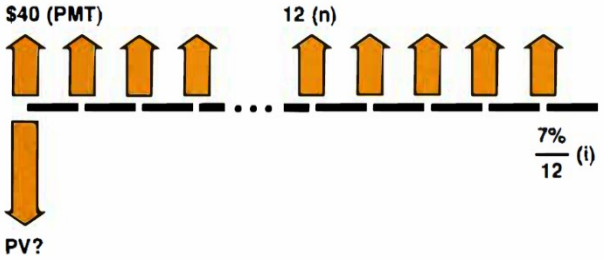
Example for finding the interest rate for an annuity due:



Equipment worth $12 000 is leased for 8 years with monthly payments in advance of $200. The equipment is assumed to have no salvage value at the end of the lease. What yield rate does this represent?

Solution:

12 per/a	12.00	months per year,
8 ⊠ n_{PER}	96.00	payment periods in total,
12000 PV	12 000.00	start value of equipment,
−200 PMT	−200.00	payments;
0 FV i%/a	13.07	% annual yield.

Example for finding the payment amount for an annuity due:

The owner of a building presently worth $70 000 intends to lease it for 20 years at the end of which time he assumes the building will be worthless (i.e., has no residual value). How much must the quarterly payments (in advance) be to achieve a 10% annual yield?

Solution:

4 per/a	4.00	quarters per year,
20 ⊠ n_{PER}	80.00	payment periods in total,
10 i%/a	10.00	% annual target yield,
−70000 PV	−70 000.00	present value of the building;
0 FV PMT	1982.27	quarterly payments.

Example for finding the present value for an annuity due:

The owner of a downtown parking lot has achieved full occupancy and a 7% annual yield by renting parking spaces for $40 per month payable in advance.

Several regular customers want to rent their spaces on an annual basis. What annual rent, also payable in advance, will maintain a 7% annual yield rate?

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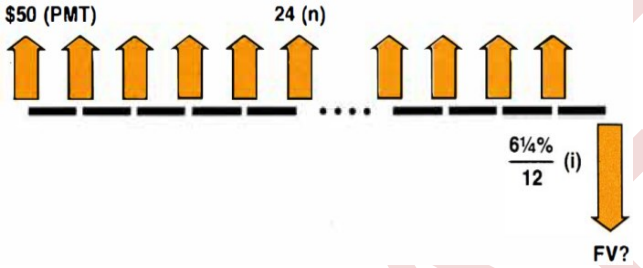
Solution:

12 per/a	12.00	months per year,
12 n_{PER}	12.00	payment periods in total,
7 i%/a	7.00	% annual target yield,
40 PMT	-40.00	monthly payments,
0 FV PV	-464.98	equivalent annual payment.

Example for finding the future value for an annuity due:



If you can afford to deposit $50 per month in an account with 6 ¼ % interest compounded monthly, how much will you have 2 years from now?

Solution: [314]

12 per/a	12.00	months per year,
2 ⓧ n_{PER}	24.00	payment periods in total,
6.1.4 i%/a	6.25	% annual target yield,
50 PMT	50.00	monthly payments;
0 PV FV	-1 281.34	balance after two years.

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[314] *TN:* The signs or arrow directions are debatable here, in my opinion. Maybe another point of view?

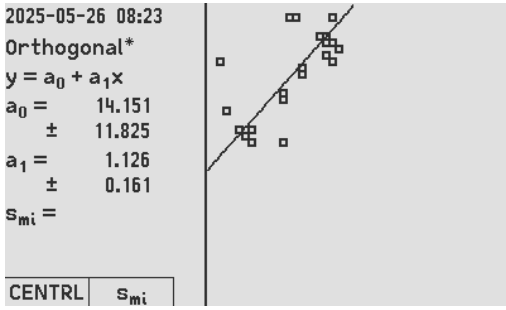
APPENDIX 4: MORE ABOUT PLOT AND 2D DATA

You are free to apply PLOT also to sets of data points not representing pairs of closely correlated measurements as described in *Section 2*. The following real-world **example** is a set of daily high and low temperatures recorded in degrees Fahrenheit for the first 25 days of May 2025 at Laredo (Tx, USA):

97	92	85	85	96	101	93	86	87	87	90	98	105
74	71	66	71	74	79	71	65	64	66	62	61	72
105	102	101	101	98	102	99	102	101				
73	77	78	79	79	77	78	78	78				

With these data keyed in, PLOT will display:

… and CENTRL will return:



Note that no s_{mi} is computed here just since this set contains less than 30 data points – your *WP43* cannot know it would make absolutely no sense here, that is your task.

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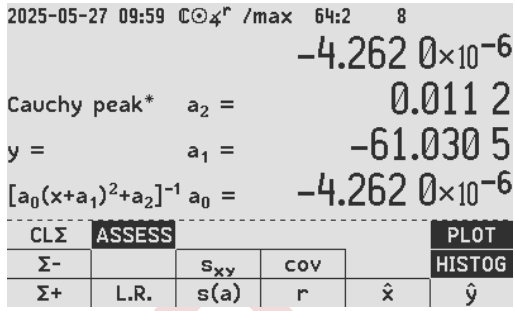
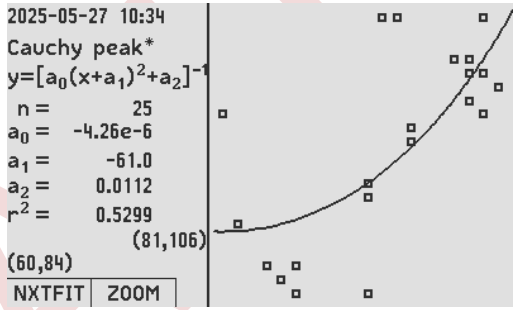
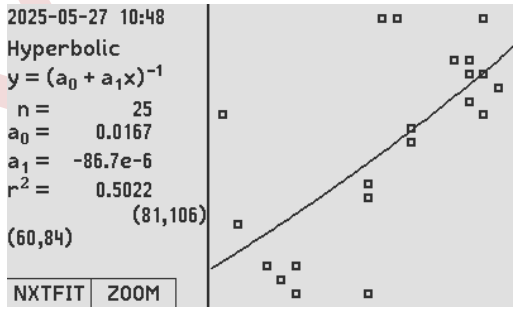
Being at it, you can as well check the correlation between highs and lows. If you allow your *WP43* to apply all fit models (BESTF **000**) and call L.R., it will return



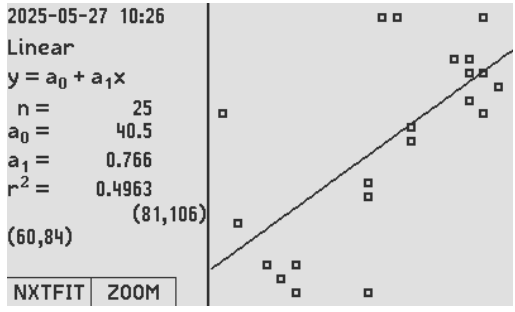
… which you will have hardly expected. Let's call ASSESS to check the fit:



This fit model doesn't make any sense for your data, however. Just browse the results for all the other models via NXTFIT or ▼ and find the hyperbolic delivering the greatest r^2 of all 2-parameter models for your data:



But its results are actually not far away from linear, so you might choose this model for your publication (although its coefficient of determination isn't really breathtaking):



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APPENDIX 5: GRAPHICS

In *Section 2* and *App. 4*, graphic output via ASSESS and PLOT was shown. Furthermore, your *WP43* features the elementary graphic commands CLLCD, AGRAPH, PIXEL, and POINT. The latter three are stored in P.FN'P.FN2; the first three are inherited from the *HP-42S*.

PIXEL sets the pixel with coordinates $x = \mathrm{IP}(x)$ and $y = \mathrm{IP}(y)$. The origin, i.e. pixel (0; 0) is located in the bottom left corner of the screen, the pixel (399; 239) top right.

CLLCD clears all pixels with $x \geq \mathrm{IP}(x)$ and $y \geq \mathrm{IP}(y)$, i.e. the entire screen northeast of the point specified. Input of [0; 0; …] will erase the whole screen.

POINT sets a square of 3×3 pixels around the coordinates $\boldsymbol{x} = \mathrm{IP}(x)$ and $\boldsymbol{y} = \mathrm{IP}(y)$. This is better visible than just a single pixel.

AGRAPH puts a vertical pattern specified by a 64-bit *short integer* in the source ***s*** to the *LCD*. The point with coordinates $\boldsymbol{x} = \mathrm{IP}(x)$ and $\boldsymbol{y} = \mathrm{IP}(y)$ is the bottom pixel of this column (valid inputs are $0 \leq \boldsymbol{x} < 399$ and $0 \leq \boldsymbol{y} < 175$). AGRAPH increments ***x*** automatically by 1 which is handy (see an instance below). So one 64 px high graphic row starting in column 0 may require 400 such *integers* – the more blank space is found in this row the less *integers* are needed for specifying it.

Since most keyboard commands refresh the screen immediately, graphics should be generated by routines. PIXEL, CLLCD, and POINT are self-explanatory. AGRAPH may need a little demonstration:

Example:

This little program draws an open square with its bottom left corner at (150; 100). It may be provided on your *WP43* already. Press

GTO PROG PICTURE
P/R and read (or key in):

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```
LBL 'PICTURE'
   REM 'initializing'
   FF FF FF FF FF FF FF FF16
   STO 00
   C0 00 00 00 00 00 00 03 16
   STO 01
   62
   STO 02
   100
   ENTER↑
   150
   CLLCD
   INC X
   AGRAPH 00
   AGRAPH 00
LBL 01
   REM 'drawing loop'
   AGRAPH 01
   DSE 02
   GTO 01
   REM 'finalizing'
   AGRAPH 00
   AGRAPH 00
   RTN
```

set up two pattern registers for vertical …

… and horizontal framing

initialize a loop counter

erase the screen NE of (150; 100)

draw left frame
… 2 pixels wide

loop: draw 1 point of top and bottom frame
loop: decrement the loop counter
loop: if > 0 then run this loop again

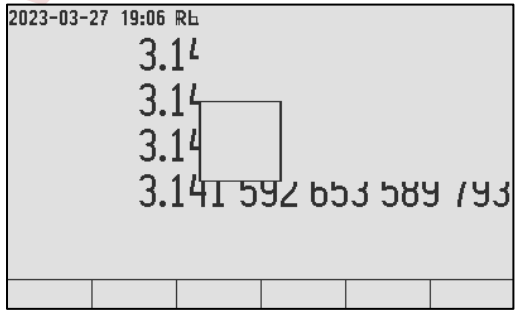
else draw right frame like above

Now do this:

P/R leave *PEM*

π FILL fill the screen

XEQ PROG PICTURE
and you will get a picture like this:



Note the *status bar* may overwrite the top 20 pixels; calling *menus* or pressing EXIT will overwrite the *menu section* – the whole graphic, however, will disappear without any trace when you call a function like ENTER↑, updating the *stack*. Remember to add SNAP to your routine if you want a more permanent output of your graphic here (cf. p. 252).

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Another **example** (you may find it preinstalled on your *WP43* as well):

GTO **PROG** SPIRAL , P/R and read (or key in):

```
LBL 'SPIRAL'
   RCL I
   x=0.?          is there a nonzero value i provided?
   1.             default for i
   STO I
   RCL J
   x=0.?          is there a nonzero value j provided?
   0.3            default for j
   |x|            ensure positive j
   STO J
   RAD
   LocR 04        allocate 4 local registers to this routine
   1
   STO .00        initialize the counter r.00
   0
   ENTER↑
   CLLCD          clear the entire LCD (i.e. everything 'northeast' of (0; 0))
   REM 'start of drawing loop'
LBL 01
   RCL .00
   500.
   /              start with 0.002 in first run
   ENTER↑
   ENTER↑         [r.00/500,  r.00/500,  r.00/500,  …]
   sin
   x⇄y
   RCL× J         [j×r.00/500,  sin(r.00/500),  r.00/500,  …]
   eˣ
   ×              [e^(j×r.00/500)×sin(r.00/500),  r.00/500,  …]
   RCL× J
   STO .01        [j×e^(j×r.00/500)×sin(r.00/500),  r.00/500,  …]
   x⇄y
   ENTER↑         [r.00/500,  r.00/500,  j×e^(…))×sin(…),  …]
   cos
   x⇄y
   RCL× J         [j×r.00/500,  cos(r.00/500),  j×e^(…))×sin(…),  …]
   eˣ
   ×              [e^(j×r.00/500)×cos(r.00/500),  j×e^(…))×sin(…),  …]
   RCL× I
   STO .02        [i×e^(j×r.00/500)×cos(r.00/500),  j×e^(…))×sin(…),  …]
```

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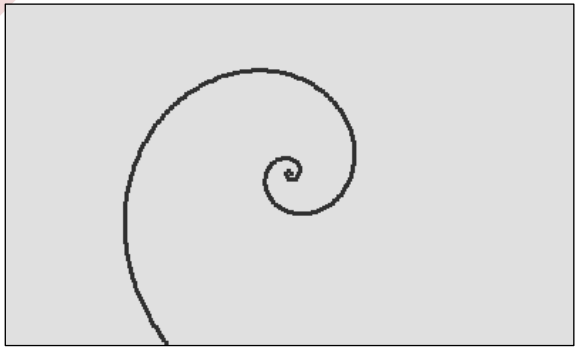
```
   →POL
   STO .03        store the radius
   234            limit value
   x≤? Y          if radius y is beyond the limit
   RTN             then return
   RCL .01           else continue with y
   120
   +              add the center coordinate for y
   ROUNDI         for an integer number of pixels
   RCL .02        recall x
   200
   +              add the center coordinate for x
   ROUNDI         for an integer number of pixels
   SF 'IGN1ER'    ignore 1 error
   POINT           while drawing the next 3x3 px. point at (x; y)
   CF 'IGN1ER'    watch errors thereafter again
   PAUSE 00       update display
   RCL .03        recall the radius
   1/x
   SDL 03         multiply times 1000
   IP
   INC X
   STO+ .00       add this to r.00
   GTO 01         run this drawing loop again until above limit is reached
END
```

Now press [P/R] to leave *PEM*, then enter

[XEQ] [PROG] SPIRAL and wait (for ~ 60 s with power supplied by the battery only). You will get a picture like this:



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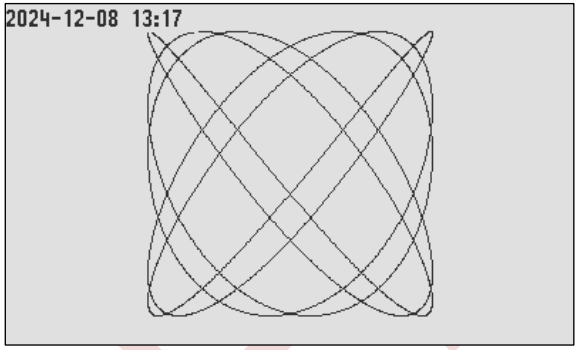
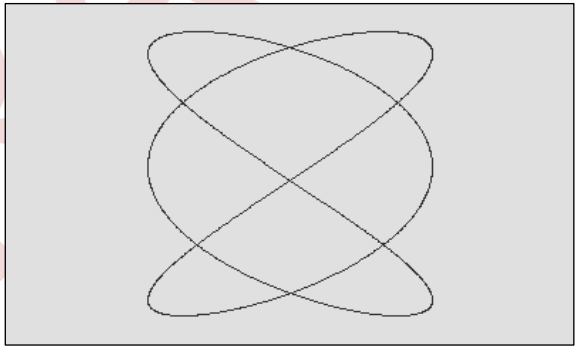
Another graphic program **example** provided is LISSAJ. It is fed by the *registers* **I**, **J**, and **K**.

With ***i*** = 5, ***j*** = 6, and ***k*** = 3 you will get this:



Date and time will be displayed while drawing since LISSAJ will take longer than 1 *minute* on your *WP43*. You can accelerate it by setting FASTFN.

With ***i*** = 3, ***j*** = 2, and ***k*** = 1 you will get this picture instead:



Check the code (53 steps taking 121 *bytes* in total) for the reasons. Note you can as well enter non-integer values for ***i***, ***j***, or ***k***.

You will find more programs in `testPgms.bin`. Feel free to try them.

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APPENDIX 6: SOUND

Beyond BEEP and TONE which are known from *HP-42S*, *WP43* features six new commands operating on the buzzer built in:

BUZZ	allows for playing a tone of defined frequency (in *hertz*) and duration (in *milliseconds*);
PLAY	plays a sequence of buzzer tones stored in a 2- or 3-column matrix of arbitrary size, each tone specified by its frequency and duration and (optionally) its volume;
VOL	sets the buzzer to a defined volume (1 … 11), while
VOL+	increments the buzzer volume by one step and
VOL−	decrements it;
VOL?	returns the buzzer volume currently set.

All buzzer commands work on the calculator hardware only. Please see the *IOI* for more information about them and their parameters.

Enjoy playing!

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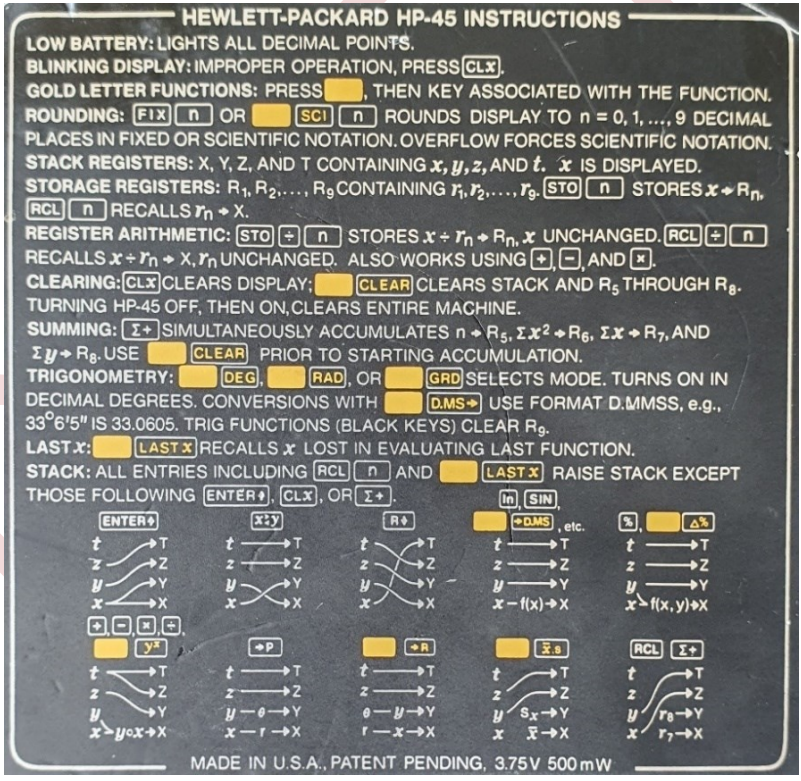
APPENDIX 7: POWER SUPPLY

Your *WP43* is powered by a single *CR2032* coin cell (3 V). It is not rechargeable. Insert it with + facing up (away from the board). With a good battery installed, your *WP43* may be also powered through its *USB* port – running at even higher speed then. When the battery becomes low, replace it with a new quality cell (cf. pp. 16f and see *ReMA A*).

WARNING: Removing the battery for longer than some *seconds* may erase all your data and programs in *RAM* – thus, SAVE them before!

See here what sufficed for explaining the basic functionality of *HP's* first 'advanced scientific' calculator, the *HP-45* of 1973, on its back:



Though it featured only 59 built-in functions, nine (volatile!) storage registers including four statistical sums, x̄ and s – neither *menus*, *catalogs*, browsers, *data types*, *variables*, regressions, diagrams, advanced operations, applications, programming, nor keyboard customizing – but your *WP43* provides them all and much more.

APPENDIX 8: TIME LINE OF QUOTED MANUALS

HP-35 OM ……....	1972
HP-45 OH	1973
HP-55 OH, HP-21 OH, HP-25 OH	1975
HP-27 OH, HP-67 OHPG	1976
HP-97 OHPG, HP-32 OH, HP-33 OH …….....	1978
HP-34C OHPG	1979
HP-41C/41CV OHPG, Solving Problems With Your HP Calculator	1980
HP-16C Computer Scientist OH, HP-15C Advanced Functions H.	1982
HP-15C OH	1985
HP-22S Science Student Applications, HP-27S OM, HP-42S OM, HP-42S Programming Examples and Techniques	1988
HP-21S OM	1990
HP-15C OH and *Advanced Functions Handbook* reprints	2011, 2023

See f. 18 on p. 23 for where you can get these manuals.

For comparison:

In 1948, a mechanical four-function pocket (!) calculator (the *Curta*) sold for 125 US$, equivalent to 1637 US$ of today.

In 1976, an electronic pocket calculator featuring *Continuous Memory* was a breathtaking innovation (you could use it as a notebook, too!); and a grand total of 72 built-in functions, 8 *GP registers*, and 49 merged program steps sufficed for professional engineers and scientists

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doing their work – and of course they did for students striving for their Ph.D. But note you had to program linear regressions, correlations, and forecasting manually if and when you needed them, e.g., for calibrating – the respective routine suggested by *HP* took 44 precious steps (i.e. almost all). Also note that 200 US$ of 1976 correspond to an investment of 1124 US$ of today.

APPENDIX 9: RELEASE NOTES

	Date	Release notes
0	29.11.12	**Official project start** with first publication of the *43S* concept and layout on one of the forums of the *MoHPC* (*https://www.hpmuseum.org/cgi-sys/cgiwrap/hpmuseum/archv021.cgi?read=234685#234685*). Though internal discussions started in September 2011, and there are even found far older traces of a '*43S*' denoting a '*Super HP-42S*', though in various more or less fictional cases – pure *vapourware*™.
0.1	2.2.14 23.5.15	Manual set up based on the one of *WP 34S*. Passed to *Jake Schwartz*, *Eric Smith*, and *Richard Ottosen* for first information. See the vivid discussions starting in February 2014 in this thread *https://www.hpmuseum.org/forum/thread-593.html* (it was stopped at 10.1.16 but still is among the three threads most viewed on that forum after over nine years).
0.2	3.10.15	Update based on *Jake's* feedback and further thoughts, distributed to *Eric*, *Jake*, *Marcus von Cube*, and *Paul Dale.*
0.3	21.3.16	Split the manual in three; moved LBL onto the keyboard, renamed STOM to STOCFG, RCLM to RCLCFG, SERR to s_m, and SERR$_w$ to s_{mw}; refined the *Key Response Table*. Passed to *Michael Steinmann* for information.
0.4 … 0.28	1.5.25	See the full *Release Notes* in the *ReM*. ***WP43* pilot units produced by *SwissMicros* were distributed in October 2022.**
0.28.1	8.5.25	Refined the *IOI* to contain (almost) all *items*. Added another example. Refined and corrected.
0.28.2	14.7.25	Split LoadPE into LoadPr and LoadEq. Extended LoadR and the *OM* and *ReM* indices. Changed RTN+1 to RTN.SKP. Deleted LOADST, refined LOAD… and I/O. Put CONST in CATALOG. Added Indian output format, generic EDIT, auxiliary constants, keystroke numbers, *App. 4* about PLOT, two chapters to *App. H*, two to *App. I*, one to *App. J*, *App. K* about the pilots, and *App. N*. Improved the explanations of CPX?, OFF, SAVEST, and *App. C*. Refined and corrected.
0.28.3	22.8.25	Added EDIT and ! to the bezel, moved USB and DMCP. Elaborated about angles. Refined and corrected.

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INDEX

The index following lists names, special terms and keywords used in this manual. Furthermore, it points to the most prominent of the over 200 examples included, marked '(ex.)'. You find all the individual *items* provided on your *WP43* explained in *ReMS 1*; since this will also point you to further explanations if applicable, particular *items* are listed below only if they are extensively covered in this *OM*. The *Table of Contents* is recommended as well – chapter headings are not repeated in the index.

Of *living persons*, just their prenames are listed below. *Historic and fictional characters* (e.g., from *HP's examples*) are quoted with their full names. The original names of the ancient Greek philosophers and scientists mentioned in this *OM* can be found in f. 216 on p. 205. ((page numbers will be updated before v1.0))

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